Optimal Financial Portfolio Design - readme

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1 Problem description

On OPD problem is defined by a triplet (v, b, r). The task is to find a matrix $P \in \{0,1\}^{v \times b}$ such that:

- $max_{i \neq j}(P_{i*} \cdot P_{j*}) = \lambda$
- $\forall i \sum P_{i*} = r$

while minimising the value of λ.

Or in other terms, we are looking for v subsets of cardinality r from a given set of cardinality b such that the cardinality of the largest interesection of differents subsets (denoted as λ) is minimised.

The task was first described by Flener [1].

2 Model design

The model is quite straightforward given the definition of OPD problem. There are 3 variables:

- `portfolios` - a binary matrix (array with 2 indices) equivalent to P
- `portfolios_sets` - array of sets, a subset representation of the problem
- `lambda` - λ as defined in the definition of OPD

First conditions are also quite straightforward. There are 2 conditions to ensure that the sum of rows of P is r and the subsets have cardinality r. Then there are 2 conditions that λ is the maximum dot product between lines and intersection between 2 subsets as per definition of the problem.

Then there are some conditions, that are more complex. First, the bounds on the λ variable. The lower bound is calculated based on a Theorem 1 from a paper by Flener [2]. This result says, that

$$\lambda \geq \frac{\left\lceil \frac{rv}{b} \right\rceil^2 mod(rv, b) + \left\lfloor \frac{rv}{b} \right\rfloor^2 (b - mod(rv, b)) - rv}{v(v-1)}.$$

Following that there is a symmetry breaking condition, that says, that the matrix P must have both rows and columns in lexicographic order which is achieved by using constraint `lex2`. This can be done thanks to the Theorem 2 from paper by Kiltzian [3]. This result says, that if a 2-d matrix model with symmetries in both columns and rows has a solution then it has a solution that:

- has rows ordered by their sums,
- has columns ordered lexicographically,
- and has rows with same sums ordered lexicographically.

Last there is a channeling constraint that joins the set and matrix representations of the problem. This may help the solver by allowing it to use 2 types of conditions. This is done using predicate `link_set_to_booleans` which links a `set of int` and an `array of bool` in such way, that value with index i in the array is `true` in and only iff the set contains i.

3 Results

To understand how each component of the model influences efficiency of the solver several experiments were run. This is called an "ablation study".

The inputs used were the two smallest ones (`small_bibd_06_50_25.dzn` and `small_bibd_06_60_30.dzn`) as bigger inputs were too computationally expensive. Also the search was automatically stopped after 3 hours, this arbitrary limit spans from the limited amount of time I had for running these experiments and lack of computational resources. The experiments were run on a VPS where nothing else was running so the OS scheduler could not really interfere with the results.

Based on the data collected (see Table 1), having better bounds on λ makes the solver roughly twice faster. Compared to that, symmetry breaking and having a set-based model seem to be really important for the efficiency as the solver was not able to solve the problem within the allocated timeframe.

	`small_bibd_06_50_25.dzn`	`small_bibd_06_60_30.dzn`
base	357.397	3173.41
only matrix model	N/A	N/A
no symmetry breaking	N/A	N/A
worse bound on λ ($\lambda \geq 0$)	675.644	6235.26

Table 1: Tabulated results of the "ablation study" - time to solve in seconds, N/A means no solution found before timelimit

References

[1] Flener, P., Pearson, J., and Reyna, L. G. Financial portfolio optimisation. In *Principles and Practice of Constraint Programming – CP 2004* (Berlin, Heidelberg, 2004), M. Wallace, Ed., Springer Berlin Heidelberg, pp. 227–241.

[2] Flener, P., Pearson, J., Reyna, L. G., and Sivertsson, O. Design of financial cdo squared transactions using constraint programming. *Constraints 12*, 2 (jun 2007), 179–205.

[3] Kiziltan, Z., and Smith, B. M. Symmetry-breaking constraints for matrix models. In *In Proc. of SymCon'02, the CP'02 Workshop on Symmetry in Constraints* (2002).