Optimal Financial Portfolio Design - readme

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1 Problem description

On OPD problem is defined by a triplet (v, b, r). The task is to find a matrix $P \in \{0,1\}^{v \times b}$ such that:

- $max_{i \neq j}(P_{i*} \cdot P_{j*}) = \lambda$
- $\forall i \sum P_{i*} = r$

while minimising the value of λ. Or in other terms, we are looking for v subsets of cardinality r from a given set of cardinality b such that the cardinality of the largest interesection of differents subsets (denoted as λ) is minimised.

The task was first described by Flener [1].

2 Model design

The model is quite straightforward given the definition of OPD problem. There are 3 variables:

- `portfolios` - a binary matrix (array with 2 indices) equivalent to P
- `portfolios_sets` - array of sets, a subset representation of the problem
- `lambda` - λ as defined in the definition of OPD

First conditions are also quite straightforward. There are 2 conditions to ensure that the sum of rows of P is r and the subsets have cardinality r. Then there are 2 conditions that λ is the maximum dot product between lines and intersection between 2 subsets as per definition of the problem.

Then there are some conditions, that are more complex. First, the bounds on the λ variable. The lower bound is calculated based on a Theorem 1 from a paper by Flener [2].

Following that there is a symmetry breaking condition, that says, that the matrix P must have both rows and columns in lexicographic order. This can be done thanks

	small_bibd_06_50_25.dzn	small_bibd_06_60_30.dzn
base	357.397	3173.41
only matrix model	N/A	N/A
no symmetry breaking	N/A	N/A
worse bound on λ	675.644	6235.26

Table 1: Tabulated results of the ”ablation study” - time to solve in seconds, N/A means no solution found before timelimit

to the Theorem 2 from paper by Kiltzian [3] which applies because the row-sums are constant.

Last there is a channeling constraint that joins the set and matrix representations of the problem. This may help the solver by allowing it to use 2 types of conditions.

3 Results

To understand how each component of the model influences efficiency of the solver several experiments were run. This is called an ”ablation study”.

The inputs used were the two smallest ones (`small_bibd_06_50_25.dzn` and `small_bibd_06_60_30.dzn`) as bigger inputs were too computationally expensive. Also the search was automatically stopped after 3 hours, this arbitrary limit spans from the limited amount of time I had for running these experiments and lack of computational resources. The experiments were run on a VPS where nothing else was running so the OS scheduler could not really interfere with the results.

References

[1] Flener, P., Pearson, J., and Reyna, L. G. Financial portfolio optimisation. In *Principles and Practice of Constraint Programming – CP 2004* (Berlin, Heidelberg, 2004), M. Wallace, Ed., Springer Berlin Heidelberg, pp. 227–241.

[2] Flener, P., Pearson, J., Reyna, L. G., and Sivertsson, O. Design of financial cdo squared transactions using constraint programming. *Constraints 12*, 2 (jun 2007), 179–205.

[3] Kiziltan, Z., and Smith, B. M. Symmetry-breaking constraints for matrix models. In *In Proc. of SymCon'02, the CP'02 Workshop on Symmetry in Constraints* (2002).