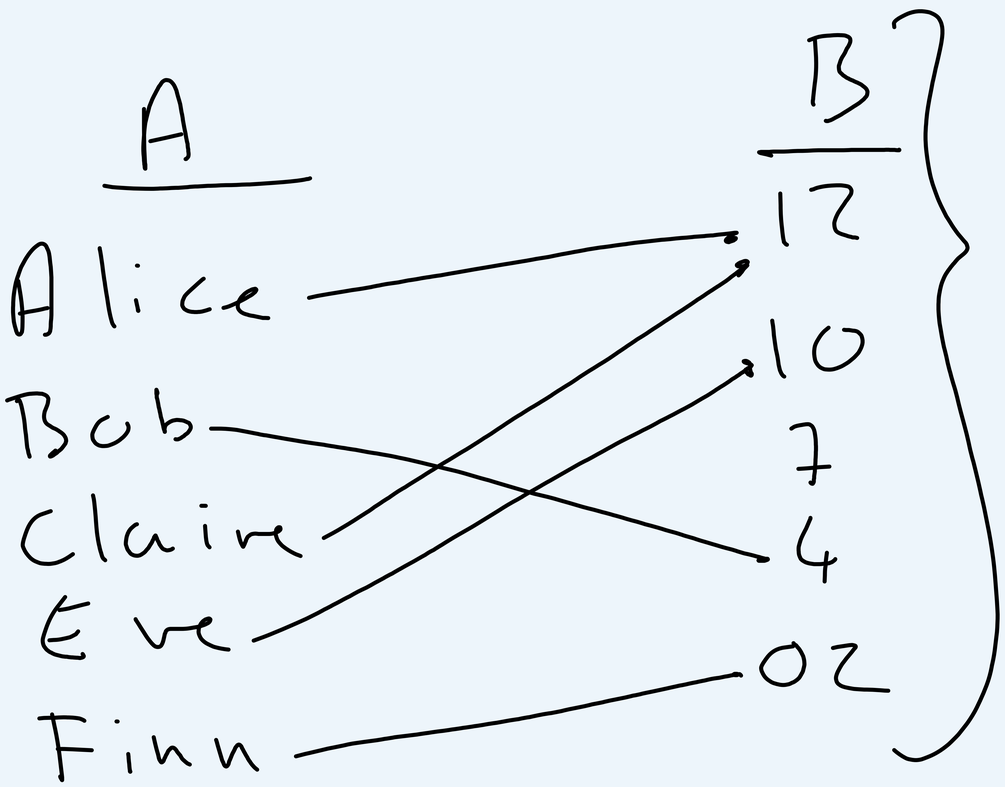
Functions:



Functions:

Let A and B be sets, a function assigns exactly one element of B to each element of A.

$f: A \longrightarrow B$

$f(a) = b$, $f(x) = y$

functions = Mapping = transformation

often:

$$f(x) = 2x + 7$$

$$f: A \to B$$

Domain — Codomain

If $f(a) = b$, b = image of a, and a is called the pre-image

Set of all images is called the range

ex:

$f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$

Domain = $\mathbb{Z}$, Codomain = $\mathbb{Z}$, Range = $\{0, 1, 4, 9, 16, \ldots\}$

One-to-one function = injective function:

f is injective if it maps every element of A to a unique element in b:

- No element in B is mapped to by two or more elements of A.

X^2

Onto Functions (surjective)

f is surjective if every element of B is mapped to by some element of A.

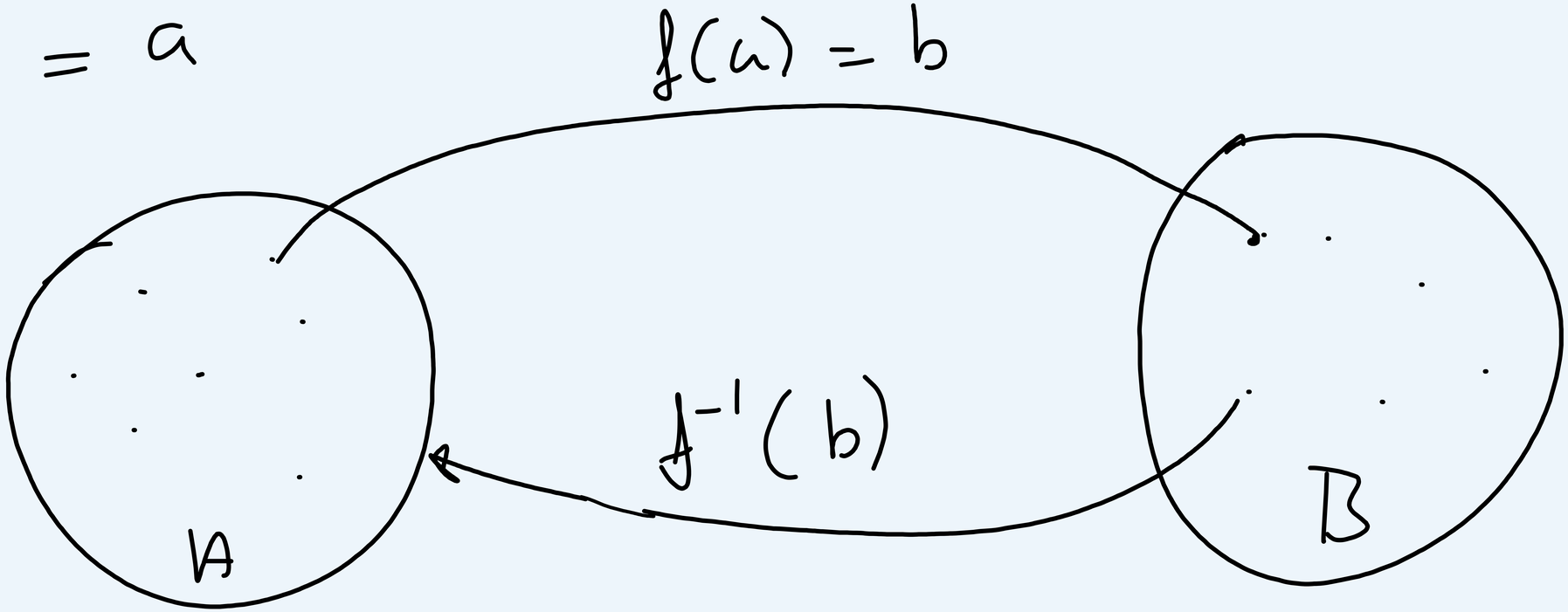
- No element of B is left out.
- Codomain = Range.

Injective + Surjective = Bijective:

Bijective functions have inverses!

$f(a) = b$

$f^{-1}(b) = a$



$f(x) = x+1 \Longrightarrow y = x+1 \Longleftrightarrow x = y-1$

$f^{-1}(y) = y-1$

x^2

Composite:

$$f(x) = 2x + 7$$

$$g(x) = x^2$$

$$(f \circ g)(x) = f(g(x)) = 2x^2 + 7$$

$$(g \circ f)(x) = (2x + 7)^2$$

$$f \cdot f^{-1} = x$$

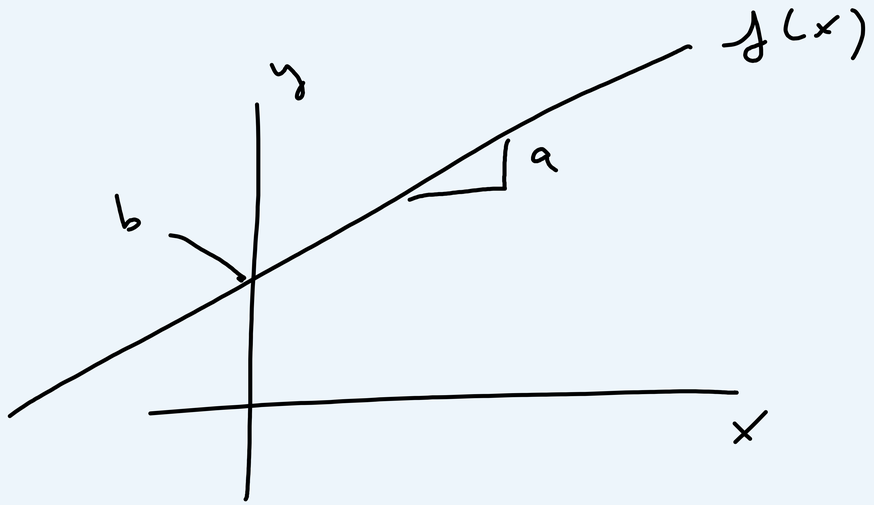
Linear Functions:

$$f(x) = ax + b$$

$$f(x) = b, \quad x = 0$$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

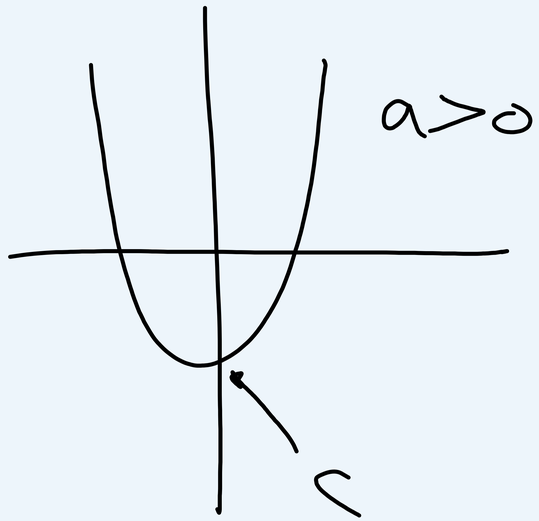
$$y_2 = y_1 + a(x_2 - x_1)$$

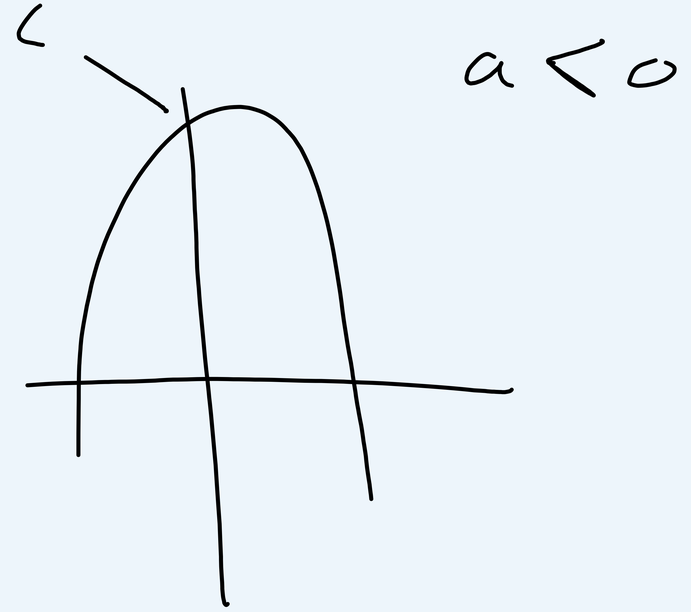


Quadratic:

$$ax^2 + bx + c = y$$

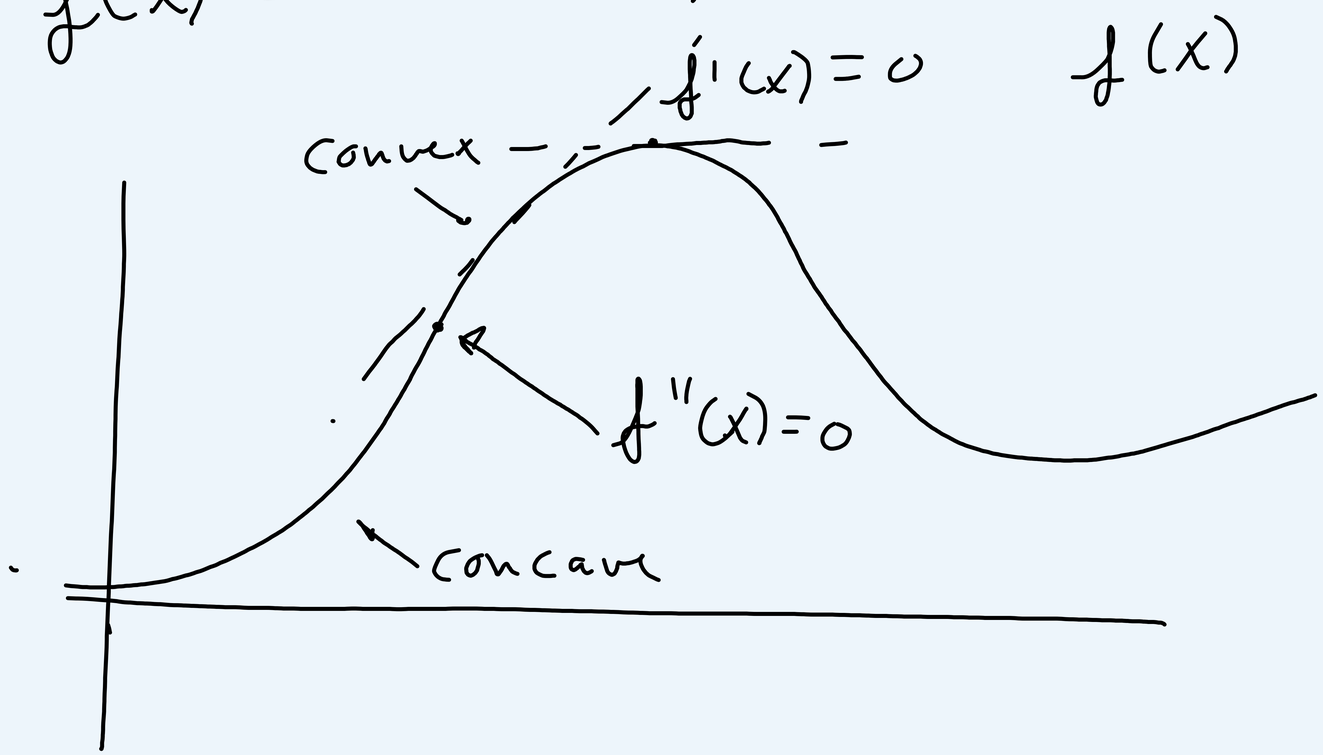
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





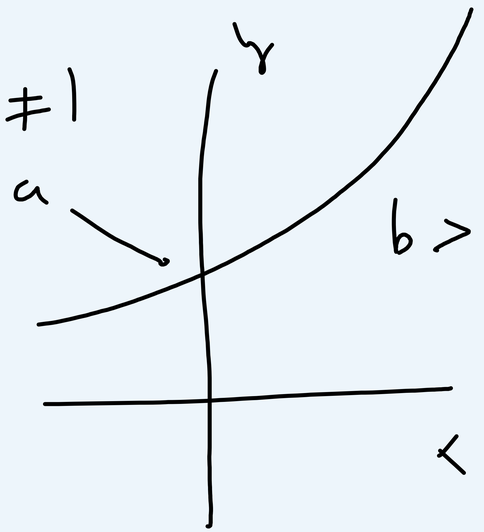
Power Functions

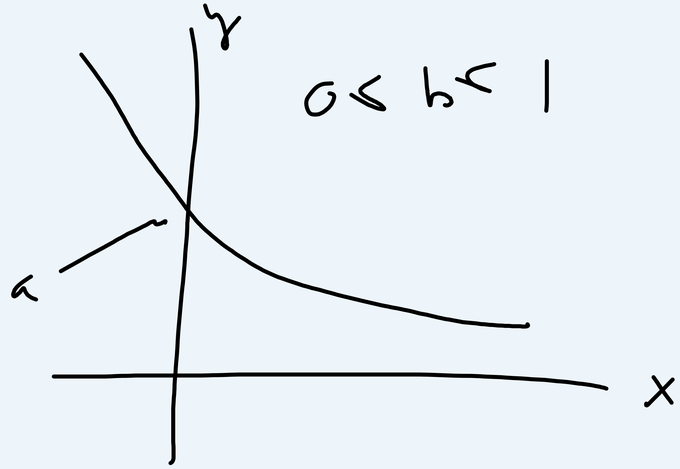
$f(x) = a x^n$, $a, n \neq 0$



Exponential functions:

$$f(x) = a\,b^x \quad , a \neq 0, b > 0, b \neq 1$$





Special bases:

$$y = a \cdot 10^x$$

$$y = a \cdot 2^x$$

$$y = a \cdot e^x$$

Logarithms:

$3 = 10^{0.4771}$

$5 = 10^{0.6989}$

$15 = 10^{1.1760}$

The exponents are "base-10 logarithms of 3, 5, 15.

$\log_{10} 3 = 0.4771$

$\log_{10} 5 = 0.6989$

$\log_{10} 15 = 1.1760$

$a^n \cdot a^m = a^{n+m}$

$$3 \cdot 5 = 10^{0.4771} \cdot 10^{0.6989} = 10^{0.4771+0.6989} = 10^{1.1760}$$

$$\log(3 \cdot 5) = \log 3 + \log 5$$

$$\frac{a^n}{a^m} = a^{n-m}$$

ex:

$$\log \frac{15}{3} = \log 15 - \log 3$$

ex:

$$\log(3^4) = \log(3 \cdot 3 \cdot 3 \cdot 3) = \log 3 + \log 3 + \log 3 + \log 3$$
$$= 4 \cdot \log 3$$

If $x = 10^y$, y is the base-10 logarithm of x

$x = 2^y$, —— 11 —— - 2 —— 11 ——

ex:

$3 = \log_2 8 \Longrightarrow 2^{3} = 8$

$4 = \log_3 81 \Longrightarrow 3^{4} = 81$

$2 = \log_{10} 100 \Longrightarrow 10^{2} = 100$

$$\log_b x = y \Longrightarrow b^{y} = x$$

$\log_e x = \ln x$

$\log_e x = y \Longrightarrow e^y = x$

$\ln x = y \longrightarrow e^y = x$

$f(x) = e^x$

$g(x) = \ln x$

$f \circ g(x) = e^{\ln x} = x$

ex:

$7^{3x} = 983 \Longrightarrow \log 7^{3x} = \log 983$

$3x \cdot \log 7 = \log 983 \Rightarrow 3x = \dfrac{\log 983}{\log 7} =$

$x = \dfrac{\log 983}{3 \cdot \log 7} = \underline{\underline{1.1803}}$

$b^{\log_b \cdots}$

ex:

$$\log_2(x-1) + \log_2(x-3) = 3$$

$$\log_2\left((x-1)\cdot(x-3)\right) = 3$$

$$2^{\log_2((x-1)\cdot(x-3))} = 2^3$$

$$(x-1)(x-3) = 2^3$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4\cdot 1\cdot(-5)}}{2}$$

$$= \begin{cases} 5 \\ -1 \end{cases}$$

$$\log_2(5-1) + \log_2(5-3) \quad \checkmark$$

$$\log_2(-1-1) + \log_2(-1-3)$$