Response to reviewers

March 29, 2022

I would like to thank all the reviewers for their help and comments and for the time they took to review the manuscript. Their suggestions were extremely helpful and we believe this has resulted in an improved manuscript. Below is a point by point response and summary of changes made.

- **Reviewer 1 - Comment 2:** *Combine Sections 2 and 5 together to demonstrate the underlying queueing models and the EMS - ED interface counterpart. The current arrangement is not easy to follow for finding the right explanations for each part of the assumptions adopted in the article.*

- **Reviewer 3 - Comment 1:** *Sections 1 and 2 are difficult to read. It would be better to focus on the main contribution and its motivation in Section 1, and to put the literature review in a separate Section.*

The paper has been completely restructured. Sections 2 and 5 have been combined and the game theoretic model is now presented along with the results in one section. In addition, section 1 contains only the introduction while the literature review has moved to section 2. Finally, the queueing model section now comes before the game theoretic section. The current structure is: Introduction; Literature Review; A queueing model for the ED-EMS interface; Strategic manipulation of the ED-EMS interface; Results.

- **Reviewer 3 - Comment 2:** *Since the application in mind is the EMS-ED interface, it would be better to write the paper immediately from this perspective, instead of first abstracting it and then applying it to that problem.*

- **Reviewer 3 - Comment 8:** *Figs 7-11: Row player and Column Player are unclear: hospital 1 and hospital 2 would be a better choice I presume.*

The paper is presented with the application of the model as the focus. All sections are now presented from the perspective of the EMS-ED interface. In the game theoretic section instead of row/column player we now use hospital A and hospital B.

- Reviewer 1 - Comment 4: *To highlight the contributions of this work, it is necessary to compare the novelty of this work with previous results in the literature.*

In the new literature review section we now directly compare the results from previous work with the results from this work to highlight the contributions of this work.

- Reviewer 3 - Comment 6: *Figure 3: about solving the Markov chain: the Markov chain is truncated to solve it. Therefore, probably N and M are not too small. What about the numerical complexity? Because if you just solve it, you have to invert the matrix? Is there some structure in the Markov chain that can be exploited if N and M would be large?*

- Reviewer 3 - Comment 7: *Figures 4-6: why do you compare your calculated values with simulation? Is it to evaluate the truncations? Or are you not sure about the formulas?*

This observation was really helpful and lead to us conducting more numerical experiments to clarify the point of truncating the state space. As the values of N and M increase, the truncated Markov chain becomes more complex to solve. We have touched upon building a closed-form formula for the steady state probabilities but that is still a work in progress and not something we are able to include in this manuscript. Figures 4-6 are there to demonstrate the accuracy of our constructed model against its discrete event simulation equivalent and these have been expanded on. An additional paragraph is added that talks about truncating the model and the old figures have been replaced with ones that compare the results from the truncated Markov model with the results from the untruncated simulation model. We feel that these extra numeric results do clarify the paper and are thankful for the reviewer comments.

- Reviewer 3 - Comment 3: *In section 2, a diagram of the full system (including the two queues and the EMS) would increase readability.*

A full system diagram is now added in the game theoretic model section.

- Reviewer 3 - Comment 9: *Page 22 scenario at bottom of the page: this scenario does not seem like a realistic scenario: the mean number of patients offered per time unit is larger than that can be treated, meaning that many patients would be rejected by the hospital. Please comment on the reality of the selected scenarios*

The queueing systems in the particular example are meant to be extremely busy systems. In the new manuscript it is more clearly stated.

- Reviewer 3 - Comment 5: *A key modelling assumption is that a fraction p_A of patients is distributed to hospital A. Some comments on how realistic this is should be added. Do distributers have a choice? Don't they just have to choose the nearest hospital? Do they really do it randomly, don't they take into account queues at the hospitals (e.g. join the shortest ambulance queue)? what about urgency of patients? I would expect patients that come in are prioritized since they are probably more urgent.*

An additional paragraph has been added that touches upon the possibility of more complex queueing models where the hospital distance or patient's priority type are considered.

- Reviewer 3 - Comment 4: *The authors claim their model has applications in a range of sectors and settings, but have given no examples.*

Examples of other applications have been added in the conclusion section of the manuscript.

- Reviewer 1 - Comment 3: *In the game-theoretic formulation and analysis part, it is better to present the essential components of a normal-form representation, including the players in the game, the strategies available to each player and the payoff received by each player for each combination of strategies that could be chosen by the players, etc.*

In the new manuscript the normal form game is explicitly defined in the game theoretic section. The players, strategies and payoffs are clearly defined.

- Reviewer 1 - Comment 1: *Rewrite the Highlights (for review) of the paper, and really capture the sense of work.*

Highlights have been rewritten to be more explicit in terms of the work that was done.

- Reviewer 1 - Comment 5: *There are some mistakes/typos with English grammar and writing in the article.*

We have thoroughly checked the grammar and spelling of the manuscript.