In[1]:=
```
ClearAll[Evaluate[Context[]<>"*"]];
ClearSystemCache[];
```

## PT kernels & check

In[3]:=
$$\mathcal{I}[n1\_, n2\_] := \frac{\text{Gamma}\left[\frac{3}{2} - n1\right] \text{Gamma}\left[\frac{3}{2} - n2\right] \text{Gamma}\left[-\frac{3}{2} + n1 + n2\right]}{8\,\pi^{3/2}\,\text{Gamma}[n1]\,\text{Gamma}[3 - n1 - n2]\,\text{Gamma}[n2]};$$

In[4]:=
$$J2[k\_, \nu1\_, \nu2\_] := \frac{1}{(2\pi)^3}\,\frac{\text{Gamma}[3/2 - \nu1]\,\text{Gamma}[3/2 - \nu2]\,\text{Gamma}[\nu1 + \nu2 - 3/2]}{\text{Gamma}[\nu1]\,\text{Gamma}[\nu2]\,\text{Gamma}[3 - \nu1 - \nu2]}\,\pi^{3/2}\,k^{3-2\nu1-2\nu2};$$

$$A1[\nu1\_, \nu2\_] := \frac{1}{2}\,(J2[1, \nu1 - 1, \nu2] - J2[1, \nu1, \nu2 - 1] + J2[1, \nu1, \nu2]);$$

$$A2[\nu1\_, \nu2\_] := \frac{1}{8}\,(-J2[1, -2 + \nu1, \nu2] + 2\,J2[1, -1 + \nu1, -1 + \nu2] + 2\,J2[1, -1 + \nu1, \nu2] - J2[1, \nu1, -2 + \nu2] + 2\,J2[1, \nu1, -1 + \nu2] - J2[1, \nu1, \nu2]);$$

$$B2[\nu1\_, \nu2\_] := \frac{1}{8} \times (3\,J2[1, -2 + \nu1, \nu2] - 6\,J2[1, -1 + \nu1, -1 + \nu2] + 2\,J2[1, -1 + \nu1, \nu2] + 3\,J2[1, \nu1, -2 + \nu2] - 6\,J2[1, \nu1, -1 + \nu2] + 3\,J2[1, \nu1, \nu2]);$$

In[8]:=
```
perm12 = Permutations[{q1, q2}];
perm123 = Permutations[{q1, q2, q3}];
```

In[10]:=
```
Clear[Fn, Gn, n, m];
Fn[n_, v_] =
  If[n == 1, 1, Sum[Gn[m, v[[1 ;; m]]]/((2 n + 3) (n - 1)) ((2 n + 1) al[Total[v[[1 ;; m]]], Total[v[[m + 1 ;; n]]]] ×
      Fn[n - m, v[[m + 1 ;; n]]] + 2 be[Total[v[[1 ;; m]]], Total[v[[m + 1 ;; n]]]] ×
      Gn[n - m, v[[m + 1 ;; n]]]), {m, 1, n - 1}]];
Gn[n_, v_] = If[n == 1, 1, Sum[Gn[m, v[[1 ;; m]]]/((2 n + 3) (n - 1))
    (3 al[Total[v[[1 ;; m]]], Total[v[[m + 1 ;; n]]]] × Fn[n - m, v[[m + 1 ;; n]]] +
     2 n be[Total[v[[1 ;; m]]], Total[v[[m + 1 ;; n]]]] ×
      Gn[n - m, v[[m + 1 ;; n]]]), {m, 1, n - 1}]];
```

In[13]:=
```
F2s[q1_, q2_] = Simplify[Sum[
    1 / Length[perm12] Fn[2, {perm12[[i, 1]], perm12[[i, 2]]}], {i, 1, Length[perm12]}]];
F3s[q1_, q2_, q3_] = Simplify[Sum[1 / Length[perm123]
     Fn[3, {perm123[[i, 1]], perm123[[i, 2]], perm123[[i, 3]]}], {i, 1, Length[perm123]}]];
```

In[15]:=
```
G2s[q1_, q2_] = Simplify[Sum[
    1 / Length[perm12] Gn[2, {perm12[[i, 1]], perm12[[i, 2]]}], {i, 1, Length[perm12]}]];
G3s[q1_, q2_, q3_] = Simplify[Sum[1 / Length[perm123]
     Gn[3, {perm123[[i, 1]], perm123[[i, 2]], perm123[[i, 3]]}], {i, 1, Length[perm123]}]];
```

In[17]:=
```
alf[k1_, k2_] = (k1 + k2).k1 / (k1.k1);
bef[k1_, k2_] = (k1 + k2).(k1 + k2) (k1.k2) / 2 / (k1.k1) / (k2.k2) ;
alfm[k1_, k2_] = 1 + 1/2 (mag[k1 + k2]^2 - mag[k1]^2 - mag[k2]^2) / mag[k1]^2;
befm[k1_, k2_] = mag[k1 + k2]^2 (mag[k1 + k2]^2 - mag[k1]^2 - mag[k2]^2) / (4 mag[k1]^2 mag[k2]^2);
sigma2[k1_, k2_] = 1/4 (mag[k1 + k2]^2 - mag[k1]^2 - mag[k2]^2)^2 / (mag[k1]^2 mag[k2]^2) - 1;
```

## NG shape

In[22]:=
```
Northo = (840 π^2 - 7363 - 189 × (20 π^2 - 193)) / (29 114 - 2940 π^2);
p = 27 / (743 / (7 × (20 π^2 - 193)) - 21);
kT = mag[k1] + mag[k2] + mag[k3];
e2 = mag[k1] × mag[k2] + mag[k2] × mag[k3] + mag[k3] × mag[k1];
e3 = mag[k1] × mag[k2] × mag[k3];
Δ = (kT - 2 mag[k1]) (kT - 2 mag[k2]) (kT - 2 mag[k3]);
Γ = 2/3 e2 - 1/3 (mag[k1]^2 + mag[k2]^2 + mag[k3]^2);
```

In[29]:=
```
S[k1_, k2_, k3_] = Simplify[1/Northo ((1 + p) Δ/e3 - p Γ^3/e3^2)]
```

Out[29]=
$$\Big(-\big(\big(-7363+840\,\pi^2\big)\,\text{mag}[k1]\times\text{mag}[k2]\times(\text{mag}[k1]-\text{mag}[k2]-\text{mag}[k3])\times(\text{mag}[k1]+\text{mag}[k2]-\text{mag}[k3])\times\text{mag}[k3]\times(\text{mag}[k1]-\text{mag}[k2]+\text{mag}[k3])\big)+7\times\big(-193+20\,\pi^2\big)\,\big(\text{mag}[k1]^2+(\text{mag}[k2]-\text{mag}[k3])^2-2\,\text{mag}[k1]\times(\text{mag}[k2]+\text{mag}[k3])\big)^3\Big)\Big/\big(\big(29\,114-2940\,\pi^2\big)\,\text{mag}[k1]^2\,\text{mag}[k2]^2\,\text{mag}[k3]^2\big)$$

In[30]:=
```
Simplify[S[k, k, k]]
```

Out[30]=
```
1
```

## $F_2$

$$Z_2 \supset b_1 \times f_{NL} \times F_2[k_1, k_2] \to k^3 \text{ as the dimension, } \nu = -0.8$$
$$\mu^0 : \text{PNEWabb1F2} \ldots$$

In[31]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

In[31]:=
```
PNEWintegrand = (2 F2s[q, k - q] × S[q, k - q, k] /. {al → alfm, be → befm}) /.
    {mag[0] → 0, mag[-q] → q, mag[q] → q, mag[k] → k, mag[k - q] → kmq} // Expand;
```

In[32]:=
```
TabNEW = Table[{-1/2 q D[PNEWintegrand[[i]], q]/PNEWintegrand[[i]], -1/2 kmq D[PNEWintegrand[[i]], kmq]/PNEWintegrand[[i]],
    PNEWintegrand[[i]] /. {q → 1, kmq → 1, k → 1}}, {i, 1, Length[PNEWintegrand]}];
```

In[33]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2] / J2[1, n1, n2],
    {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[34]:=
```
PNEWabb1F2[n1_, n2_] = sum;
```

In[35]:=
```
Simplify[PNEWabb1F2[n1, n2] - PNEWabb1F2[n2, n1]]
```

Out[35]= 0

In[36]:=
```
Export[NotebookDirectory[] <> "F2_term.m", PNEWabb1F2[n1, n2]];
```

## $G_2$

$$Z_2 \supset f_{NL} \times f \times G_2[k_1, k_2] \to k^3 \text{ as the dimension, } \nu = -0.8$$
$$\mu^2 : \text{PNEWabG2}$$

---

In[38]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

---

In[37]:=
```
PNEWintegrand = (2 G2s[q, k - q] × S[q, k - q, k] /. {al → alfm, be → befm}) /.
    {mag[0] → 0, mag[-q] → q, mag[q] → q, mag[k] → k, mag[k - q] → kmq} // Expand;
```

In[38]:=
```
TabNEW = Table[{-1/2 q D[PNEWintegrand[[i]], q]/PNEWintegrand[[i]], -1/2 kmq D[PNEWintegrand[[i]], kmq]/PNEWintegrand[[i]],
    PNEWintegrand[[i]] /. {q → 1, kmq → 1, k → 1}}, {i, 1, Length[PNEWintegrand]}];
```

In[39]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2] / J2[1, n1, n2],
    {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[40]:=
```
PNEWabG2[n1_, n2_] = sum;
```

In[41]:=
```
Simplify[PNEWabG2[n1, n2] - PNEWabG2[n2, n1]]
```

Out[41]= 0

In[42]:=
```
Export[NotebookDirectory[] <> "G2_term.m", PNEWabG2[n1, n2]];
```

## $b_2$

$$Z_2 \supset \frac{b_2}{2} \times f_{NL} \to k^3 \text{ as the dimension, } \nu = -1.25 :$$

notice that the factor of $\frac{b_2}{2}$ is outside. I.e. I do not have an overall $\frac{1}{2}$ ...

$$\mu^0 : \text{Pnewabb2}$$

In[44]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

In[43]:=
```
PNEWintegrand =
  (2 × 1 × 𝒮[q, k - q, k] /. {al → alfm, be → befm}) /. {mag[0] → 0, mag[-q] → q,
     mag[q] → q, mag[k] → k, mag[k - q] → kmq, mag[k + q] → kmq} // Expand;
```

In[44]:=
```
TabNEW = Table[{-1/2 q D[PNEWintegrand[[i]], q]/PNEWintegrand[[i]], -1/2 kmq D[PNEWintegrand[[i]], kmq]/
    PNEWintegrand[[i]] /. {q → 1, kmq → 1, k → 1}}, {i, 1, Length[PNEWintegrand]}];
```

In[45]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2] / J2[1, n1, n2],
    {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[46]:=
```
Pnewabb2[n1_, n2_] = sum;
```

In[47]:=
```
Simplify[Pnewabb2[n1, n2] - Pnewabb2[n2, n1]]
```

Out[47]=
```
0
```

In[48]:=
```
Export[NotebookDirectory[] <> "b2_term.m", Pnewabb2[n1, n2]];
```

## $\mathcal{G}_2$

$Z_2 \supset b_{\mathcal{G}_2} \times f_{NL} \times \mathcal{G}_2[k_1, k_2] \to k^3$ as the dimension,

$\nu = -0.8$ (notice that it is different from the above)

$\mu^0$ : Pnewab$\mathcal{G}$2

In[50]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

In[49]:=
```
PNEWintegrand = (2 × sigma2[q, k - q] × 𝒮[q, k - q, k] /. {al → alfm, be → befm}) /.
    {mag[0] → 0, mag[-q] → q, mag[q] → q, mag[k] → k, mag[k - q] → kmq} // Expand;
```

In[50]:=
```
TabNEW = Table[{-1/2 q D[PNEWintegrand[[i]], q]/PNEWintegrand[[i]], -1/2 kmq D[PNEWintegrand[[i]], kmq]/
    PNEWintegrand[[i]] /. {q → 1, kmq → 1, k → 1}}, {i, 1, Length[PNEWintegrand]}];
```

In[51]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2] / J2[1, n1, n2],
    {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[52]:=
```
Pnewab𝒢2[n1_, n2_] = sum;
```

In[53]:=
```
Simplify[Pnewab𝒢2[n1, n2] - Pnewab𝒢2[n2, n1]]
```

Out[53]=
```
0
```

In[54]:=
```
Export[NotebookDirectory[] <> "calG2_term.m", Pnewab𝒢2[n1, n2]];
```

## $b_1 f$

$$Z_2 \supset b_1 \times f_{NL} \times \left(\frac{f \times \mu \times k}{2}\right) \times \left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2}\right) \to \nu = -0.8,$$

overall becomes $b_1 \times f_{NL} \times \frac{f}{2}$ with again $k^3$ as dimension ! Careful for the 1 / 2 !

$\mu^2$ : PNEWabb1$f$

---

In[56]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

---

In[55]:=
```
PNEWintegrand = (2 × (((1/2 (mag[k]^2 + mag[q]^2 - mag[k - q]^2))/mag[q]^2 +
    (mag[k]^2 - (1/2 (mag[k]^2 + mag[q]^2 - mag[k - q]^2)))/mag[k - q]^2) × S[q, k - q, k]) /.
  {al → alfm, be → befm}) /. {mag[0] → 0, mag[-q] → q, mag[q] → q,
  mag[k] → k, mag[k - q] → kmq} // Expand;
```

In[56]:=
```
TabNEW = Table[{-1/2 q D[PNEWintegrand[[i]], q]/PNEWintegrand[[i]],
   -1/2 kmq D[PNEWintegrand[[i]], kmq]/PNEWintegrand[[i]],
   PNEWintegrand[[i]] /. {q → 1, kmq → 1, k → 1}}, {i, 1, Length[PNEWintegrand]}];
```

In[57]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2]/J2[1, n1, n2],
   {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[58]:=
```
PNEWabb1f[n1_, n2_] = sum;
```

In[59]:=
```
Simplify[PNEWabb1f[n1, n2] - PNEWabb1f[n2, n1]]
```

Out[59]=
```
0
```

In[60]:=
```
Export[NotebookDirectory[] <> "b1f_term.m", PNEWabb1f[n1, n2]];
```

## $f^2$ terms

$$Z_2 \supset f_{NL} \times \left(\frac{f \times \mu \times k}{2}\right) \times f \times \left(\frac{\mu_1 \times \mu_2^2}{k_1} + \frac{\mu_2 \times \mu_1^2}{k_2}\right) \to \nu = -0.8,$$

overall becomes $f_{NL} \times \frac{f^2}{2}$ with again $k^3$ as dimension ! Careful about the 1 / 2 ...

$\mu^2$ : PNEWab$f$2$\mu$2

$\mu^4$ : PNEWab$f$2$\mu$4

In[61]:=
$$\text{kdotq} = \frac{1}{2}\left(\text{mag}[k]^2 + \text{mag}[q]^2 - \text{mag}[k-q]^2\right);$$
$$\text{qdotkmq} = \frac{1}{2}\left(\text{mag}[k]^2 + \text{mag}[q]^2 - \text{mag}[k-q]^2\right) - \text{mag}[q]^2;$$
$$\text{kdotkmq} = \text{mag}[k]^2 - \text{kdotq};$$

In[64]:=
$$\mathcal{A}\text{prime} = \frac{2}{3}\times\left(\frac{\text{qdotkmq}\times\text{kdotkmq}}{\text{mag}[q]^2\,\text{mag}[k-q]^2} + \frac{\text{qdotkmq}\times\text{kdotq}}{\text{mag}[k-q]^2\,\text{mag}[q]^2}\right) + \frac{1}{3}\times\left(\frac{\text{kdotq}}{\text{mag}[q]^2} + \frac{\text{kdotkmq}}{\text{mag}[k-q]^2}\right);$$

In[65]:=
$$\mathcal{B}\text{prime} = \frac{\text{kdotq}}{\text{mag}[q]^2}\times\frac{\text{kdotkmq}^2}{\text{mag}[k-q]^2\,\text{mag}[k]^2} + \frac{\text{kdotkmq}}{\text{mag}[k-q]^2}\times\frac{\text{kdotq}^2}{\text{mag}[k]^2\,\text{mag}[q]^2};$$

## $\mu^2$

In[68]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

In[66]:=
$$\text{PNEWintegrand} = \left(2\times\left(\frac{3}{2}\times(\mathcal{A}\text{prime} - \mathcal{B}\text{prime})\times\mathcal{S}[q, k-q, k]\right) /. \{\text{al}\to\text{alfm}, \text{be}\to\text{befm}\}\right) /.$$
$$\{\text{mag}[0]\to 0, \text{mag}[-q]\to q, \text{mag}[q]\to q, \text{mag}[k]\to k, \text{mag}[k-q]\to \text{kmq}\} \text{ // Expand};$$

In[67]:=
$$\text{TabNEW} = \text{Table}\left[\left\{-\frac{1}{2}\,q\,\frac{\text{D}[\text{PNEWintegrand}[\![i]\!], q]}{\text{PNEWintegrand}[\![i]\!]}, -\frac{1}{2}\,\text{kmq}\,\frac{\text{D}[\text{PNEWintegrand}[\![i]\!], \text{kmq}]}{\text{PNEWintegrand}[\![i]\!]},\right.\right.$$
$$\left.\left.\text{PNEWintegrand}[\![i]\!] /. \{q\to 1, \text{kmq}\to 1, k\to 1\}\right\}, \{i, 1, \text{Length}[\text{PNEWintegrand}]\}\right];$$

In[68]:=
$$\text{sum} = \text{Sum}[\text{TabNEW}[\![i, 3]\!]\times J2[1, \text{TabNEW}[\![i, 1]\!] + n1, \text{TabNEW}[\![i, 2]\!] + n2] / J2[1, n1, n2],$$
$$\{i, 1, \text{Length}[\text{TabNEW}]\}] \text{ // FunctionExpand // Simplify};$$

In[69]:=
```
PNEWabf2μ2[n1_, n2_] = sum;
```

In[70]:=
```
Simplify[PNEWabf2μ2[n1, n2] - PNEWabf2μ2[n2, n1]]
```

Out[70]=
```
0
```

In[71]:=
```
Export[NotebookDirectory[] <> "f2mu2_term.m", PNEWabf2μ2[n1, n2]];
```

## $\mu^4$

In[75]:=
```
Clear[PNEWintegrand, TabNEW, sum];
```

In[72]:=
$$\text{PNEWintegrand} =$$
$$\left(2\times\left(-\frac{3}{2}\times\left(\mathcal{A}\text{prime} - \frac{5}{3}\times\mathcal{B}\text{prime}\right)\times\mathcal{S}[q, k-q, k]\right) /. \{\text{al}\to\text{alfm}, \text{be}\to\text{befm}\}\right) /.$$
$$\{\text{mag}[0]\to 0, \text{mag}[-q]\to q, \text{mag}[q]\to q, \text{mag}[k]\to k, \text{mag}[k-q]\to \text{kmq}\} \text{ // Expand};$$

In[73]:=
$$\text{TabNEW} = \text{Table}\left[\left\{-\frac{1}{2}\,q\,\frac{\text{D}[\text{PNEWintegrand}[\![i]\!], q]}{\text{PNEWintegrand}[\![i]\!]}, -\frac{1}{2}\,\text{kmq}\,\frac{\text{D}[\text{PNEWintegrand}[\![i]\!], \text{kmq}]}{\text{PNEWintegrand}[\![i]\!]},\right.\right.$$
$$\left.\left.\text{PNEWintegrand}[\![i]\!] /. \{q\to 1, \text{kmq}\to 1, k\to 1\}\right\}, \{i, 1, \text{Length}[\text{PNEWintegrand}]\}\right];$$

In[74]:=
```
sum = Sum[TabNEW[[i, 3]] × J2[1, TabNEW[[i, 1]] + n1, TabNEW[[i, 2]] + n2] / J2[1, n1, n2],
      {i, 1, Length[TabNEW]}] // FunctionExpand // Simplify;
```

In[75]:=
```
PNEWabf2μ4[n1_, n2_] = sum;
```

In[76]:=
```
Simplify[PNEWabf2μ4[n1, n2] - PNEWabf2μ4[n2, n1]]
```

Out[76]=
```
0
```

In[77]:=
```
Export[NotebookDirectory[] <> "f2mu4_term.m", PNEWabf2μ4[n1, n2]];
```