In[1]:=
```
ClearAll[Evaluate[Context[]<>"*"]];
ClearSystemCache[];
```

Definitions

In[3]:=
$$kT = k1 + k2 + k3;$$
$$e2 = k1 \times k2 + k2 \times k3 + k3 \times k1;$$
$$e3 = k1 \times k2 \times k3;$$

In[6]:=
$$K1 = kT;$$
$$K2 = e2^{1/2};$$
$$K3 = e3^{1/3};$$

In[9]:=
$$bisp = \frac{24\, K3^6 - 8\, K2^2\, K3^3\, K1 - 8\, K2^4\, K1^2 + 22\, K3^3\, K1^3 - 6\, K2^2\, K1^4 + 2\, K1^6}{K1^3\, K3^9};$$

In[10]:=
$$bisp = \frac{24\, e3^2 - 8\, e2\, e3\, kT - 8\, e2^2\, kT^2 + 22\, e3\, kT^3 - 6\, e2\, kT^4 + 2\, kT^6}{kT^3\, e3};$$

In[11]:=
$$shape = bisp (* \times e3^2 *);$$

In[12]:=
```
S[x1_, x2_] = (Simplify[shape /. {k1 → x1 × k3, k2 → x2 × k3}]);
```

In[13]:=
```
norm = S[1, 1]
```

Out[13]=
$$-\frac{34}{9}$$

In[14]:=
$$toplot\pi dotd\pi 2[\{x1\_, x2\_\}] = \frac{S[x1, x2]}{norm} \text{ // Simplify}$$

Out[14]=
$$-\frac{1}{17\, x1\, x2\, (1 + x1 + x2)^3} 9 \left(x1^6 + 3\, x1^5\, (1 + x2) - x1^4\, \left(1 - 6\, x2 + x2^2\right) + \left(-1 + x2^2\right)^2\, \left(1 + 3\, x2 + x2^2\right) + 3\, x1\, (-1 + x2)^2 \left(1 + 4\, x2 + 4\, x2^2 + x2^3\right) - 3\, x1^3\, \left(2 + 3\, x2 + 3\, x2^2 + 2\, x2^3\right) - x1^2\, \left(1 + 9\, x2 + 12\, x2^2 + 9\, x2^3 + x2^4\right)\right)$$

In[15]:=
```
Export[NotebookDirectory[] <> "dotpinablapisquared.txt", toplotπdotdπ2[{x1, x2}]];
```

In[16]:=
$$thing = \left(\frac{k1}{k2} + \frac{k1}{k3} + \frac{k2}{k1} + \frac{k2}{k3} + \frac{k3}{k1} + \frac{k3}{k2}\right) - \left(\frac{k1^2}{k2 \times k3} + \frac{k2^2}{k1 \times k3} + \frac{k3^2}{k1 \times k2}\right) - 2;$$

In[17]:=
```
S[x1_, x2_] = (Simplify[thing /. {k1 → x1 × k3, k2 → x2 × k3}]);
```

In[18]:=
```
norm = S[1, 1]
```

Out[18]=
1

In[19]:=
$$\text{toplotequil}[\{x1\_, x2\_\}] = \frac{S[x1, x2]}{\text{norm}} \text{ // Simplify}$$

Out[19]=
$$\frac{-x1^3 + x1\,(-1 + x2)^2 + x1^2\,(1 + x2) - (-1 + x2)^2\,(1 + x2)}{x1\,x2}$$

In[20]:=
$$\text{thing} = \frac{k1 \times k2 \times k3}{(k1 + k2 + k3)^3};$$

In[21]:=
$$S[x1\_, x2\_] = (\text{Simplify}[\text{thing} \mathbin{/.} \{k1 \to x1 \times k3, k2 \to x2 \times k3\}]);$$

In[22]:=
$$\text{norm} = S[1, 1]$$

Out[22]=
$$\frac{1}{27}$$

In[23]:=
$$\text{toplot}\pi\text{dotcubed}[\{x1\_, x2\_\}] = \frac{S[x1, x2]}{\text{norm}} \text{ // Simplify}$$

Out[23]=
$$\frac{27\,x1\,x2}{(1 + x1 + x2)^3}$$

In[24]:=
$$p = \frac{27}{-21 + \frac{743}{7 \times (20\,\pi^2 - 193)}};$$

$$\Delta = (kT - 2\,k1)\,(kT - 2\,k2)\,(kT - 2\,k3);$$

$$\Gamma = \frac{2}{3}\,e2 - \frac{1}{3}\,\left(k1^2 + k2^2 + k3^2\right);$$

In[27]:=
$$\text{thing} = \left((1 + p)\,\frac{\Delta}{e3^3} - p \times \frac{\Gamma^3}{e3^4}\right) \times e3^2;$$

In[28]:=
$$S[x1\_, x2\_] = (\text{Simplify}[\text{thing} \mathbin{/.} \{k1 \to x1 \times k3, k2 \to x2 \times k3\}]);$$

In[29]:=
$$\text{norm} = S[1, 1]$$

Out[29]=
$$\frac{-7363 + 840\,\pi^2 - 189 \times \left(-193 + 20\,\pi^2\right)}{29\,114 - 2940\,\pi^2}$$

In[30]:=
$$\text{toplotortho}[\{x1\_, x2\_\}] = \frac{S[x1, x2]}{\text{norm}} \text{ // Simplify}$$

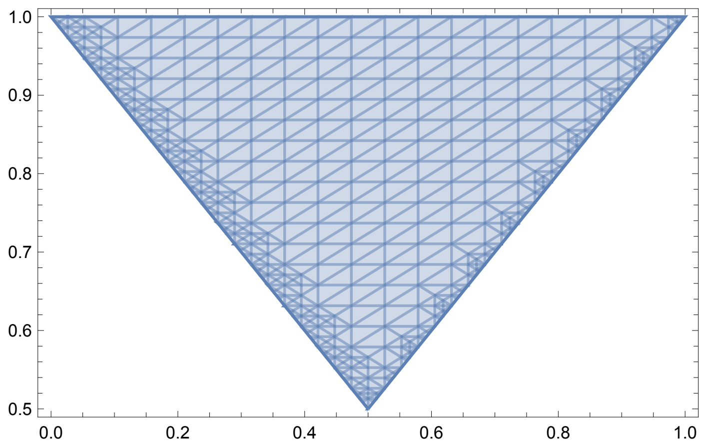
Out[30]=
$$\left(-\left(\left(-7363 + 840\,\pi^2\right)\,x1\,(-1 + x1 - x2) \times (1 + x1 - x2)\,x2\,(-1 + x1 + x2)\right) + 7 \times \left(-193 + 20\,\pi^2\right)\,\left(x1^2 + (-1 + x2)^2 - 2\,x1\,(1 + x2)\right)^3\right) \Big/ \left(\left(29\,114 - 2940\,\pi^2\right)\,x1^2\,x2^2\right)$$

In[31]:=
```
Export[NotebookDirectory[] <> "orthogonal.txt", toplotortho[{x1, x2}]];
```

Region and cosine definitions

In[32]:=
```
RegionPlot[x2 > Abs[x1 - 1] && x1 < x2,
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio]
```

Out[32]=



In[33]:=
```
dot[S1_, S2_] := NIntegrate[S1[{x1, x2}] × S2[{x1, x2}]
   Boole[x2 > Abs[x1 - 1] && x1 < x2], {x1, 0, 1}, {x2, 1 / 2, 1}];
cos[S1_, S2_] := dot[S1, S2] / Sqrt[dot[S1, S1] × dot[S2, S2]];
```

I have that the single-field bispectrum is $f_{\mathrm{NL},\dot{\pi}(\nabla\pi)^2}\, B_{\dot{\pi}(\nabla\pi)^2} + f_{\mathrm{NL},\dot{\pi}^3}\, B_{\dot{\pi}^3}$. I can also write it as $f_{\mathrm{NL,equil.}}\, B_{\mathrm{equil.}} + f_{\mathrm{NL,ortho.}}\, B_{\mathrm{ortho.}}$ -> suppose I write $B_{\dot{\pi}(\nabla\pi)^2} = \mathrm{Cos}[\theta]\, B_{\mathrm{equil.}} + \mathrm{Sin}[\theta]\, B_{\mathrm{ortho.}}$, and with an angle φ for the other (notice that equil. and ortho. are orthonormal) -> what happens? I can relate the f_{NL} by some matrices of scalar products. That's the point of Leonardo...

In[35]:=
```
invM = Rationalize[(
   dot[toplotequil,toplotπdotdπ2]/dot[toplotequil,toplotequil]   dot[toplotequil,toplotπdotcubed]/dot[toplotequil,toplotequil]
   dot[toplotortho,toplotπdotdπ2]/dot[toplotortho,toplotortho]   dot[toplotortho,toplotπdotcubed]/dot[toplotortho,toplotortho]
  ), 10^-32]
```

Out[35]= $\left\{\left\{\frac{513\,865\,931}{494\,001\,065}, \frac{227\,154\,851}{187\,668\,044}\right\}, \left\{-\frac{18\,794\,399}{475\,638\,518}, -\frac{81\,528\,755}{464\,062\,707}\right\}\right\}$

In[36]:=
```
invM = N[invM, 6]
```

Out[36]= {{1.04021, 1.21041}, {-0.0395140, -0.175685}}

Out[◦]= {{1.04021, 1.21041}, {-0.039514, -0.175685}}

In[37]:=
```
invM = (
   dot[toplotequil,toplotπdotdπ2]/dot[toplotequil,toplotequil]   dot[toplotequil,toplotπdotcubed]/dot[toplotequil,toplotequil]
   dot[toplotortho,toplotπdotdπ2]/dot[toplotortho,toplotortho]   dot[toplotortho,toplotπdotcubed]/dot[toplotortho,toplotortho]
  )
```

Out[37]= {{1.04021, 1.21041}, {-0.039514, -0.175685}}

In[38]:=
```
invM.{1, 0}
```

Out[38]= {1.04021, -0.039514}

In[39]:=
```
invM.{0, 1}
```

Out[39]= {1.21041, -0.175685}

In[40]:=
```
dotpinablapisquaredcombo[{x1_, x2_}] =
  (invM.{1, 0}).{toplotequil[{x1, x2}], toplotortho[{x1, x2}]}
```

Out[40]=
$$\frac{1.04021\left(-x1^3+x1\,(-1+x2)^2+x1^2\,(1+x2)-(-1+x2)^2\,(1+x2)\right)}{x1\,x2}-\frac{1}{x1^2\,x2^2}$$
$$0.000405842\left(-\left(\left(-7363+840\,\pi^2\right)x1\,(-1+x1-x2)\times(1+x1-x2)\,x2\,(-1+x1+x2)\right)+7\times\left(-193+20\,\pi^2\right)\left(x1^2+(-1+x2)^2-2\,x1\,(1+x2)\right)^3\right)$$

In[41]:=
```
dotpinablapisquaredcombo[{1, 1}]
```

Out[41]= 1.0007

In[42]:=
```
dotpinablapisquaredcombo[{x1_, x2_}] =
  Rationalize[dotpinablapisquaredcombo[{x1, x2}] / dotpinablapisquaredcombo[{1, 1}], 10^-32]
```

Out[42]=
$$\frac{1}{57\,043\,016}\,57\,003\,219$$
$$\left(\frac{513\,865\,931\left(-x1^3+x1\,(-1+x2)^2+x1^2\,(1+x2)-(-1+x2)^2\,(1+x2)\right)}{494\,001\,065\,x1\,x2}-\frac{1}{1\,691\,226\,281\,x1^2\,x2^2}\right.$$
$$\left.686\,371\left(-\left(\left(-7363+840\,\pi^2\right)x1\,(-1+x1-x2)\times(1+x1-x2)\,x2\,(-1+x1+x2)\right)+7\times\left(-193+20\,\pi^2\right)\left(x1^2+(-1+x2)^2-2\,x1\,(1+x2)\right)^3\right)\right)$$

In[43]:=
```
dotpicubedcombo[{x1_, x2_}] =
  (invM.{0, 1}).{toplotequil[{x1, x2}], toplotortho[{x1, x2}]}
```

Out[43]=
$$\frac{1.21041\left(-x1^3+x1\,(-1+x2)^2+x1^2\,(1+x2)-(-1+x2)^2\,(1+x2)\right)}{x1\,x2}-\frac{1}{x1^2\,x2^2}$$
$$0.00180443\left(-\left(\left(-7363+840\,\pi^2\right)x1\,(-1+x1-x2)\times(1+x1-x2)\,x2\,(-1+x1+x2)\right)+7\times\left(-193+20\,\pi^2\right)\left(x1^2+(-1+x2)^2-2\,x1\,(1+x2)\right)^3\right)$$

In[44]:=
```
dotpicubedcombo[{1, 1}]
```

Out[44]= 1.03472

In[45]:=
```
dotpicubedcombo[{x1_, x2_}] = Rationalize[dotpicubedcombo[{x1, x2}] / dotpicubedcombo[{1, 1}], 10^-32]
```

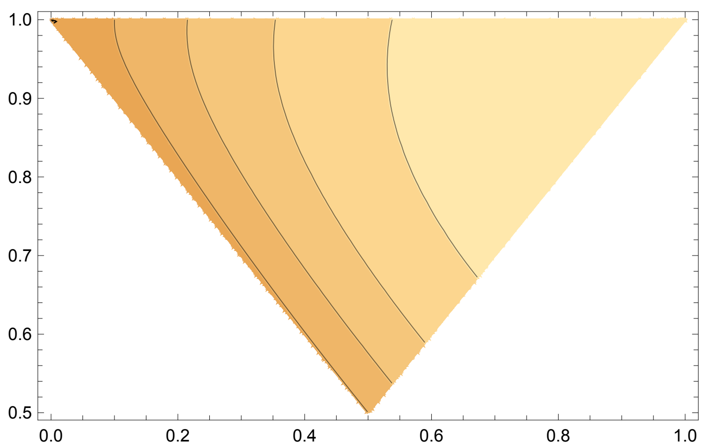
Out[45]=

$$\frac{1}{28\,794\,413}\,27\,828\,138\left(\frac{227\,154\,851\left(-x1^3+x1\,(-1+x2)^2+x1^2\,(1+x2)-(-1+x2)^2\,(1+x2)\right)}{187\,668\,044\,x1\,x2}-\frac{1}{1\,498\,979\,025\,x1^2\,x2^2}\,2\,704\,802\left(-\left(\left(-7363+840\,\pi^2\right)x1\,(-1+x1-x2)\times(1+x1-x2)\,x2\,(-1+x1+x2)\right)+7\times\left(-193+20\,\pi^2\right)\left(x1^2+(-1+x2)^2-2\,x1\,(1+x2)\right)^3\right)\right)$$

In[46]:=
```
(**)Export[NotebookDirectory[] <> "dotpinablapisquaredcombo.txt",
  dotpinablapisquaredcombo[{x1, x2}]];(**)
Export[NotebookDirectory[] <> "dotpicubedcombo.txt", dotpicubedcombo[{x1, x2}]];
```

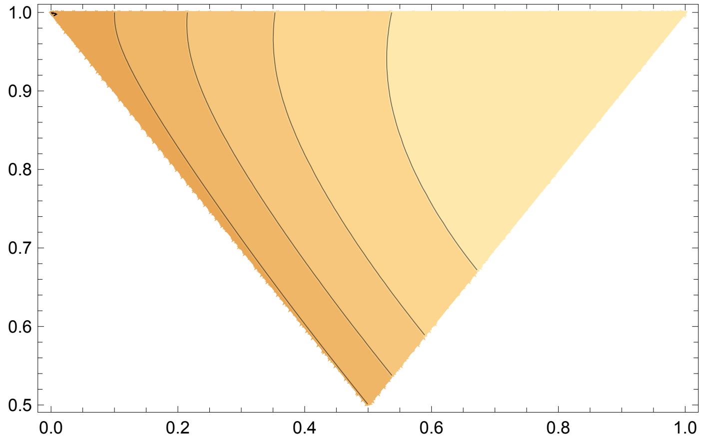
In[48]:=
```
ContourPlot[dotpinablapisquaredcombo[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio,
 RegionFunction → Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2], PlotRange → {-1, 1}]
```

Out[48]=



In[49]:=
```
ContourPlot[toplotπdotdπ2[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio,
 RegionFunction → Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2], PlotRange → {-1, 1}]
```

Out[49]=



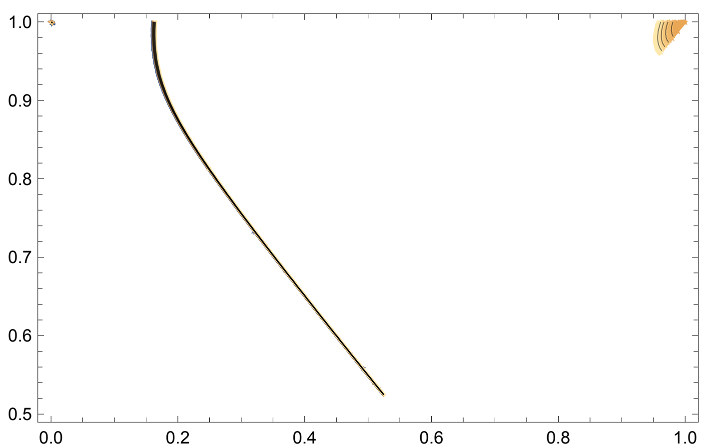
In[50]:=
```
(* I had made a mistake in the plots for ORTHOGONAL,
this plot above shows that I did not make any mistake in saying that the
  combination for the π̇(∇π)^2 operator is essentially indistinguishable
  from the others. It still pays to do the plot though... *)
```

In[51]:=
```
cos[toplotπdotdπ2, dotpinablapisquaredcombo]
```

Out[51]= 1.

In[52]:=
```
ContourPlot[toplotπdotdπ2[{x1, x2}] - dotpinablapisquaredcombo[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio, RegionFunction →
  Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2], PlotRange → {-10^-5, 10^-5}]
```

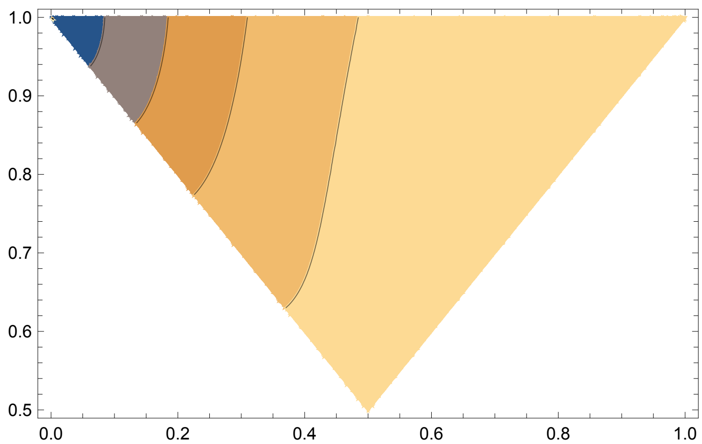
Out[52]=



In[53]:=
```
(* so these are essentially the same,
I won't care to plot them in python... Now I
 have issues in plotting them. If we want to plot this,
I can say the plot is essentially indistinguishable from the plot of the
    nonseparable shape of the operator with spatial derivatives... So one plots
  the two non-separable shapes, and then the separable orthogonal below...
  Which I indeed show to be very similar... *)
```

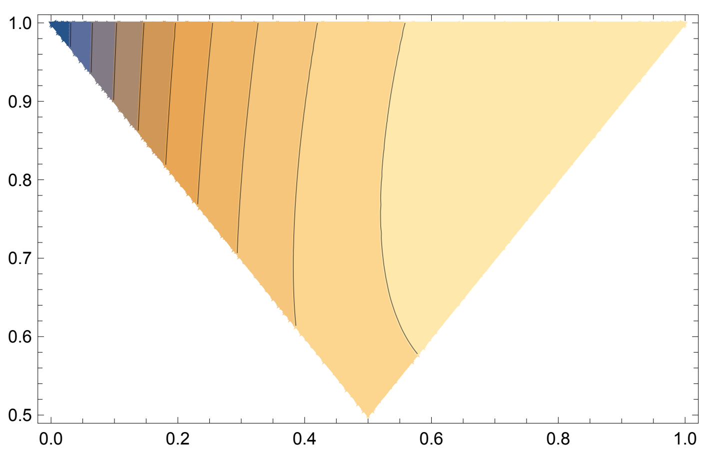
In[54]:=
```
ContourPlot[dotpicubedcombo[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio,
 RegionFunction → Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2]]
```

Out[54]=



In[55]:=
```
ContourPlot[toplotπdotcubed[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio,
 RegionFunction → Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2]]
```

Out[55]=



In[56]:=
```
cos[toplotπdotcubed, dotpicubedcombo]
```

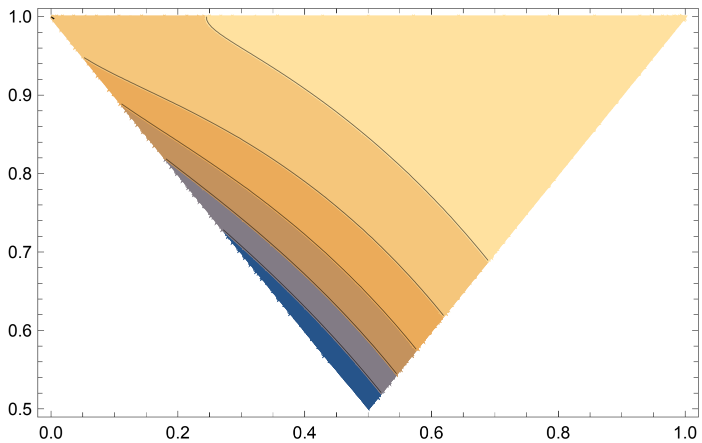
Out[56]= 0.999507

In[57]:=
```
(* perfect... Essentially total overlap... The constant-
 shape lines seem different? But I do not really know how they are computed,
i.e. what the lines in the plots above are... Well,
they are the lines of constant shape... *)
```

In[58]:=
```
(* orthogonal... *)
```

In[59]:=
```
ContourPlot[toplotortho[{x1, x2}],
 {x1, 0, 1}, {x2, 1 / 2, 1}, AspectRatio → 1 / GoldenRatio,
 RegionFunction → Function[{x1, x2, z}, x2 > Abs[x1 - 1] && x1 < x2], PlotRange → {-5, 1}]
```

Out[59]=



In[60]:=
```
(* check... *)
```

In[61]:=
```
invM
```

Out[61]= {{1.04021, 1.21041}, {-0.039514, -0.175685}}

In[62]:=
```
M = Inverse[invM]
```

Out[62]= {{1.30213, 8.97121}, {-0.292867, -7.70977}}

In[63]:=
```
M.{fequil, fortho}
```

Out[63]= {1.30213 fequil + 8.97121 fortho, -0.292867 fequil - 7.70977 fortho}

In[64]:=
```
invMLEO = {{1.040, 1.210}, {0.1079, -0.06572}}
```

Out[64]= {{1.04, 1.21}, {0.1079, -0.06572}}

In[65]:=
```
MatrixForm[invMLEO]
```

Out[65]//MatrixForm=

$$\begin{pmatrix} 1.04 & 1.21 \\ 0.1079 & -0.06572 \end{pmatrix}$$

In[66]:=
```
Inverse[invMLEO]
```

Out[66]= {{0.330404, 6.08322}, {0.542462, -5.22855}}