Bearings and Distances
Angles of Elevation & Depression
Longitude and Latitude

WAEC General Mathematics Comprehensive Lecture Notes

Academic Lecture Series

1 Bearings and Distances

1.1 Introduction to Bearings

Definition 1.1. A **bearing** is the direction or path along which something moves or along which it points, measured as an angle from a reference direction (usually North).

1.2 Types of Bearings

Definition 1.2. 1. Compass Bearings (Cardinal Points):

- Use cardinal directions: N, S, E, W
- Measured as angles from North or South
- Format: Nθ°E, Sθ°W, etc.
- Examples: N30°E, S45°W, N60°W

2. Three-Figure Bearings (True Bearings):

- Measured clockwise from North
- Always use three digits (000° to 360°)
- Format: θ°
- Examples: 045°, 135°, 270°

Key Point

Cardinal Directions:

- North (N) = 000° or 360°
- East (E) = 090°
- South (S) = 180°
- West (W) = 270°

1.3 Converting Between Bearing Types

Worked Example

Example 1.1: Convert the following compass bearings to three-figure bearings:

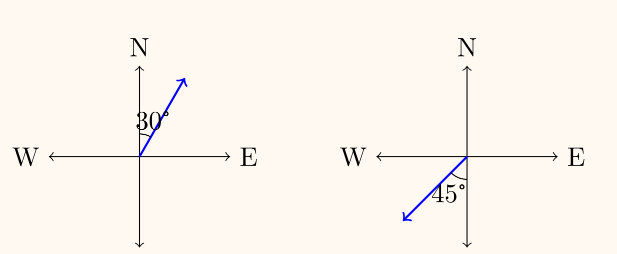
(a) N30°E

(b) S45°W

(c) N60°W

(d) S25°E

Solution:



(a) N30°E = 030° (b) S45°W = 225°

(a) N30°E: Start from North (000°), go 30° clockwise (towards East)

$$= 030$$

(b) S45°W: Start from South (180°), go 45° clockwise (towards West)

$$= 180 + 45 = 225$$

(c) N60°W: Start from North (000°), go 60° anticlockwise (or 360° - 60°)

$$= 360 - 60 = 300$$

(d) S25°E: Start from South (180°), go 25° anticlockwise (or 180° - 25°)

$$= 180 - 25 = 155$$

Worked Example

Example 1.2: Convert 135° to compass bearing.
Solution: 135° is between 090° (East) and 180° (South).
Angle from South = 180° - 135° = 45°
Since it's east of south: **S45°E**

1.4 Back Bearings

Definition 1.3. The **back bearing** is the bearing from point B to point A when the bearing from A to B is known.

Rule:

- If bearing < 180: Back bearing = Bearing + 180°
- If bearing ≥ 180: Back bearing = Bearing - 180°

Or simply: Back bearing = (Bearing + 180°) mod 360°

Worked Example

Example 1.3: Find the back bearings:

(a) Bearing of B from A is 065°

(b) Bearing of Q from P is 245°

Solution:
(a) 065° + 180° = 245°
Bearing of A from B = 245°
(b) 245° - 180° = 065°
Bearing of P from Q = 065°

1.5 Distance and Bearing Problems

Worked Example

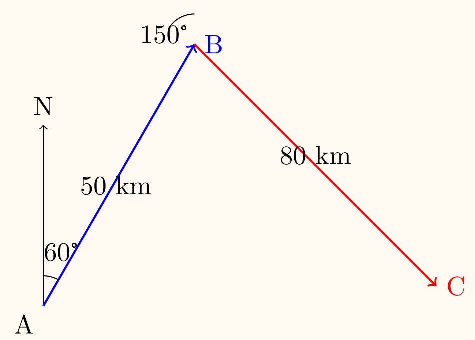
Example 1.4: A ship sails 50 km on a bearing of 060° from port A to point B. It then sails 80 km on a bearing of 150° to point C.

(a) Sketch the path

(b) Find the distance AC

(c) Find the bearing of C from A

Solution:
(a)



(b) Using the angle at B:

- Bearing from A to B = 060°
- Bearing from B to C = 150°
- Angle ABC = 150° - 60° = 90°

Since angle at B is 90°, use Pythagoras:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 50^2 + 80^2 \\ &= 2500 + 6400 \\ &= 8900 \\ AC &= \sqrt{8900} \\ &= 94.3 \text{ km} \end{aligned}$$

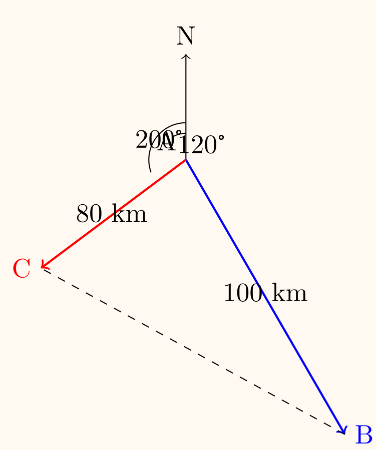
(c) Find angle CAB using trigonometry:

$$\begin{aligned} \tan(\text{angle CAB}) &= \frac{BC}{AB} = \frac{80}{50} = 1.6 \\ \text{angle CAB} &= \tan^{-1}(1.6) = 58 \end{aligned}$$

Bearing of C from A = 060° + 58° = 118°

Worked Example

Example 1.5: Town B is 100 km from town A on a bearing of 120°. Town C is 80 km from A on a bearing of 200°. Find the distance BC.
Solution:



Angle BAC = 200° - 120° = 80°
Using cosine rule:

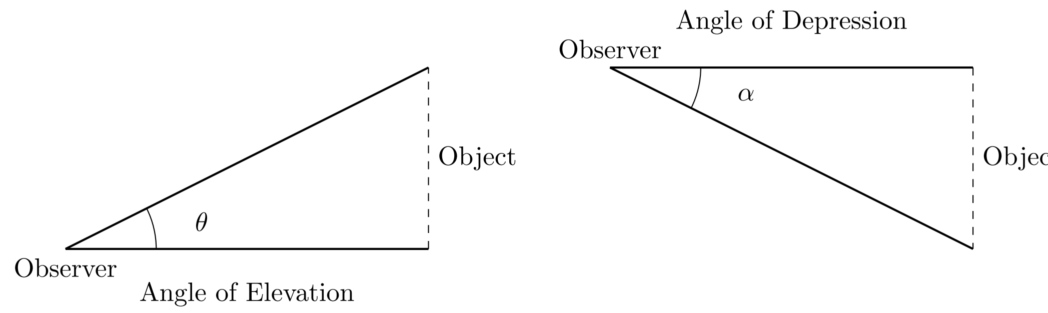
$$\begin{aligned} BC^2 &= AB^2 + AC^2 - 2(AB)(AC)\cos(80) \\ &= 100^2 + 80^2 - 2(100)(80)\cos(80) \\ &= 10000 + 6400 - 16000(0.1736) \\ &= 16400 - 2777.6 \\ &= 13622.4 \\ BC &= 116.7 \text{ km} \end{aligned}$$

2 Angles of Elevation and Depression

2.1 Definitions

Definition 2.1. Angle of Elevation: The angle between the horizontal line of sight and the line of sight upward to an object above the horizontal.

Angle of Depression: The angle between the horizontal line of sight and the line of sight downward to an object below the horizontal.



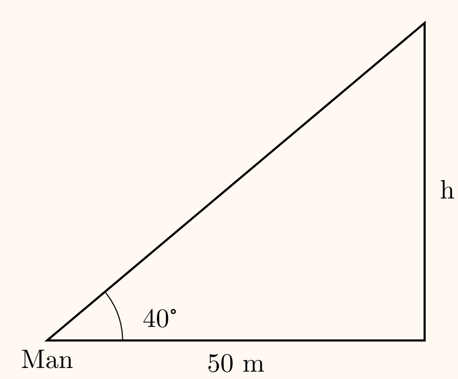
Key Point

If you are looking UP at something, it's an angle of elevation.
If you are looking DOWN at something, it's an angle of depression.
The angle of elevation from point A to B equals the angle of depression from B to A (alternate angles).

2.2 Solving Elevation/Depression Problems

Worked Example

Example 2.1: A man standing 50 m from the base of a building observes that the angle of elevation to the top is 40°. Find the height of the building.
Solution:



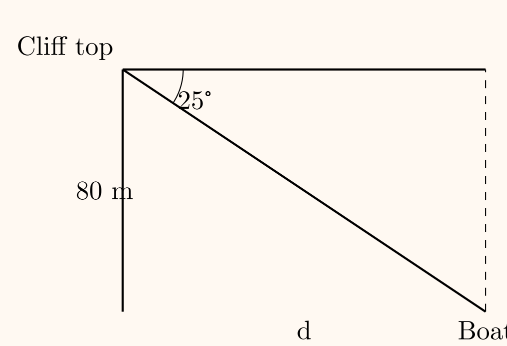
Using trigonometry:

$$\begin{aligned} \tan(40) &= \frac{h}{50} \\ h &= 50 \times \tan(40) \\ &= 50 \times 0.839 \\ &= 41.95 \text{ m} \end{aligned}$$

The building is approximately 42 m tall.

Worked Example

Example 2.2: From the top of a cliff 80 m high, the angle of depression to a boat at sea is 25°. How far is the boat from the base of the cliff?
Solution:



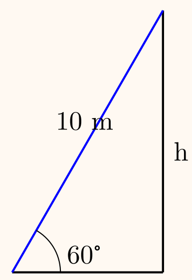
Angle of depression = 25° (from horizontal)
Angle at boat = 25° (alternate angles)
Using trigonometry:

$$\begin{aligned} \tan(25) &= \frac{80}{d} \\ d &= \frac{80}{\tan(25)} \\ &= \frac{80}{0.466} \\ &= 171.7 \text{ m} \end{aligned}$$

The boat is approximately 172 m from the cliff base.

Worked Example

Example 2.3: A ladder 10 m long leans against a wall. The angle of elevation of the ladder is 60°. How high up the wall does the ladder reach?
Solution:



Using trigonometry:

$$\sin(60) = \frac{h}{10}$$
$$h = 10 \times \sin(60)$$
$$= 10 \times 0.866$$
$$= 8.66 \text{ m}$$

The ladder reaches approximately 8.7 m up the wall.

2.3 Combined Problems

Worked Example

Example 2.4: From a point on level ground 100 m from the base of a tower, the angle of elevation to the top of the tower is 35°. From a point 50 m closer to the tower, what is the angle of elevation?
Solution:
First find the height of the tower:

$$\tan(35) = \frac{h}{100}$$
$$h = 100 \times \tan(35)$$
$$= 100 \times 0.700$$
$$= 70 \text{ m}$$

Now from 50 m away:

$$\tan(\theta) = \frac{70}{50}$$
$$= 1.4$$
$$\theta = \tan^{-1}(1.4)$$
$$= 54.5$$

The angle of elevation from 50 m is approximately 55°.

3 Longitude and Latitude

3.1 Basic Concepts

Definition 3.1. Latitude: Angular distance north or south of the equator, measured in degrees (0° to 90°).

- North of equator: N latitude (0° to 90°N)
- South of equator: S latitude (0° to 90°S)
- Equator: 0° latitude

Longitude: Angular distance east or west of the Prime Meridian (Greenwich), measured in degrees (0° to 180°).

- East of Prime Meridian: E longitude (0° to 180°E)
- West of Prime Meridian: W longitude (0° to 180°W)
- Prime Meridian: 0° longitude

Key Point

Important Facts:

- Radius of Earth ≈ 6400 km (or 3440 nautical miles)
- Circumference of Earth ≈ 40,000 km
- 1° of arc on a great circle = $\frac{40000}{360}$ km ≈ 111 km
- Lines of latitude are circles parallel to equator
- Lines of longitude are semicircles (meridians) from pole to pole
- All meridians are great circles
- Only the equator is a great circle among latitudes

3.2 Distance Along Lines of Latitude

Important Formula

Distance between two points on the same latitude:
If two points P and Q are on latitude $\theta°$ with longitudes λ_1 and λ_2:
1. Difference in longitude:

$$\Delta\lambda = |\lambda_2 - \lambda_1|$$

2. Radius at latitude θ:

$$r = R\cos(\theta)$$

where R = radius of Earth
3. Distance PQ:

$$d = \frac{2\pi r \times \Delta\lambda}{360} = \frac{2\pi R\cos(\theta) \times \Delta\lambda}{360}$$

Or: $d = 111 \times \cos(\theta) \times \Delta\lambda$ km

Worked Example

Example 3.1: Find the distance between two points A(60°N, 30°W) and B(60°N, 50°E) measured along their circle of latitude. (Take $R = 6400$ km, $\pi = \frac{22}{7}$)
Solution:
Both points are on latitude 60°N
Difference in longitude:

$$\Delta\lambda = 30 + 50 = 80$$

(Add because one is W and one is E)
Radius at 60°N:

$$r = R\cos(60)$$
$$= 6400 \times 0.5$$
$$= 3200 \text{ km}$$

Distance:

$$d = \frac{2\pi r \times 80}{360}$$
$$= \frac{2 \times \frac{22}{7} \times 3200 \times 80}{360}$$
$$= \frac{1126400}{2520}$$
$$= 4470.5 \text{ km}$$

3.3 Distance Along Lines of Longitude

Important Formula

Distance along a meridian (same longitude):
If two points P and Q have the same longitude with latitudes ϕ_1 and ϕ_2:
Difference in latitude:

$$\Delta\phi = |\phi_2 - \phi_1|$$

Distance:

$$d = \frac{2\pi R \times \Delta\phi}{360} = \frac{\pi R \times \Delta\phi}{180}$$

Or: $d = 111 \times \Delta\phi$ km

Worked Example

Example 3.2: Find the distance between P(30°N, 40°E) and Q(50°S, 40°E). (Take Earth's radius = 6400 km)
Solution:
Same longitude (40°E), so distance is along a meridian.
Difference in latitude:

$$\Delta\phi = 30 + 50 = 80$$

(Add because one is N and one is S)
Distance:

$$\begin{aligned} d &= \frac{2\pi R \times 80}{360} \\ &= \frac{2 \times \frac{22}{7} \times 6400 \times 80}{360} \\ &= \frac{2252800}{2520} \\ &= 8940.5 \text{ km} \end{aligned}$$

Or using: $d = 111 \times 80 = 8880$ km

3.4 Shortest Distance (Great Circle Route)

Important Formula

Shortest distance between two points on Earth's surface:
The shortest distance is along a great circle. For two points on the same meridian or on the equator, the great circle route is along that line.
For general points, use spherical geometry formulas (typically beyond WAEC scope, but students should know the concept).

3.5 Position of Places

Worked Example

Example 3.3: A plane flies due north from point A(20°N, 15°E) through a distance of 2220 km. Find the position of the final point B.
Solution:
Flying due north means longitude remains constant at 15°E.
Distance = 2220 km
Change in latitude:

$$\begin{aligned} \Delta\phi &= \frac{2220}{111} \\ &= 20 \end{aligned}$$

New latitude:

$$\begin{aligned} \text{Latitude of B} &= 20N + 20 \\ &= 40N \end{aligned}$$

Therefore: B is at (40°N, 15°E)

Worked Example

Example 3.4: Two towns P and Q are both on latitude 52°N. Their longitudes are 30°W and 70°W respectively. Calculate the distance between P and Q along their circle of latitude. (Take $R = 6400$ km, $\pi = 3.142$)
Solution:
Both on latitude 52°N
Difference in longitude:

$$\Delta\lambda = 70 - 30 = 40$$

Radius at 52°N:

$$\begin{aligned} r &= 6400 \times \cos(52) \\ &= 6400 \times 0.6157 \\ &= 3940.5 \text{ km} \end{aligned}$$

Distance:

$$\begin{aligned} d &= \frac{2\pi r \times 40}{360} \\ &= \frac{2 \times 3.142 \times 3940.5 \times 40}{360} \\ &= \frac{987732.8}{360} \\ &= 2743.7 \text{ km} \end{aligned}$$

3.6 Time Zones

Key Point

Time and Longitude:

- Earth rotates 360° in 24 hours
- Therefore: 15° of longitude = 1 hour
- Or: 1° of longitude = 4 minutes
- Moving east: time increases
- Moving west: time decreases

Worked Example

Example 3.5: When it is 12:00 noon at town A(30°E), what is the time at town B(75°E)?
Solution:
Difference in longitude:

$$\Delta\lambda = 75 - 30 = 45$$

Time difference:

$$\text{Time} = 45 \times 4 \text{ min} = 180 \text{ min} = 3 \text{ hours}$$

Since B is east of A, time at B is ahead:

$$\text{Time at B} = 12:00 + 3:00 = 15:00 \text{ (3:00 PM)}$$

Worked Example

Example 3.6: A plane leaves town P(15°W) at 9:00 AM and arrives at town Q(45°E) at 5:00 PM the same day. If the plane flew at an average speed of 800 km/h, calculate:

(a) The time difference between P and Q

(b) The actual flight time

(c) The distance covered

Solution:
(a) Difference in longitude:

$$\Delta\lambda = 15 + 45 = 60$$

Time difference:

$$\text{Time} = 60 \times 4 \text{ min} = 240 \text{ min} = 4 \text{ hours}$$

(b) Departure time at P: 9:00 AM
Arrival time at Q: 5:00 PM = 17:00
Time elapsed: 17:00 - 9:00 = 8 hours
But Q is 4 hours ahead of P, so:

$$\text{Actual flight time} = 8 - 4 = 4 \text{ hours}$$

(c) Distance:

$$\begin{aligned} d &= \text{speed} \times \text{time} \\ &= 800 \times 4 \\ &= 3200 \text{ km} \end{aligned}$$

4 Practice Exercises

4.1 Bearings and Distances

Exercise 4.1. Convert the following to three-figure bearings:

(a) N45°E

(b) S30°W

(c) N70°W

(d) S15°E

Exercise 4.2. The bearing of B from A is 125°. Find the bearing of A from B.

Exercise 4.3. A ship sails 60 km on a bearing of 040° from port P to point Q. It then sails 80 km on a bearing of 130° to point R.

(a) Find the angle PQR

(b) Calculate the distance PR

4.2 Angles of Elevation and Depression

Exercise 4.4. From a point 80 m from the foot of a tower, the angle of elevation to the top is 38°. Calculate the height of the tower.

Exercise 4.5. From the top of a lighthouse 50 m high, the angle of depression to a ship is 15°. How far is the ship from the base of the lighthouse?

Exercise 4.6. A 12 m ladder leans against a wall making an angle of 65° with the ground. How high up the wall does it reach?

4.3 Longitude and Latitude

Exercise 4.7. Calculate the distance along the parallel of latitude 40°N between two points with longitudes 20°E and 65°E. (Take $R = 6400$ km)

Exercise 4.8. Find the distance along a meridian between points at 25°N and 45°S. (Use 111 km per degree)

Exercise 4.9. When it is 10:00 AM at place A(30°W), what time is it at place B(60°E)?

5 Solutions to Selected Exercises

Solution to Exercise 1:

(a) N45°E = 045°

(b) S30°W = 180° + 30° = 210°

(c) N70°W = 360° - 70° = 290°

(d) S15°E = 180° - 15° = 165°

Solution to Exercise 2:

$$\begin{aligned}\text{Back bearing} &= 125 + 180\\ &= 305\end{aligned}$$

Solution to Exercise 3(a): Angle at Q = 130° - 40° = 90°
Solution to Exercise 3(b): Since angle = 90°:

$$\begin{aligned}PR^2 &= 60^2 + 80^2\\ &= 3600 + 6400\\ &= 10000\\ PR &= 100 \text{ km}\end{aligned}$$

Solution to Exercise 4:

$$\begin{aligned}\tan(38) &= \frac{h}{80}\\ h &= 80 \times \tan(38)\\ &= 80 \times 0.781\\ &= 62.5 \text{ m}\end{aligned}$$

Solution to Exercise 5:

$$\begin{aligned}\tan(15) &= \frac{50}{d}\\ d &= \frac{50}{\tan(15)}\\ &= \frac{50}{0.268}\\ &= 186.6 \text{ m}\end{aligned}$$

Solution to Exercise 6:

$$\begin{aligned}\sin(65) &= \frac{h}{12}\\ h &= 12 \times \sin(65)\\ &= 12 \times 0.906\\ &= 10.9 \text{ m}\end{aligned}$$

Solution to Exercise 7:

$$\begin{aligned}\Delta\lambda &= 65 - 20 = 45\\ r &= 6400 \times \cos(40)\\ &= 6400 \times 0.766\\ &= 4902.4 \text{ km}\\ d &= \frac{2\pi r \times 45}{360}\\ &= \frac{2 \times 3.142 \times 4902.4 \times 45}{360}\\ &= 3864.5 \text{ km}\end{aligned}$$

Solution to Exercise 8:

$$\begin{aligned}\Delta\phi &= 25 + 45 = 70\\ d &= 111 \times 70\\ &= 7770 \text{ km}\end{aligned}$$

Solution to Exercise 9:

$$\begin{aligned}\Delta\lambda &= 30 + 60 = 90\\ \text{Time difference} &= 90 \times 4 \text{ min}\\ &= 360 \text{ min} = 6 \text{ hours}\\ \text{Time at B} &= 10:00 + 6:00\\ &= 16:00 \text{ (4:00 PM)}\end{aligned}$$

6 Summary of Key Formulas

Important Formula

Bearings:

- Back bearing = Bearing ± 180°
- Use cosine/sine rules for distances

Angles of Elevation/Depression:

$$\begin{aligned}\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}}\\ \sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}}\\ \cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}}\end{aligned}$$

Longitude and Latitude:

$$\begin{aligned}\text{Along latitude: } d &= 111 \times \cos(\phi) \times \Delta\lambda \text{ km}\\ \text{Along longitude: } d &= 111 \times \Delta\phi \text{ km}\\ \text{Time difference} &= \Delta\lambda \times 4 \text{ minutes}\end{aligned}$$

7 Examination Tips

Strategy for WAEC Exams

Bearings:

1. Always draw diagrams
2. Mark North direction at each point
3. Use appropriate triangle rules
4. Check if angles are at the correct vertex

Elevation/Depression:

1. Draw right-angled triangles
2. Label known sides and angles
3. Choose appropriate trig ratio
4. Check calculator is in degree mode
5. Include units in final answer

Longitude/Latitude:

1. Note if points are N/S of equator
2. Note if points are E/W of Prime Meridian
3. Add longitudes if on opposite sides of Prime Meridian
4. Subtract if on same side
5. Remember: 1° = 111 km on great circle
6. Use $r = R\cos(\phi)$ for latitude circles
7. Time: East is ahead, West is behind

End of Lecture Notes

Practice with diagrams - visualization is key!