There are two ways of viewing Morse theory. The classical way is essentially just a proof that one can prove that a smooth manifold admits a cell decomposition by using a Morse function.

**Definition 1.** Let X be a compact manifold. A function

$$f:X\to\mathbb{R}$$

is Morse if  $df \pitchfork 0_X$  (the intersection is transverse, see transverse intersection/Definition 1.)

This is based on [schwarzMorseHomology1993].