This is based on [DK97]. Let V be a four dimensional real vector space with an inner product $\langle , \rangle$. Let $\not{S}^{\pm}$ denote the spinor bundles associated to V.

Definition 1. Let $\mathcal{H}$ be a Hermitian inner product space of dimension k, and E_∞ be a Hermitian inner product space of dimension n with determinant form. Let

$$T \in V^* \otimes \mathfrak{u}(k)$$

and $P \in \mathrm{Hom}(E \to \not{S}^+ \otimes \mathcal{H})$. (By $\mathrm{Hom}(E, \cdot)$ we mean in the category of inner product spaces, so P is isometric.) Then the tuple $(V, \mathcal{H}, E, T, P)$ is called preADHM data.

Definition 2. Fix some preADHM data $(V, \mathcal{H}, E, T, P)$. Then we can define a map

$$R : \not{S}^- \otimes \mathcal{H} \oplus E \to \not{S}^+ \otimes \mathcal{H}$$

given by

$$R = \sum_{i=1}^{4} (T_i - x_i I) \otimes \gamma^*(e_i) \oplus P$$

Definition 3. The preADHM data $(V, \mathcal{H}, E, T, P)$ is ADHM if for all $x \in V$ the map R given in Definition 2 is surjective, and T, P satisfies the moment map condition

$$\mu(T, P) = [T, T]^+ + (PP^*)^+ = 0.$$

Theorem 4 ([Ati+78])**.** There is a one to one correspondence between ADHM data (up to natural equivalence) and ASD instantons on $\mathbb{R}^4$.

To prove one direction, we can define an n dimensional bundle over $\mathbb{R}^4$ as the kernel of R_x (where we view R as a map between trivial bundles, depending on the parameter x). Then the induced connection, where we pick the trivial flat connection on $\mathbb{R}^4$, in the sense of `InducedConnections/`Definition 1

References

[Ati+78] M. F. Atiyah et al. “Construction of Instantons”. In: *Physics Letters A* 65 (Mar. 1, 1978), pp. 185–187. ISSN: 0375-9601. DOI: `10.1016/0375-9601(78)90141-X`. URL: `https://ui.adsabs.harvard.edu/abs/1978PhLA...65..185A` (visited on 02/14/2023).

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