Chapter 1

Physics Flow

1.1 Definition

Definition 1.1.1 (Pure separable state and pure entangled state)**.** *Define* $|\Psi\rangle_{AB}$ *as the joint state of the composite system A and B, then*

- $|\Psi\rangle_{AB}$ *is separable if and only if* $\exists$ $|\Psi\rangle_A$, $|\Psi\rangle_B$ *such that* $|\Psi\rangle_{AB} = |\Psi\rangle_A \bigotimes |\Psi\rangle_B$.
- $|\Psi\rangle_{AB}$ *is entangled if and only if* $|\Psi\rangle_{AB}$ *is Not separable.*

Example 1.1.1. *Entangled: EPR Pair* $|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$.

Separable:

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \tag{1.1}$$

$$= \frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A) \bigotimes |1\rangle_B. \tag{1.2}$$

Definition 1.1.2 (Mixed separable state and mixed entangled state)**.** *Define* ρ_{AB} *as the joint state of the composite system A and B, then*

- ρ_{AB} *is separable if and only if* $\exists$ $\left\{p_j, \rho_A^j, \rho_B^j\right\}$ *such that* $\rho_{AB} = \sum_j p_j \rho_A^j \bigotimes \rho_B^j$.
- ρ_{AB} *is entangled if and only if* ρ_{AB} *is Not separable.*

Example 1.1.2.
- $\rho_{AB} = \frac{1}{2}|0\rangle\langle 0|_A \bigotimes |1\rangle\langle 1|_B + \frac{1}{2}|+\rangle\langle +|_A \bigotimes |-\rangle\langle -|_B$ *is separable*
- *Classical-quantum states,* $\rho_{AB} = \sum_j p_j |j\rangle\langle|_A \bigotimes \rho_B^j$
- *EPR Pair*

$$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \tag{1.3}$$

it is computational hard to test whether a given state is entangled or separable.

1.2 Theorem