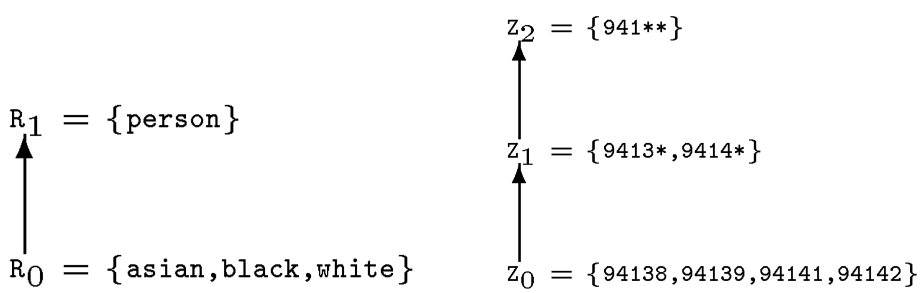
Methodology, Ethics and Practice of Data Privacy
Course Exercise #1

April 13 2021

1. (10') Try to explain why recursive (c, l)-diversity guards against all adversaries who possess at most $l - 2$ statements of the form "Bob does not have heart disease".

2. (15') Consider domains R_0 (Race) and Z_0 (ZIP code) whose generalization hierarchies are illustrated in Fig. 1a and Fig. 1b independently. Assume $QI = \{\text{Race, ZIP}\}$ to be a quasi-identifier. Consider private table PT illustrated in table 1, please give all possible 2-anonymity using **full domain generalization** and **suppression** under the condition that the maximum number of suppressed records ($MaxSup$) is less than or equal to 1. (If it is not generalized, 4 records need to be suppressed, which does not meet the requirement of $MaxSup \leq 1$, illustrated in table 2).

3. (15') [The t-closeness Principle] An equivalence class is said to have t-closeness if the distance between the distribution of a sensitive attribute in this class and distribution of the attribute in the whole table is no more than a threshold t. A table is said to have t-closeness if all equivalence classes have t-closeness.



(a) The generalization hierarchy of R_0 (b) The generalization hierarchy of Z_0

Figure 1: Generalization hierarchies

Race:R_0	ZIP:Z_0
asian	94138
asian	94138
asian	94142
asian	94142
black	94138
black	94141
black	94142
white	94138

Table 1: PT

Race:R_0	ZIP:Z_0
asian	94138
asian	94138
asian	94142
asian	94142

Table 2: Suppression for table PT

(a) Given the anonymized table (table 3), where the quasi-identifier attributes are *ZIP Code* and *Age* and the sensitive attribute is *Salary*. Please give the value of t so that table 3 satisfies t-closeness. Please use **Earth Mover's distance (EMD)** to calculate the distance between two distributions.

Hint. The overall distribution of the Income attribute is $\boldsymbol{Q} = \{3k, 4k, 5k, 6k, 7k, 8k, 9k, 10k, 11k\}$ (We use the notation $\{v_1, v_2, \cdots, v_m\}$ to denote the uniform distribution where each value in $\{v_1, v_2, \cdots, v_m\}$ is equally likely.) The first equivalence class in table 3 has distribution $\boldsymbol{P}_1 = \{3k, 5k, 9k\}$.

[Earth Mover's distance (EMD)]. The *Salary* is the numerical attribute. Numerical attribute values are ordered. Let the attribute domain be $\{v_1, v_2, \cdots, v_m\}$, where v_i is the i^{th} smallest value. Let $\boldsymbol{P} = \{p_1, p_2, \cdots, p_m\}$ and $\boldsymbol{Q} = \{q_1, q_2, \cdots, q_m\}$ be distributions. we use *Ordered Distance* to calculate the distance between two values. Let $r_i = p_i - q_i (i = 1, 2, \cdots, m)$, then EMD between $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be calculate as:

$$\begin{aligned} D[\boldsymbol{P}, \boldsymbol{Q}] &= \frac{1}{m-1}(|r_1| + |r_1 + r_2| + \cdots + |r_1 + r_2 + \cdots + r_{m-1}|) \\ &= \frac{1}{m-1}\sum_{i=1}^{m} |\sum_{j=1}^{i} r_j| \end{aligned} \tag{1}$$

[Ordered Distance] *Ordered Distance* between two values is based on the number of values between them in the total order, i.e., $ordered\_list(v_i, v_j) = \frac{|i-j|}{m-1}$.

4. (25') Given the following private table (table 4):

   Please answer the following questions:

   (a) (5') Given the health condition as the sensitive attribute, please name the quasi-identifier attributes.

ZIP Code	Age	Salary
4767*	≤ 40	3K
4767*	≤ 40	5K
4767*	≤ 40	9K
4790*	≥ 40	6K
4790*	≥ 40	11K
4790*	≥ 40	8K
4760*	≤ 40	4K
4760*	≤ 40	7K
4760*	≤ 40	10K

Table 3: The anonymized table.

Name	Age	Gender	Nationality	Salary	Condition
Ann	35	F	Japanese	40K	Viral Infection
Bluce	27	M	American	38K	Flu
Cary	41	F	India	45K	Heart Disease
Dick	32	M	Korean	38K	Flu
Eshwar	52	M	Japanese	61K	Heart Disease
Fox	22	M	American	22K	Flu
Gary	36	M	India	34K	Flu
Helen	26	F	Chinese	26K	Cancer
Irene	18	F	American	16K	Viral Infection
Jean	25	F	Korean	38K	Cancer
Ken	38	M	American	55K	Viral Infection
Lewis	47	M	American	64K	Heart Disease
Martin	24	M	American	37K	Viral Infection

Table 4: Private table.

(b) (15') Let the valid range of age be $\{0, \cdots, 120\}$. Given the health condition as the sensitive attribute, design a cell-level generalization solution to achieve k-Anonymity, where $k = 2$. Please give the generalization hierarchies, released table and calculation of the loss metric (LM) of your solution.

(c) (5') Please design a k-anonymization algorithm to optimize the loss metric.

5. (20') Suppose that private information x is a number between 0 and 1000. This number is chosen as a random variable X such that 0 is 1%-likely whereas any non-zero is only about 0.1%-likely:

$$P[X = 0] = 0.01, P[X = k] = 0.00099, k = 1 \cdots 1000 \tag{2}$$

Suppose we want to randomize such a number by replacing it with a new random number $y = R(x)$ that retains some information about the original number x. Here are three possible methods to do it:

(a) Given x, let $R_1(x)$ be x with 20% probability, and some other number (chosen uniformly at random in $\{0, \cdots, 1000\}$) with 80% probability.

(b) Given x,let $R_2(x)$ be $(x + \delta) mod\ 1001$, where δ is chosen uniformly at random in $\{-100 \cdots 100\}$.

(c) Given x, let $R_3(x)$ be $R_2(x)$ with 50% probability, and a uniformly random number in $\{0, \cdots, 1000\}$ otherwise.

Please answer the following questions:

(a) (15') Compute prior and posterior probabilities of two properties of X: 1) $X = 0$; 2) $X \in \{200, \cdots, 800\}$ using the above three methods respectively. The posterior probabilities only need to be computed when $R_i(X) = 0,\ i = 1, 2, 3$, respectively.

(b) (5') Which method is better? Why?

6. (15') [(α, β)-Privacy] Let R be an algorithm that takes as input $u \in D_U$ and outputs $v \in D_V$. R is said to allow an upward (α, β)-privacy breach with respect to a predicate ϕ if for some probability distribution f,

$$\exists u \in D_U, \exists v \in D_V\ s.t.\ P_f(\Phi(u)) \le \alpha\ and\ P_f(\Phi(u)|R(u) = v) \ge \beta \tag{3}$$

Similarly, R is said to allow a downward (α, β)-privacy breach with respect to a predicate Φ if for some probability distribution f,

$$\exists u \in D_U, \exists v \in D_V\ s.t.\ P_f(\Phi(u)) \ge \beta\ and\ P_f(\Phi(u)|R(u) = v) \le \alpha \tag{4}$$

R is said to satisfy (α, β)-privacy if it does not allow any (α, β)-privacy breach for any predicate Φ. The necessary and sufficient conditions for R to satisfy (α, β)-privacy for any prior distribution and any property ϕ: γ-amplifying

$$\forall v \in D_V, \forall u_1, u_2 \in D_U, \frac{P(R(u_1) = v)}{P(R(u_2) = v)} \le \gamma \tag{5}$$

(a) Let R be an algorithm that is γ-amplifying. Please proof that R does not permit an (α, β)-privacy breach for any adversarial prior distribution if

$$\gamma \le \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}. \tag{6}$$