$$\frac{k!}{i!\,(k-i)!}$$

一. Possion 分布

$$P(X=k)=e^{-\lambda}\cdot\frac{\lambda^k}{k!}$$

有① $\underset{(\lambda_1)}{X_1}+\underset{(\lambda_2)}{X_2} \sim P(\lambda_1+\lambda_2)$

证明: $$P(X_1+X_2=k)=\sum_{i=0}^{k} P(X_1=i,\ X_2=k-i) = \sum_{i=0}^{k} e^{-\lambda_1}\frac{(\lambda_1)^i}{i!}\cdot e^{-\lambda_2}\frac{(\lambda_2)^{k-i}}{(k-i)!} = e^{-(\lambda_1+\lambda_2)}\cdot\frac{\sum_{i=0}^{k} C_k^i\,\lambda_1^i(\lambda_2)^{k-i}}{k!}$$
$$= e^{-(\lambda_1+\lambda_2)}\cdot\frac{(\lambda_1+\lambda_2)^k}{k!}$$

② $EX=\lambda$, $Var(X)=\lambda$ 或 $\lambda\underbrace{\sum_{k=0}^{\infty} e^{-\lambda}\cdot\frac{\lambda^k}{k!}}_{\text{概率和}}=\lambda\cdot 1=\lambda$

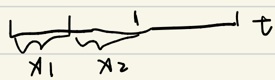
I: $$EX=\sum_{k=0}^{\infty} k\cdot e^{-\lambda}\cdot\frac{\lambda^k}{k!} = e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!} = \lambda e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!} = \lambda e^{-\lambda}\cdot e^{\lambda}=\lambda$$

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$$

II: $$Var(X)=EX^2-(EX)^2=\sum_{k=0}^{\infty} k^2\cdot e^{-\lambda}\cdot\frac{\lambda^k}{k!}-\lambda^2 = \sum_{k=1}^{\infty} k\cdot e^{-\lambda}\cdot\frac{\lambda^k}{(k-1)!}-\lambda^2$$
$$=\sum_{k=0}^{\infty}(k+1)e^{-\lambda}\cdot\frac{\lambda^{k+1}}{k!}-\lambda^2 = \underbrace{\sum_{k=1}^{\infty}\frac{\lambda^{k+1}}{(k-1)!}\cdot e^{-\lambda}}_{=\lambda^2\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}} + \underbrace{\sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^{k+1}}{k!}}_{=\lambda\sum_{k=0}^{\infty} e^{-\lambda}\cdot\frac{\lambda^k}{k!}} - \lambda^2$$
$$=\lambda^2+\lambda-\lambda^2=\lambda$$

二. Poisson 过程.

$$P(N(t)=k)=e^{-\lambda t}\cdot\frac{(\lambda t)^k}{k!}$$



$P(X>t\mid X>s)=P(X>t-s)$ (指数分布的无记忆性)

$$P(X_1\le t)=1-P(X_1>t)=1-P(\text{在}[0,t]\text{里发生0个})=1-e^{-\lambda t}$$

$$P(X_2\le s\mid X_1=t)=P(\text{在}[t,t+s]\text{内发生了}1,2,3,\cdots)$$
$$=P(\text{在}[0,s]\text{内发生了}1,2,3\cdots)=1-P(\text{在}[0,s]\text{内发生 0次})$$
$$=1-e^{-\lambda s}$$ (见书P17中部)

$(t>s)$

$$P(X_1\le s \mid N(t)=1) = \frac{P\{X_1\le s,\ N(t)=1\}}{P(N(t)=1)} = \frac{P\{[0,s]间有一个，\underbrace{[s,t]}_{[0,s-t]}间没有\}}{P(N(t)=1)}$$

⇐ 独立增量 定理2.1

$$= \frac{\lambda s\, e^{-\lambda s}\cdot e^{-\lambda(t-s)}}{\lambda t\, e^{-\lambda t}} = \frac{s}{t}$$

类比 n个均匀分布的 s_i 的顺序统计量.

$$(s_1,\cdots,s_n) \Rightarrow n!\,\frac{1}{t^n} \quad (有\ n!\ 种排列)$$

非齐(次)时 Poisson 分布