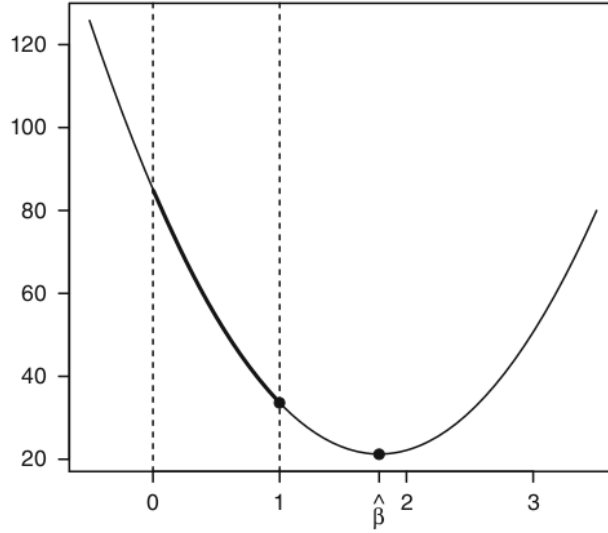


1.

$$\begin{aligned} i) \quad \mathbf{C} &= (0, 5, -7, 0), & d &= -2 \\ ii) \quad \mathbf{C} &= (0, 2, -1, -1, 0), & d &= 0 \end{aligned}$$

2. This plot shows the SSE in function of β :



The F-test score is $\frac{SSE_{H_0} - SSE}{SSE}$ where $SSE = SSE(\hat{\beta})$ that is the y in the plot where $x = \hat{\beta}$ and $SSE_{H_0} = SSE(\beta = 1)$

The plot for the numerator of Wald test is the same of the previous because the numerator is the same except for a square difference instead of normal difference.

The main difference between F-test and Wald test is the denominator in the first is $\hat{\theta}$ while in the second is $var(\hat{\theta})$. There is also a relation between test statistics F and W : $W = rF$

3. The confidence interval for μ_0 is $\mathbf{x}'_0 \hat{\beta} \pm t_{n-p}(1 - \alpha/2) \hat{\sigma}(\mathbf{x}'_0(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)^{1/2}$ while for prediction is $\mathbf{x}'_0 \hat{\beta} \pm t_{n-p}(1 - \alpha/2) \hat{\sigma}(1 + \mathbf{x}'_0(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)^{1/2}$. The difference between the two formulas is the "1 +" near x_0 that become from the fact that the estimation try to give an confidence interval of a parameter , so with no variance, while the prediction try to estimate a aleatory variable with a expected value and a variance.

4. $f(x_0) = \mathbf{x}'_0 \hat{\beta} \pm t_{n-2}(1 - \alpha/2) \hat{\sigma}(1 + \mathbf{x}'_0(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)^{1/2}$ where $X = \begin{pmatrix} 1 & x_{11} \\ \vdots & \vdots \\ 1 & x_{n1} \end{pmatrix}$

and $p = 2$.

The length of interval is minimum when $\mathbf{x}_0 = \mathbf{0}$

5. Increasing the number of parameters is not always a good idea: we can ex-

press the SPSE (expected squared prediction error as):

$$\underbrace{n\sigma^2}_{\text{irreducible part}} + \underbrace{|M|\sigma^2}_{\text{Variance error}} + \underbrace{\sum_{i=1}^n (u_{iM} - u_i)^2}_{\text{BIAS}^2}$$

6. If we consider only \mathbf{X}_1 we have $\mathbf{X} = \mathbf{X}_1$ then

$$\begin{aligned} E[\tilde{\beta}_1] &= E[(\mathbf{X}_1' \mathbf{X}_1)^{-1} \mathbf{X}_1' \mathbf{y}] \\ &= (\mathbf{X}_1' \mathbf{X}_1)^{-1} \mathbf{X}_1' E[\mathbf{y}] \\ &= (\mathbf{X}_1' \mathbf{X}_1)^{-1} \mathbf{X}_1' (\mathbf{X}_1 \beta_1 + \mathbf{X}_2 \beta_2) \\ &= \beta_1 + (\mathbf{X}_1' \mathbf{X}_1)^{-1} \mathbf{X}_1' \mathbf{X}_2 \beta_2 \end{aligned}$$

The bias vanish if $\mathbf{X}_1' \mathbf{X}_2 = \mathbf{0}$ that in statcal words mean that \mathbf{X}_1 and \mathbf{X}_2 are uncorrelated