Week 6 exercises

April 14, 2018

1. The linear probability model for binary response is simply $P(y=1) = \pi_i = \boldsymbol{x_i'\beta} = \beta_{0i} + \beta_{1i}x_{1i} + \cdots + \beta_{ni}x_{ni}$ and require the restriction that $0 \leq \boldsymbol{X'\beta} \leq 1$

2. In the GLM we combine the probability output to the linear prediction through a function h called *response function* that it is a cumulative distribution function with co domain in [0,1]. In formula we can express the GLM as

$$P(y=1) = \pi_i = h(\eta_i) = h(\mathbf{x_i'}\beta) = h(\beta_{0i} + \beta_{1i}x_{1i} + \cdots + \beta_{ni}x_{ni})$$

$g = h^{-1}$ is the *link function* and it is used to calculate the linear predictor in function of probability: $\eta_i = g(\pi_i)$ The logit model use as response function the logistic function:

$$\pi = h(\eta) = \frac{e^{\eta}}{1+e^{\eta}}$$

. The linear predictor returns the log odds

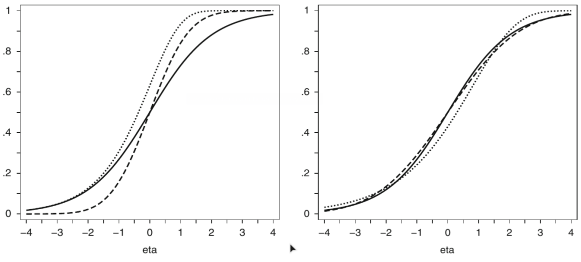
$$\mathbf{x_i'}\beta = \beta_{0i} + \beta_{1i}x_{1i} + \cdots + \beta_{ni}x_{ni} = \pi_i = \log\left(\frac{\pi}{1-\pi}\right)$$

. The probit model use instead a normal distribution cumulative function. The c-log-log model use as response function the extreme minimum-value cumulative distribution function

$$h(\eta) = 1 - e^{-e^{\eta}}$$

with the following link function

$$g(\pi) = \log(-\log(1-\pi))$$



Response functions (*left*) and adjusted response functions (*right*) in binary regression models: logit model (—), probit model (- - -), complementary log–log model (···)

3. From the latter plot we can note that the *probit* and *logit* are symmetric around the 0 while the c-log-log is not symmetric. The c-log-log is similar to the logit but tend more speedly to one. If we do a comparison with the same variance between the logit and probit we have that the coefficients differ for a values of $\frac{\pi}{3} = 1.814$

4. With a latent continuous variable we use the standard normal distribution for the errors because we don't know the variance of the latent variable so the new coefficients are $\tilde{\beta} = \frac{\beta}{\sigma}$. We can't calculate the original β since doesn't know σ but the ratio between coefficients are constants $\dfrac{\tilde{\beta}_i}{\tilde{\beta}_j} = \dfrac{\beta_i}{\beta_j}$

5. In the logit model if increase x_k by 1 we have an increments on the odds by e^{β} while the increment of probability is not the same in every point. In the probit model we have an increment equal to $\phi^{-1}(\beta)$

Solution to applied exercise

1.

$$\eta = 0.42+0.06\cdot kidsge6-1.44\cdot kidslt6-0.09\cdot age+0.21\cdot exper-0.0031\cdot exper^2+0.22\cdot educ-0.021\cdot nwifeinc$$

$$P(y=1) = \hat{\pi} = h(\hat{\eta}) = \frac{e^{\hat{\eta}}}{1+e^{\hat{\eta}}}$$

2.

$$0.42+0.06\cdot 0-1.44\cdot 0-0.09\cdot 40+0.21\cdot 0-0.0031\cdot 0+0.22\cdot 10-0.021\cdot 0 = -0.98$$

3. one year of more eduction when all the other variables are the same is a multiplicative effect of $e^{0.22}$

4. The probability is 0.27

5. -0.0174

6. 0.046

7. $0.22 * 0.5^2 = 0.055$

8. The probit coefficients are obtained by dividing by 1.84