

endless variety and unlimited complication. Nor can its existence lie within the multitude of computer printouts that begin to capture some of its incredible sophistication and detail, for at best those printouts capture but a shadow of an approximation to the set itself. Yet it has a robustness that is beyond any doubt; for the same structure is revealed—in all its perceivable details, to greater and greater fineness the more closely it is examined—independently of the mathematician or computer that examines it. Its existence can only be within the Platonic world of mathematical forms.

I am aware that there will still be many readers who find difficulty with assigning any kind of actual existence to mathematical structures. Let me make the request of such readers that they merely broaden their notion of what the term ‘existence’ can mean to them. The mathematical forms of Plato’s world clearly do not have the same kind of existence as do ordinary physical objects such as tables and chairs. They do not have spatial locations; nor do they exist in time. Objective mathematical notions must be thought of as timeless entities and are not to be regarded as being conjured into existence at the moment that they are first humanly perceived. The particular swirls of the Mandelbrot set that are depicted in Fig. 1.2c or 1.2d did not attain their existence at the moment that they were first seen on a computer screen or printout. Nor did they come about when the general idea behind the Mandelbrot set was first humanly put forth—not actually first by Mandelbrot, as it happened, but by R. Brooks and J. P. Matelski, in 1981, or perhaps earlier. For certainly neither Brooks nor Matelski, nor initially even Mandelbrot himself, had any real conception of the elaborate detailed designs that we see in Fig. 1.2c and 1.2d. Those designs were already ‘in existence’ since the beginning of time, in the potential timeless sense that they would necessarily be revealed precisely in the form that we perceive them today, no matter at what time or in what location some perceiving being might have chosen to examine them.

1.4 Three worlds and three deep mysteries

Thus, mathematical existence is different not only from physical existence but also from an existence that is assigned by our mental perceptions. Yet there is a deep and mysterious connection with each of those other two forms of existence: the physical and the mental. In Fig. 1.3, I have schematically indicated all of these three forms of existence—the physical, the mental, and the Platonic mathematical—as entities belonging to three separate ‘worlds’, drawn schematically as spheres. The mysterious connections between the worlds are also indicated, where in drawing the diagram

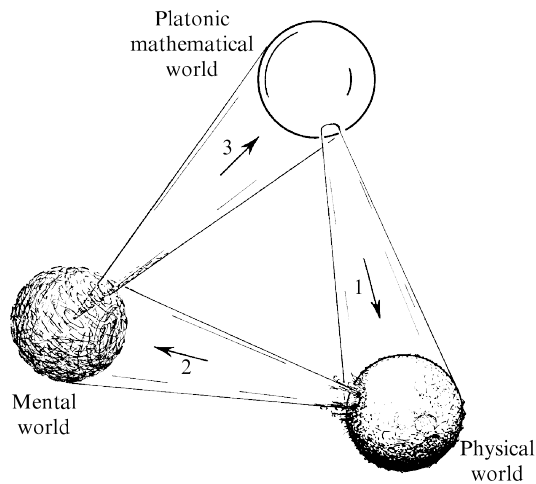


Fig. 1.3 Three 'worlds'—the Platonic mathematical, the physical, and the mental—and the three profound mysteries in the connections between them.

I have imposed upon the reader some of my beliefs, or prejudices, concerning these mysteries.

It may be noted, with regard to the *first* of these mysteries—relating the Platonic mathematical world to the physical world—that I am allowing that only a small part of the world of mathematics need have relevance to the workings of the physical world. It is certainly the case that the vast preponderance of the activities of pure mathematicians today has no obvious connection with physics, nor with any other science (cf. §34.9), although we may be frequently surprised by unexpected important applications. Likewise, in relation to the *second* mystery, whereby mentality comes about in association with certain physical structures (most specifically, healthy, wakeful human brains), I am not insisting that the majority of physical structures need induce mentality. While the brain of a cat may indeed evoke mental qualities, I am not requiring the same for a rock. Finally, for the *third* mystery, I regard it as self-evident that only a small fraction of our mental activity need be concerned with absolute mathematical truth! (More likely we are concerned with the multifarious irritations, pleasures, worries, excitements, and the like, that fill our daily lives.) These three facts are represented in the smallness of the base of the connection of each world with the next, the worlds being taken in a clockwise sense in the diagram. However, it is in the encompassing of each entire world within the scope of its connection with the world preceding it that I am revealing my prejudices.

Thus, according to Fig. 1.3, the entire physical world is depicted as being governed according to mathematical laws. We shall be seeing in later chapters that there is powerful (but incomplete) evidence in support of this contention. On this view, everything in the physical universe is indeed

governed in completely precise detail by mathematical principles—perhaps by equations, such as those we shall be learning about in chapters to follow, or perhaps by some future mathematical notions fundamentally different from those which we would today label by the term ‘equations’. If this is right, then even our own physical actions would be entirely subject to such ultimate mathematical control, where ‘control’ might still allow for some random behaviour governed by strict probabilistic principles.

Many people feel uncomfortable with contentions of this kind, and I must confess to having some unease with it myself. Nonetheless, my personal prejudices are indeed to favour a viewpoint of this general nature, since it is hard to see how any line can be drawn to separate physical actions under mathematical control from those which might lie beyond it. In my own view, the unease that many readers may share with me on this issue partly arises from a very limited notion of what ‘mathematical control’ might entail. Part of the purpose of this book is to touch upon, and to reveal to the reader, some of the extraordinary richness, power, and beauty that can spring forth once the right mathematical notions are hit upon.

In the Mandelbrot set alone, as illustrated in Fig. 1.2, we can begin to catch a glimpse of the scope and beauty inherent in such things. But even these structures inhabit a very limited corner of mathematics as a whole, where behaviour is governed by strict computational control. Beyond this corner is an incredible potential richness. How do I really feel about the possibility that all my actions, and those of my friends, are ultimately governed by mathematical principles of this kind? I can live with that. I would, indeed, prefer to have these actions controlled by something residing in some such aspect of Plato’s fabulous mathematical world than to have them be subject to the kind of simplistic base motives, such as pleasure-seeking, personal greed, or aggressive violence, that many would argue to be the implications of a strictly scientific standpoint.

Yet, I can well imagine that a good many readers will still have difficulty in accepting that all actions in the universe could be entirely subject to mathematical laws. Likewise, many might object to two other prejudices of mine that are implicit in Fig. 1.3. They might feel, for example, that I am taking too hard-boiled a scientific attitude by drawing my diagram in a way that implies that all of mentality has its roots in physicality. This is indeed a prejudice, for while it is true that we have no reasonable scientific evidence for the existence of ‘minds’ that do not have a physical basis, we cannot be completely sure. Moreover, many of a religious persuasion would argue strongly for the possibility of physically independent minds and might appeal to what they regard as powerful evidence of a different kind from that which is revealed by ordinary science.

A further prejudice of mine is reflected in the fact that in Fig. 1.3 I have represented the entire Platonic world to be within the compass of mentality. This is intended to indicate that—at least in principle—there are no mathematical truths that are beyond the scope of reason. Of course, there are mathematical statements (even straightforward arithmetical addition sums) that are so vastly complicated that no one could have the mental fortitude to carry out the necessary reasoning. However, such things would be *potentially* within the scope of (human) mentality and would be consistent with the meaning of Fig. 1.3 as I have intended to represent it. One must, nevertheless, consider that there might be other mathematical statements that lie outside even the potential compass of reason, and these would violate the intention behind Fig. 1.3. (This matter will be considered at greater length in §16.6, where its relation to Gödel’s famous incompleteness theorem will be discussed.)⁸

In Fig. 1.4, as a concession to those who do not share all my personal prejudices on these matters, I have redrawn the connections between the three worlds in order to allow for all three of these possible violations of my prejudices. Accordingly, the possibility of physical action beyond the scope of mathematical control is now taken into account. The diagram also allows for the belief that there might be mentality that is not rooted in physical structures. Finally, it permits the existence of true mathematical assertions whose truth is in principle inaccessible to reason and insight.

This extended picture presents further potential mysteries that lie even beyond those which I have allowed for in my own preferred picture of the world, as depicted in Fig. 1.3. In my opinion, the more tightly organized scientific viewpoint of Fig. 1.3 has mysteries enough. These mysteries are not removed by passing to the more relaxed scheme of Fig. 1.4. For it

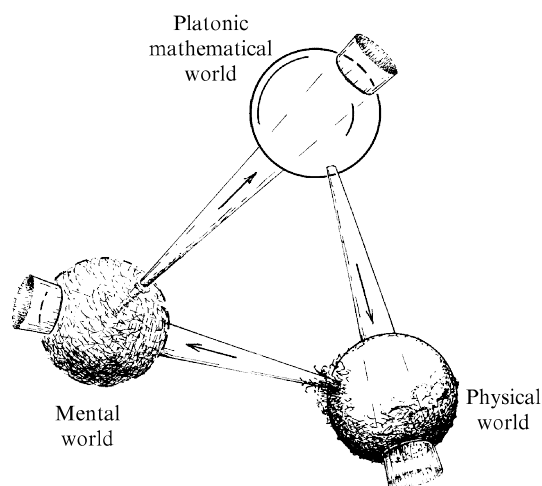


Fig. 1.4 A redrawing of Fig. 1.3 in which violations of three of the prejudices of the author are allowed for.