Definition

Subset

Notation: $A \subseteq B$, informally $A \subset B$

if $x \in A$, then $x \in B$

Definition

Proper Subet

Notation: $A \subset B$

$A \subseteq B$ and $A \neq B$

Definition

Union

Notation: $A \cup B$

$\{x \mid x \in A \text{ or } x \in B\}$

Definition

Intersection

Notation: $A \cap B$

$\{x \mid x \in A \text{ and } x \in B\}$

Definition

Disjoint Sets

A and B are disjoint if $A \cap B = \emptyset$

Definition

Equal Sets

A and B are equivalent if $A \subset B$ and $B \subset A$

Definition

Complement

For any set $A \subset U$

$$A' = \{x \in U \mid x \notin A\}$$

Definition

Difference

$$A \backslash B = A \cap B'$$

Definition

Cartesian Product

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Definition

Relation

A relation from A to B is a subset of $A \times B$

Definition

Function or Map

Notation: $f : A \to B$, where A is the *domain* and B is the *target*

A relation from A to B where $\forall a \in A, \exists!\ (a, b)$

$f(a) = b$ means $(a, b) \in f$

Definition

Image of Function

$f(A) = \{f(a) : a \in A\}$

Note: $f(A) \subset B$

DEFINITION

Surjective

Also known as *onto*

A function $f : A \to B$ for which $f(A) = B$

DEFINITION

Injective

Also known as *one-to-one*

A function $f : A \to B$ for which $f(a_1) = f(a_2)$ implies $a_1 = a_2$

DEFINITION

Bijective

A function which is both injective and surjective.

DEFINITION

Composition

For $f : A \to B$ and $g : B \to C$, the composition $g \circ f$ is defined as:

$$(g \circ f)(x) = g(f(x))$$

Notes: $g \circ f : A \to C$, and composition is associative

Definition

Composition Properties

- If g and f are surjective, $g \circ f$ is surjective
- If g and f are injective, $g \circ f$ is injective
- If g and f are bijective, $g \circ f$ is bijective

Definition

Identity Map

Notation: id_s

$$id(x) = x, id : S \to S$$

Note: $id_s = \{(x, x) : x \in S\}$

Definition

Inverse

Notation: f^{-1}

$$g = f^{-1} \iff g \circ f = id_A$$

f is invertible iff it has an inverse.

f is invertible iff it's bijective.