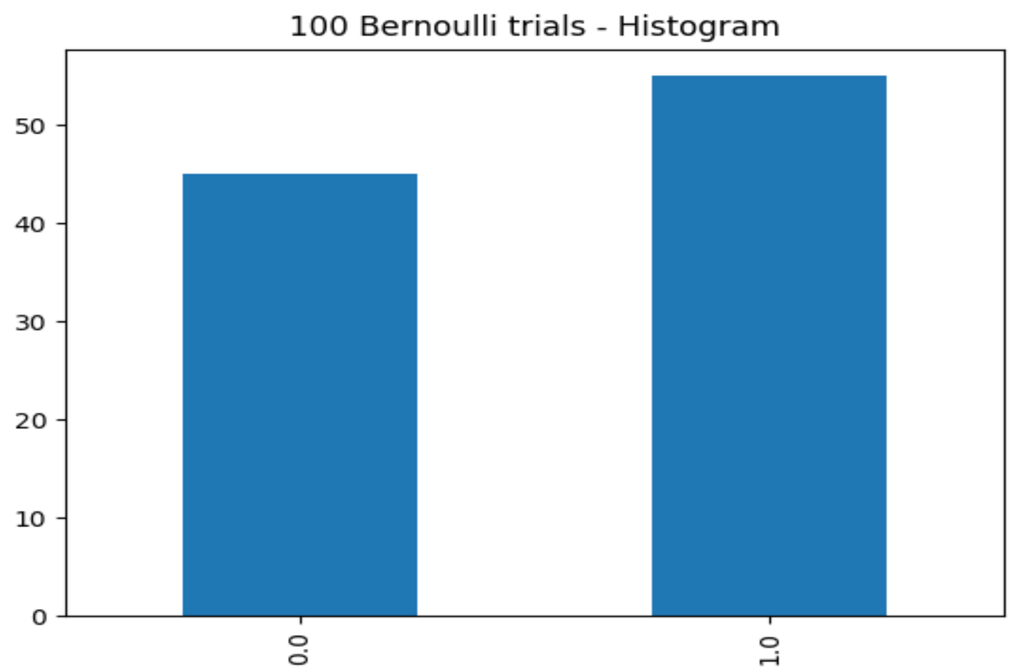
EE 511 – Simulation Methods for Stochastic Systems
Project # 1 – Abinaya Manimaran

Question 1:

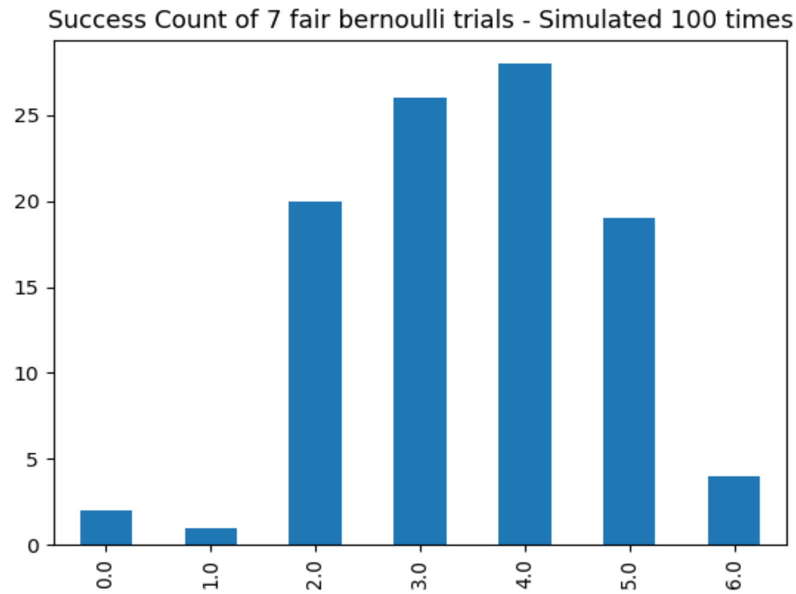
a) Simulation of fair Bernoulli trial: 100 times

- Generate random samples from uniform distribution between 0 and 1 – [0,1]
- For fair Bernoulli trials: probability to be set is 0.5 – This is the threshold
- For each value *x* drawn from uniform distribution, return Bernoulli 1 if $x <= p$
- Similarly return 0 if $x > p$, where p = 0.5
- This is repeated for 100 times and the histogram is given below
- The histogram has Bernoulli distribution



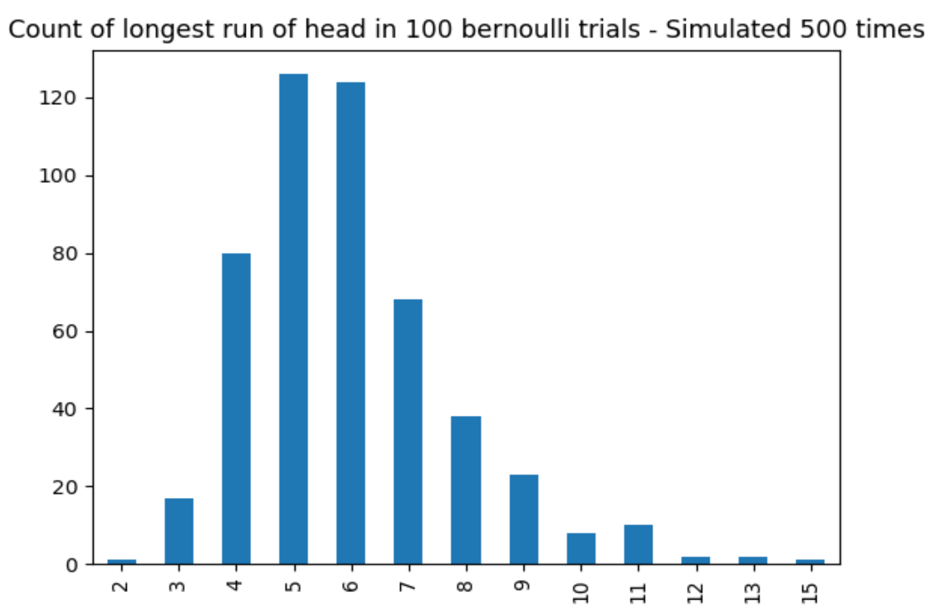
b) Routine to count the number of successes in 7 fair Bernoulli trials – 100 times

- Generate 7 random samples from uniform distribution between 0 and 1
- 7 Bernoulli samples are generated using these random uniform samples by setting a threshold probability as 0.5 (since fair Bernoulli trials)
- For each 7 fair Bernoulli trials – count the number of successes (or) sum the number of 1's
- This is repeated 100 times and the histogram is given below
- The random variable (count of number of successes) follows Binomial distribution. If we repeat that 100 times, we approximately get Normal Distribution (approximate mean $\mu = 4$)



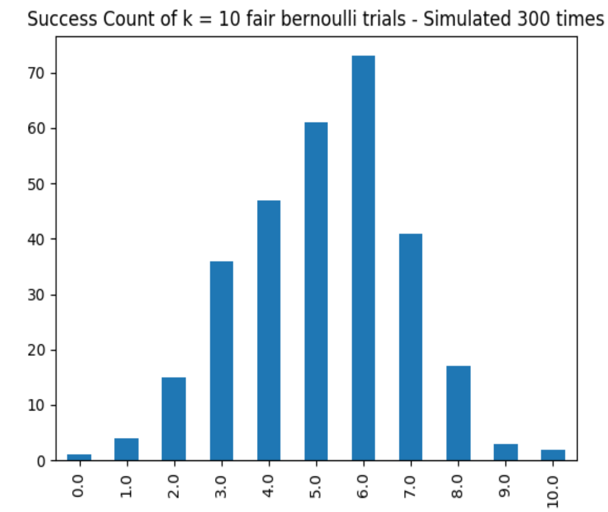
c) Routine to count the longest run of heads in 100 Bernoulli samples – 100 times

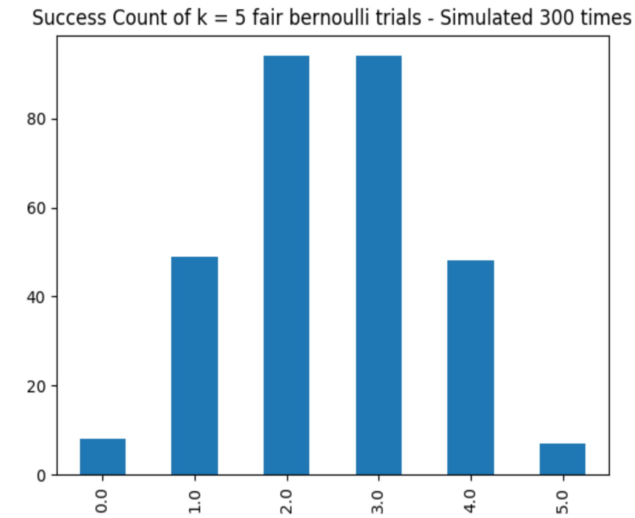
- Generate 100 random samples from uniform distribution between 0 and 1
- 100 Bernoulli samples are generated using these random uniform samples by setting a threshold probability as 0.5 (since fair Bernoulli trials)
- Find the longest run of heads in the 100 Bernoulli samples
- This is repeated 500 times and the histogram is given in below.
- The random variable (longest run of heads) follows Poisson Distribution (Treating 1's as the number of arrivals (approximate mean λ = 4)

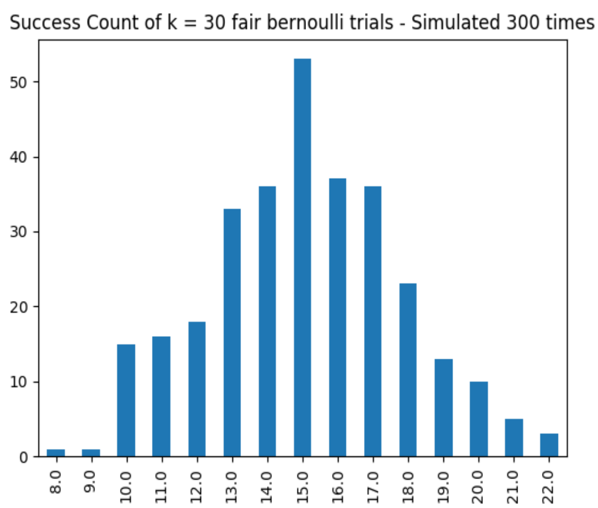


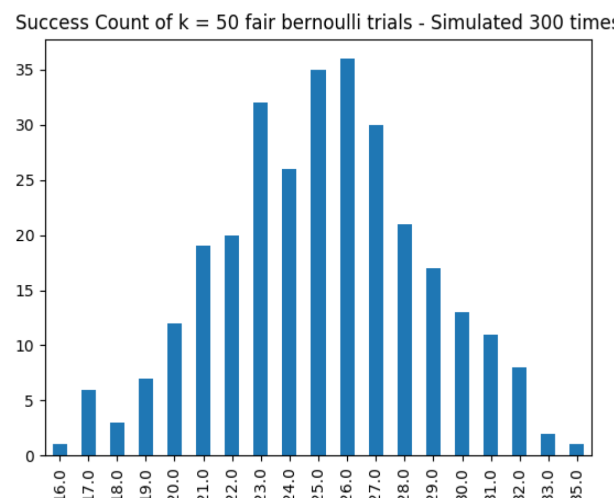
Question 2:

Repeat Bernoulli Success-counting random variable for various k values – 300 times









- Comparing the above histograms – as value of k increases the success count becomes more like a Normal Distribution.
- The range of the x axis increase as the value of k increases. This facilitates the range for Normal Distribution.
- We can see clearly from the histograms that the mean also keeps shifting to the right as the value of k increases - mean = 2, 6, 15 and 26.
- Therefore, Normal Distribution can be considered as approximation of Binomial Distribution

Question - 3

Given n = 20 people in a social network. Given any unordered pair of 2 people are connected at random and independently with success probability *p = 0.05*

a) Number of possible edges (N) is n Choose 2 = 190
b) For routine to select edges:
    - Consider selection of single edge as a Bernoulli random variable with success probability 0.05
    - Generate N uniform random variables between 0 and 1
    - N Bernoulli samples are generated using these random uniform samples by setting a threshold probability as 0.05 (success probability given)
c) Distribution of Random number of edges selected in this way:
    - Count the number of successes in this N Bernoulli samples. This will give the number of edges selected this way
    - Repeat this counting number of edges selected for 100, 500 and 1000 times
    - Histograms of these experiments are given below
    - The number of edges selected is a Binomial Distribution. But when the number of simulations are increased, Binomial distribution can be approximated to Normal Distribution. The mean is around 9 and is seen clearly in the last plot.