SAPIENZA
UNIVERSITÀ DI ROMA

NBD Homework 1

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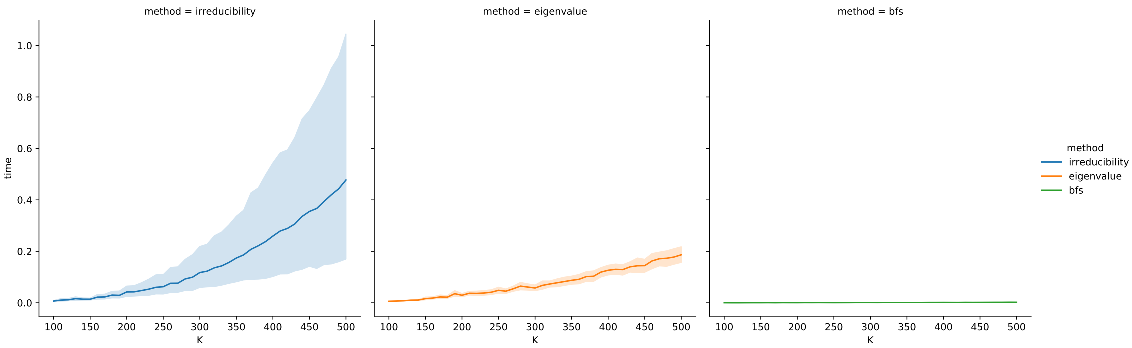
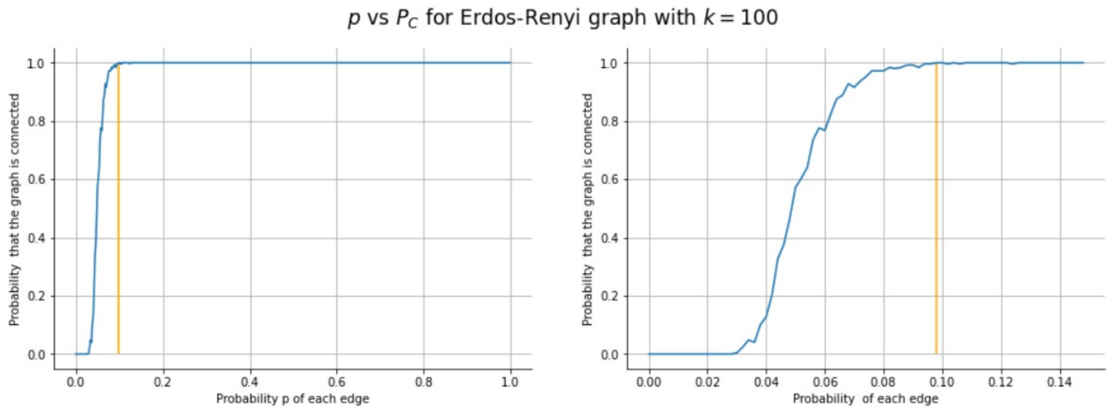
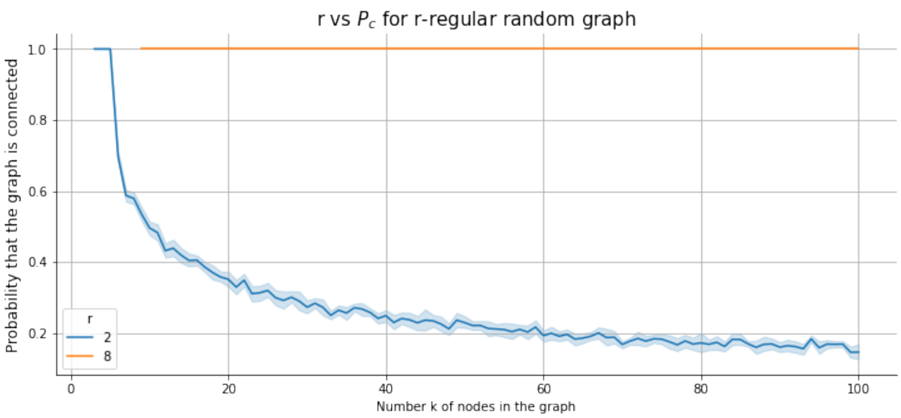
Assignment 1-Part 1



As we can see from the figures above ,the best method is the BFS which has a complexity measure of $(|N| + |E|)$ where $|N|$ denotes the number of nodes and $|E|$ the number of edges in the graph. BFS is followed by eigenvalues of the Laplacian matrix which has a complexity of $O(n^3)$, practically it can be reduced to $O(n^{2.376})$. The worst between the three is the irreducibility method ,which also has the highest complexity measure $(O(n^3))$ and, as it is shown in the plot, it has the highest variability ,displayed in the blue area.



For an Erdös-Rényi graph the probability that it is connected $(p_c(G))$ tends to 1 really quickly, when p is above a certain threshold $\widetilde{t} = 0.098$ the graph is connected with probability equal to 1.



For r-regular graphs instead we have two very different situations. When $r = 2, P_c(G)$ is a decreasing function that tends asymptotically to 0. With $r = 2$ the graph can't keep up with the creation of new nodes and maintain its connectivity while with an $r = 8$ instead even with the increasing number of nodes our graph is able to stay connected.

Assignment 1-Part 2

1. Equaling the number of resources (N,S,L) for Fat-tree and Jellyfish topology we obtain that r, the number of switch port in Jellyfish,is:

$$\begin{cases} S_j = S_F = \frac{5}{4}n^2 \\ N_j = N_F \Rightarrow (n-r)S_j = \frac{n^3}{4} \Rightarrow (n-r)\frac{5}{4}n^2 = \frac{n^3}{4} \Rightarrow 5n - 5r = n \Rightarrow r = \frac{4}{5}n \\ L_j = L_F \Rightarrow \frac{S_j r}{2} = \frac{n^3}{2} \Rightarrow \frac{5}{4}n^2 r = n^3 \Rightarrow r = \frac{4}{5}n \end{cases}$$

2. For an all-to-all traffic matrix each server communicates with every other server, implying that the number of flows generated is equal to $\nu_f = \frac{n^3(n^3-4)}{16}$.
Given this fact and knowing that the number of links between severs is $l = \frac{3}{4}n^3$ the formula for the upper bound of TH is:

$$TH \leq \frac{l}{\overline{h}\nu_f} = \frac{12}{\overline{h}(n^3-4)}$$

3. The mean shortest path lengths for server-to-server paths is

$$\overline{h} = \frac{2(\#servers\ in\ the\ same\ rack) + 4(\#servers\ in\ the\ same\ pod) + 6(\#servers\ of\ different\ pods)}{total\ number\ of\ servers - 1}$$

$$\overline{h} = \frac{6n^3 - 2n^2 - 4n - 8}{n^3 - 4}$$

n	N	S	L	$TH_{Fat-Tree}$	$TH_{Jellyfish}$
20	2000	500	4000	$2,5 * 10^{-4}$	$5,2 * 10^{-4}$
30	6750	1125	13500	$7.5 * 10^{-5}$	$1,5 * 10^{-4}$
40	16000	2000	32000	$3.15 * 10^{-5}$	$6,4 * 10^{-5}$
50	31250	3125	62500	$1.61 * 10^{-5}$	$3,2 * 10^{-5}$
60	54000	4500	108000	$9.31 * 10^{-6}$	$1,9 * 10^{-5}$

Table 1: Summary Table

As we expected the throughput for the Fat-Tree is much smaller than the one of Jellyfish due to the fact that the mean shortest path between servers $\overline{h}$ has proven to be smaller for the Jellyfish configuration.