



SAPIENZA  
UNIVERSITÀ DI ROMA

# **A Unified Framework for Distance Based Pairs Trading via Graph Laplacians**

## **Professors:**

Juri Marcucci

Daniele Bianchi

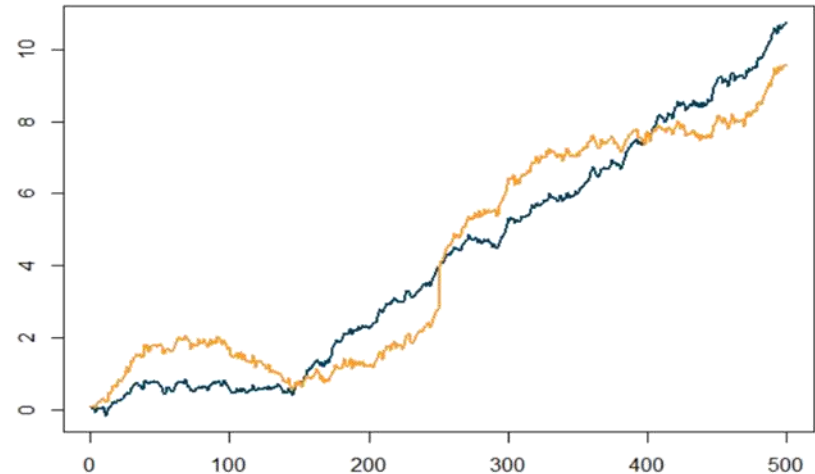
## **Student:**

Michelangelo Saveriano



## Introduction (pairs trading)

- Pairs trading idea:
  1. first find two securities whose prices move together
  2. when they diverge, buy the undervalued asset and sell the overvalued one
- In the quasi-multivariate framework, a security is traded against a basket of securities.
- In the distance-based approach various distance metrics are leveraged to identify comoving assets.





## Our contribution

- We presented a new unified framework where relationships between stocks are represented via a graphical structure.
- We introduced two novel approaches for the estimation of the comoving portfolios.
- We back-tested a total of 780 approaches to show the differences between the various techniques.
- We analyzed the evolution of the excess returns over time and how liquidity and market capitalization affect performance.



## Framework

Our framework is split in 3 phases:

1. Data preprocessing
2. Spreads evaluation
3. Trading



## Framework – Data Preprocessing

- Remove the effect of common risk factors when evaluating the idiosyncratic component of a stock
- We assume the following linear factor model:

$$r = \sum_i \beta_i f_i + \epsilon$$

- We compute the spreads using the residuals  $\epsilon$  rather than the returns  $r$



## Framework – Spreads Evaluation

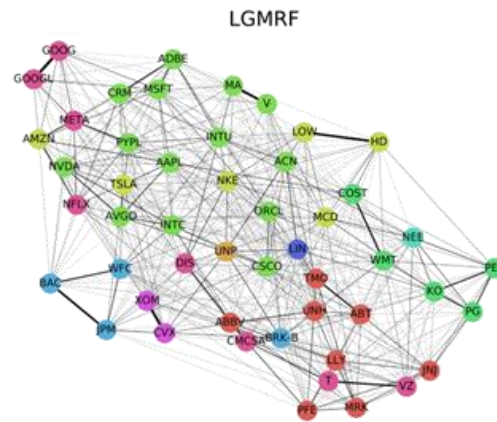
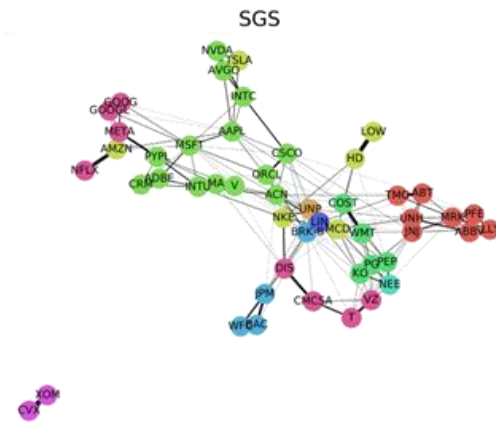
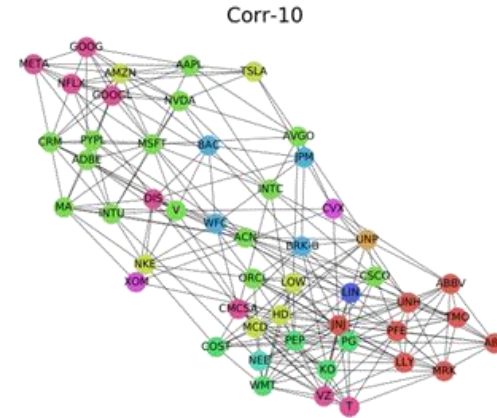
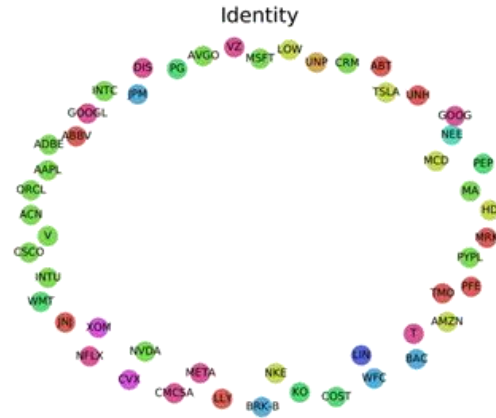
- The idiosyncratic component of a stock is represented as the spread between the returns of the stock and the ones of its comoving portfolio

$$s_t^T = r_t^T - r_t^T A = r_t^T L$$

1. Correlation-based approaches - Corr-K
2. Approaches based on the sparse inverse covariance:
  - Laplacian constrained Gaussian Markov Random Field (LGMRF)
  - Smooth Graph Signal (SGS) representation
3. Identity -  $L = I$



## Graph Visualization





## Framework - Trading

- We define the rolling spreads with window  $w$ :

$$s_t^{(w)} = \sum_{\tau=t-w}^t s_{\tau}$$

### Non-Parametric Threshold

- Given the rolling spreads standard deviation  $s_{std}^{(w)}$
- We long asset  $i$  if  $s_{t,i}^{(w)} \leq -\gamma \cdot s_{std,i}^{(w)}$
- We short asset  $i$  if  $s_{t,i}^{(w)} \geq \gamma \cdot s_{std,i}^{(w)}$

### Quantiles-based

- Given the amount of assets traded  $q$
- We long the  $q/2$  assets with the lowest spreads
- We short the  $q/2$  assets with the highest spreads





## Framework - Trading

### Quantiles-Std

- We long asset  $i$  if  $s_{t,i} \leq -\gamma \cdot s_{std,i}^{(w)}$  and it belongs to the subset of the  $q/2$  assets with the lowest spreads.
- We long asset  $i$  if  $s_{t,i} \geq \gamma \cdot s_{std,i}^{(w)}$  and it belongs to the subset of the  $q/2$  assets with the highest spreads.
- When  $\gamma = \text{None}$  we neglect the non-parametric threshold trading rule;
- When  $q = \text{None}$  we neglect the quantiles-based trading rule



## Results

- We utilized two datasets:
  - S&P500, 427 stocks from 1995 to 2022;
  - CRSP, 3218 stocks from 1961 to 2022.
- We tested a total of 780 configurations on the S&P500
- Metrics of interest:
  - Monthly Excess Returns;
  - Sharpe Ratio;
  - Sortino Ratio.
- Empirical distributions and 95% confidence intervals are computed using non-parametric bootstrap.



## Results – Preprocessing method

- **Returns vs Residuals:** The excess returns generated using residuals are less risky.

	Excess Returns	Sharpe Ratio	Sortino Ratio
Method			
residuals	0.82% (0.78%-0.86%)	<b>0.47</b> (0.45-0.49)	<b>0.76</b> (0.72-0.81)
returns	<b>0.86%</b> (0.8%-0.91%)	0.36 (0.34-0.39)	0.58 (0.54-0.62)



## Results – Spreads method

- **Identity vs Corr-50 vs LGMRF vs SGS:** Corr-K and LGMRF can considerably reduce the risk of the strategy.

	Excess Returns	Sharpe Ratio	Sortino Ratio
Method			
<b>Corr-50</b>	0.87% (0.76%-0.99%)	<b>0.47</b> (0.42-0.53)	<b>0.79</b> (0.68-0.91)
<b>Identity</b>	<b>0.99%</b> (0.83%-1.17%)	0.35 (0.31-0.4)	0.65 (0.53-0.77)
<b>LGMRF</b>	0.84% (0.73%-0.95%)	0.45 (0.39-0.5)	0.73 (0.62-0.84)
<b>SGS</b>	0.91% (0.77%-1.06%)	0.38 (0.33-0.44)	0.6 (0.5-0.71)



## Results – Preprocessing method

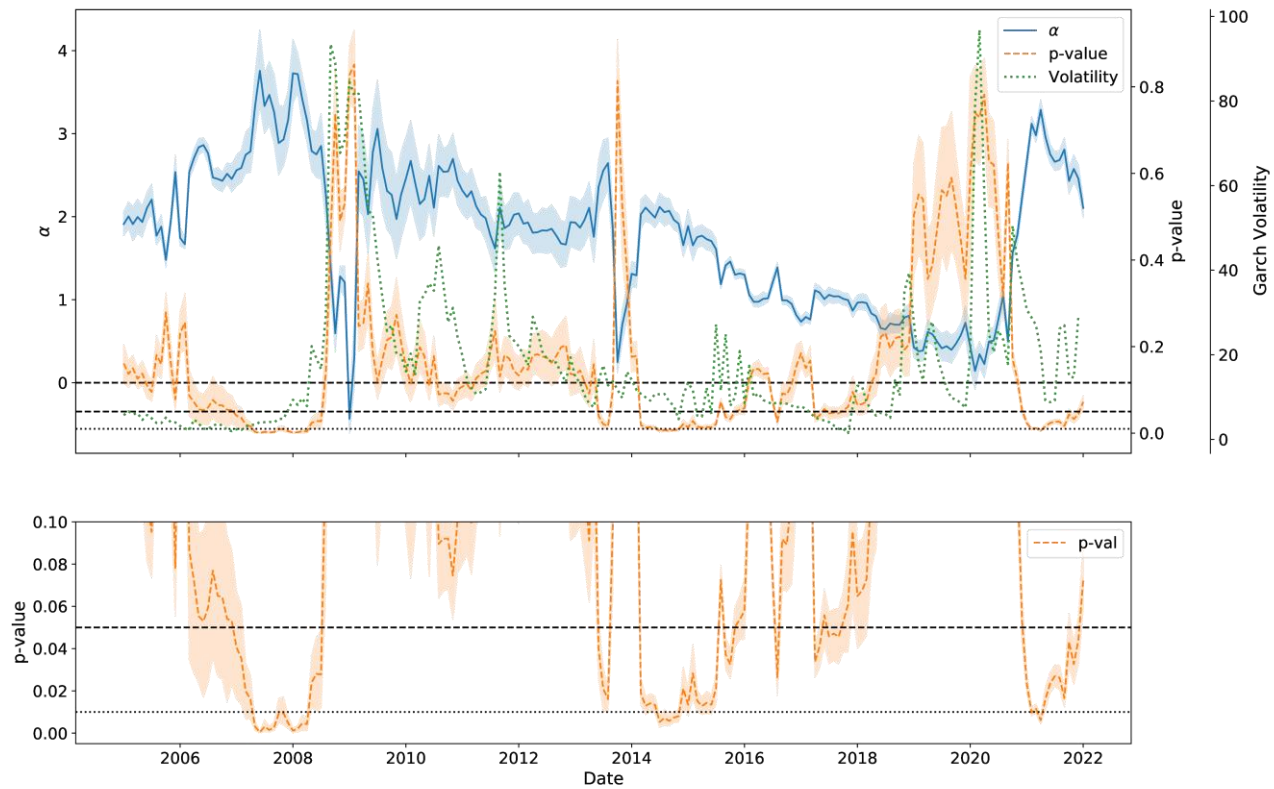
- A small rolling window and the union of the two approaches give the best performance.

Configuration	Excess Returns	Sharpe Ratio	Sortino Ratio
w=1 - $\gamma$ =None - q=0.1	0.96% (0.85%-1.09%)	0.65 (0.58-0.73)	1.0 (0.85-1.17)
w=1 - $\gamma$ =None - q=0.2	0.8% (0.71%-0.89%)	<b>0.72</b> (0.65-0.8)	1.17 (1.02-1.34)
w=2 - $\gamma$ =2 - q=0.1	<b>1.43%</b> (1.25%-1.61%)	0.64 (0.56-0.73)	<b>1.18</b> (1.01-1.37)
w=2 - $\gamma$ =2 - q=0.2	1.3% (1.13%-1.47%)	0.63 (0.54-0.73)	1.14 (0.97-1.33)
w=2 - $\gamma$ =2 - q=None	1.25% (1.09%-1.42%)	0.63 (0.54-0.72)	1.12 (0.95-1.3)



## Performance over time – Rolling $\alpha$

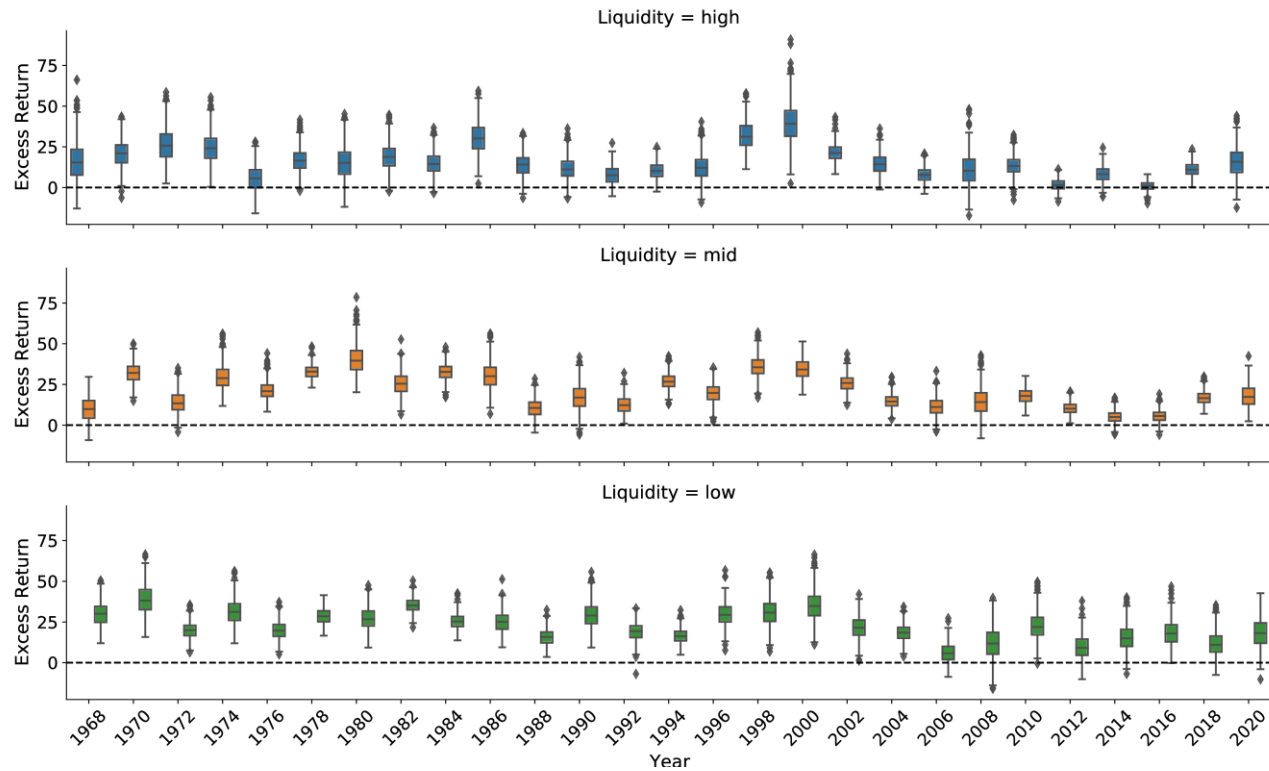
$$r = \alpha + \sum_i \beta_i f_i + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$





## Market Cap. and Liquidity

- We split the U.S. CRSP dataset into 3 tertiles based on the average monthly volumes: **Low**, **Mid**, and **High** liquidity.





## Conclusion

- We suggest using residuals and information from other stocks to reduce the risk of the trading strategies.
- We found that a low liquidity environment is beneficial to pairs trading.
- We formalized the pairs trading problem, and we presented a statistical arbitrage approach that casts some doubts on the robustness of the idea behind pairs trading.



**Thanks for your attention!**



**SAPIENZA**  
UNIVERSITÀ DI ROMA