2.

The throw of a dice can be meant as a random variable because of its uncertain result, and it is a discrete one because all the values that can be assumed by the dice are: $x = [\,1, 2, 3, 4, 5, 6\,]$.
The probability distribution of a discrete random variable is often called "probability mass function" and it can be represented as a vector whose values is the probability for each possible result, so the probability mass function of a rigged dice can be: $p(x) = [\frac{2}{15}, \frac{1}{3}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}]$.

a)

The population mean of this rigged dice is equal to the sum of its possible values times probability, hence the expected value:

$$\mu = \sum_{i=1}^{6} x_i \, p(x_i) = 3.2$$

While its variance as expected value square, by the square of distance between mean and possible value which is multiplied for each probability:

$$\sigma^2 = \sum_{i=1}^{6} (x_i - \mu)^2 \, p(x_i) = 2.69$$

b)

According to the possible throw of a dice treatment as a discrete random variable, a roll u of a rigged or fair dice can be simulated by $rand$ command. It returns an N-by-N matrix containing pseudorandom values drawn from the standard uniform distribution on the open interval (0,1) and the x_i value is assumed by the dice, given the returned value, depends on which interval of cumulative mass function belongs u. The probability mass function, denoted by $F(x_i\,)$ gives the probability that x does not exceed x_i:

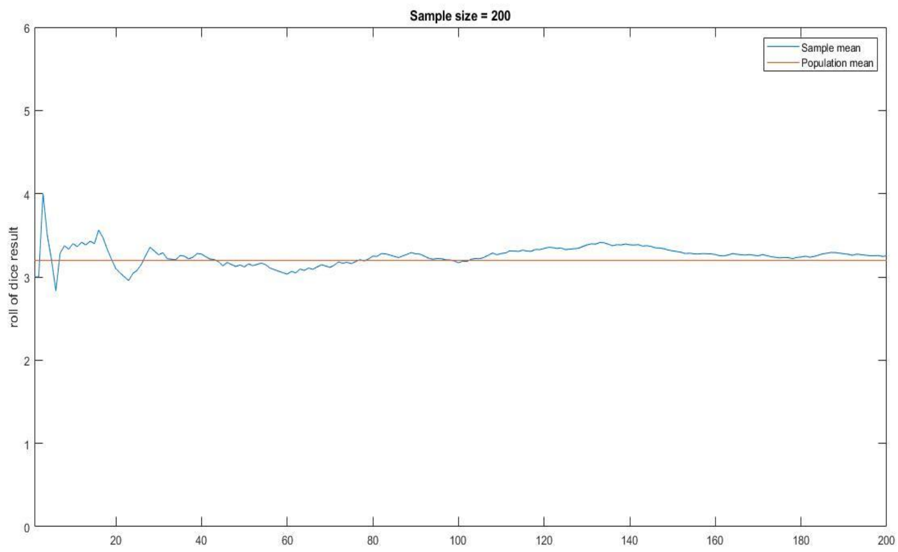
$$F(x_i\,) = \sum_{x \le x_i} p(x)$$

X	p(x)	F(x)
1	$\frac{2}{15}$	0.13
2	$\frac{1}{3}$	0.47
3	$\frac{2}{15}$	0.60
4	$\frac{2}{15}$	0.73
5	$\frac{2}{15}$	0.87
6	$\frac{2}{15}$	1

c)

If n rolls are simulated, for $n \to \infty$, the sample mean estimator $\bar{x}$ converges to the population one μ. This result is guaranteed by the Law of Large Numbers which states that an observed sample average from a large sample will be close to the true population average, and that it will get closer the larger the sample. Then, the sample mean is consistent because the estimator approaches, in probability, to the unknow parameter increasing the sample size.

$$\lim_{n\to\infty} \bar{x}_n = \left(\frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_n}{n}\right) = \mu$$



These variables have to be independent from each other, which means that a roll of dice does not affect any other rolls.
They also have the same distribution, P.D.F. and parameters, which can be verified when it uses the same dice (rigged or fair does not matter) for all the rolls.

3.

When the dice is a fair one its probability mass function is: $p(x) = \left[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]$ because there is the same probability for each possible value $x = [\,1, 2, 3, 4, 5, 6\,]$.

a)

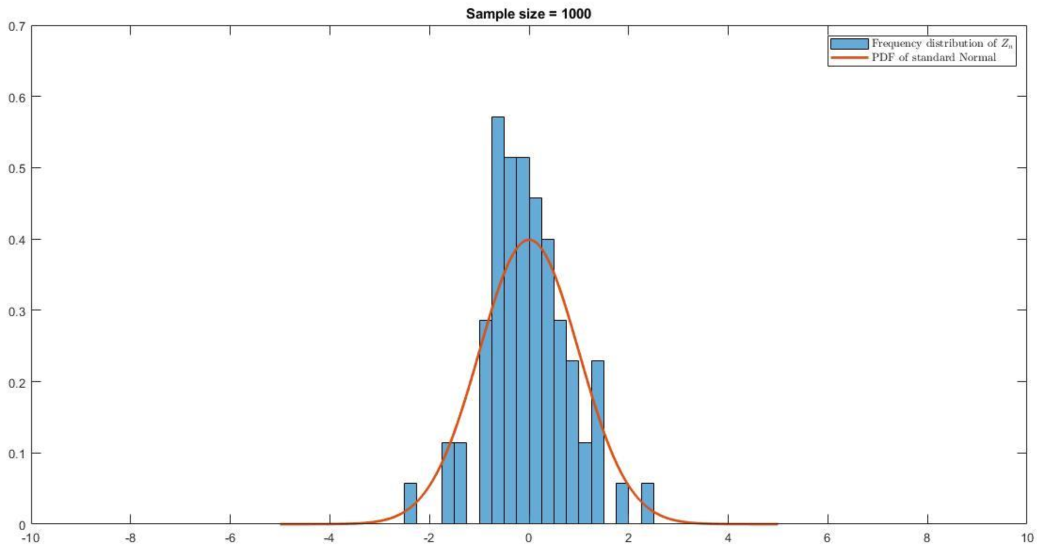
Its mean and variance can be found by the same computation for rigged one. The results are obtained: $\mu = 3.5$; $\sigma^2 = 2.92$, the fair dice variance is greater than the rigged one because of the greater uncertainty.

b)

The roll of a fair dice can be simulated by rand command multiplied for 6 because the dice can assume a discrete value from 1 to 6. Ceil command allow to get an integer number.

c)

The probability density function is a function whose value can be interpreted as providing a relative likelihood that the value of the random variable would be close to that sample, and the Central Limit Theorem guarantees the probability density function $\mathrm{Z} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-3.5}{\sqrt{2.92/n}}$ approaches to the standard Normal one: $\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(\bar{x})^2}$ for $n \to \infty$.



The variance of random variables exists and the random variables have to be i.i.d. but they are not necessarily Normal or continuous.