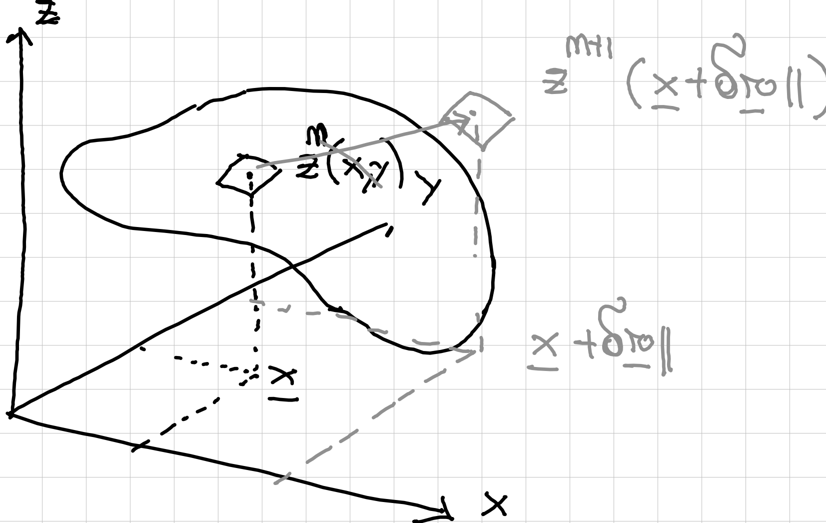

As $t \to t+\Delta t$, the volume element $\diamond$ moves by $\Delta\underline{r} = \Delta t(v^{m_i}\underline{e}_i + v^m\hat{n})$, where we decompose $\Delta\underline{r}$ as $\Delta\underline{r} = \Delta\underline{r}_\parallel + \Delta\underline{r}_\perp$, and $\Delta\underline{r}_\parallel$ lies in the xy plane, $\Delta\underline{r}_\perp$ is orthogonal to the xy plane: $\Delta\underline{r}_\perp = \hat{z}\,\Delta r_\perp$

$$z^{m+1}(\underline{x}+\Delta\underline{r}_\parallel) = z^m(\underline{x}) + \Delta r_\perp \simeq$$

$$\simeq z^{m+1}(\underline{x}) + \partial_i z^{m+1}\big|_{\underline{x}}\,\Delta r^i_\parallel \simeq$$

$$\simeq z^{m+1}(\underline{x}) + \partial_i z^{m}\big|_{\underline{x}}\,\Delta r^i_\parallel,$$ thus

$$
\begin{aligned}
z^{n+1}(\underline{x}) &= z^{n}(\underline{x}) + \Delta r_{\perp} - \partial_i z^{n}\big|_{\underline{x}} \Delta \underline{r}^{i}_{\parallel} = \\
&= z^{n}(\underline{x}) + \Delta t \left(v^{ni} \underline{e}_i + w^{n} \hat{n} \right) \cdot \hat{z} - \qquad (10) \\
&\quad - \left(\partial_i z^{n} \right)_{\underline{x}} \Delta \underline{r}^{i}_{\parallel} ,
\end{aligned}
$$

which in continuous form becomes

$$
\underline{\partial_t z} = \left(v^{i} \underline{e}_i + w \hat{n} \right) \cdot \hat{z} - \left(\partial_i z \right) \frac{\Delta \underline{r}^{i}_{\parallel}}{\Delta t} ,
$$