5.1) $\frac{1}{x^4} \cdot 4x^3 \sin(x^3) + \log(x^4)\cos(x^3)\, 3x^2$

$$\frac{4}{x} \sin(x^3) + 3x^2 \log(x^4) \cos(x^3)$$

5.2) $\frac{1}{1+\exp(-x)} = [1+\exp(-x)]^{-1}$

$$-[1+\exp(-x)]^{-2} \cdot (-1) \exp(-x)$$

$$\frac{\exp(-x)}{[1+\exp(-x)]^2} = \sigma(1-\sigma)$$

5.3) $\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$

$$\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)(\cancel{2})\left(-\frac{1}{\cancel{2}\sigma^2}\right)(x-\mu)$$

$$-\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)\left(\frac{x-\mu}{\sigma^2}\right)$$

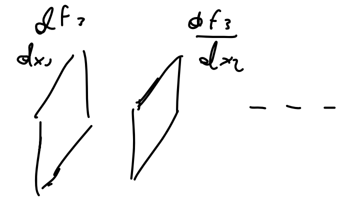
5.5) $f_1'(x) = [\cos(x_1)\cos(x_2), -\sin(x_1)\sin(x_2)]$

(1×2)

$f_2'(x,y) = [y^T, x^T]$

$(1 \times N)$

$$f_3(x) = x x^T = \begin{bmatrix} x_1^2 & x_1 x_2 & x_1 x_3 \dots & x_1 x_m \\ x_2 x_1 & x_2^2 & \dots & \dots \\ x_3 x_1 & \dots & & \\ \dots & & \dots & \\ x_4 x_1 & \dots & \dots & x_m^2 \end{bmatrix}$$

$f_3'(y) =$ 

$(N \times N \times N)$

$$\frac{df_3}{dx_1} = \begin{bmatrix} 2x_1 & x_2 & x_3 & \dots \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

5.6) a) $$\frac{df(t)}{dt} = \cos(\log(t^T t)) \cdot \frac{1}{t^T t} \cdot 2t^T$$

$$= \frac{2t^T}{t^T t} \cos(\log(t^T t))$$

b) $$\begin{bmatrix} a_{11} & & & \\ + & a_{22} & & \\ & + & a_{33} & \\ & & & \dots \end{bmatrix}$$

$a_{11} = a_{1k} \cdot X \cdot b_{1k}$

$a_{22} = a_{2k} \cdot x \cdot b_{2k}$.

$---$

$$\text{trace} = \sum_{k=0}^{B[1]\,A[0]} \sum_{i=0}^{x[0]\,A[1]} \sum_{j=0}^{b[0]\,x[1]} a_{ki}\, x_{ij}\, b_{jk}$$

$$\frac{d}{dx}\,\text{trace} = a^T b^T \Rightarrow A^T B^T$$

$(E \times F)$

5.7) a) $$\frac{1}{1 + x^T x}\left[2x^T\right]$$

$(1 \times D)$

b) $$\frac{d}{dz}\sin z \cdot \frac{d}{dx}(Ax+b)$$

$E \to E$ $E \to D$

$$\begin{bmatrix} \cos z_1 & 0 & & \\ 0 & \cos z_2 & & \\ \cdots & & \cdots & \\ & & & \cos z_m \end{bmatrix} \cdot A$$

$(E \times D)$

5.8) a)

$$-\frac{1}{2}\exp\left(-\frac{1}{2}z\right)\cdot y^T\left(S^{-1}+\left(S^{-1}\right)^T\right)\cdot I$$

1×1 $\quad$ $1\times D$ $\quad$ $D\times D$

b) $\sum_{i=0}^{D}\left(x_i^2+a^2\right) \Rightarrow 2x^T$

$(1\times D)$

c)

$$M=\frac{d\tanh(z)}{dz}=\begin{bmatrix}\frac{1}{\cosh^2(z_1)} & 0 & & \\ 0 & \frac{1}{\cosh^2(z_2)} & & \\ \dots & & \dots & \\ & & & \frac{1}{\cosh^2(z_M)}\end{bmatrix} \quad (M\times M)$$

$$\frac{d\,Ax+B}{dx}=\frac{d}{dx}\left(\sum a_{ji}\,x_i\right)+b_j \quad \forall j\in[0,M]$$

$$\begin{bmatrix} a_{00} & \dots & a_{0N} \\ a_{10} & \dots & a_{1N} \\ \dots & & \dots \\ \dots & & a_{NN}\end{bmatrix}=A$$

$(M\times N)$

$$\frac{d}{dx} f \Rightarrow M \cdot A$$

$$(M \times N)$$

5, 3) $\frac{d}{dv} \log p(x,z) - \log q(z,v)$

$$\frac{1}{p(x, t(\epsilon, v))} \cdot \frac{dp}{dv}(x, t(\epsilon, v)) \cdot \frac{dt}{dv}(\epsilon, v)$$

$$- \frac{1}{q(t(\epsilon, v), v)} \cdot \frac{dq}{dv}(t(\epsilon, v), v) \cdot \frac{dt}{dv}(\epsilon, v)$$