Implementation of the Sensitivities-Based Method (FRTB) with AAD: Application to an Equity Portfolio

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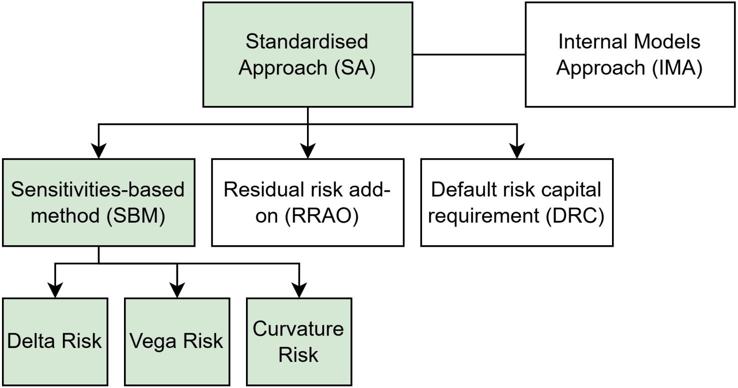
Contents

Context and Objectives

- **Context:** The FRTB regulation, within the sensitivities-based method, involves using finite differences ($\rightarrow$ derivative approximation) to calculate delta and vega.
- **Objective:** Build a Java project to replicate the sensitivities-based method by integrating Automatic Adjoint Differentiation (AAD, $\rightarrow$ exact derivatives) for an equity portfolio.

FRTB – Components



Sample Portfolio

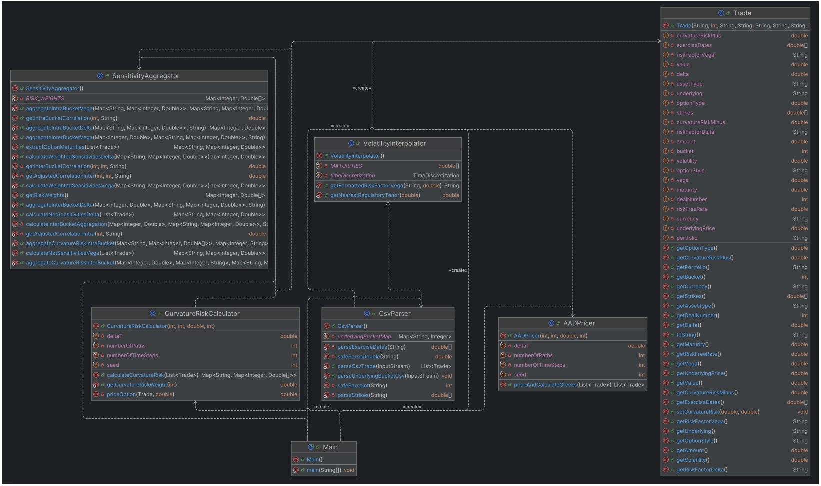
Composition:

- 2 stocks
- 2 European options
- 2 Bermuda options

Portfolio	DealNumber	AssetType	OptionStyle	Underlying	OptionType	Currency	Amount	Volatility	Strikes	UnderlyingPrice	Maturity	ExerciseDates	RiskFreeRate
EQ_PORT	1	Stock		WMT		USD	1			90			
EQ_PORT	2	Stock		JPM		USD	1			100			
EQ_PORT	3	Option	European	AAPL	1	USD	1	0.32	100	100	1.5		0.02
EQ_PORT	4	Option	European	AMZN	1	USD	1	0.25	110	90	2		0.02
EQ_PORT	5	Option	Bermudan	MSFT	1	USD	1	0.3	100;120	100	2	1.0;2.0	0.02
EQ_PORT	6	Option	Bermudan	NFLX	1	USD	1	0.32	100;90	100	2	1.5;2.0	0.02

Input table: excerpt from `sample_trades.csv`

Project UML Diagram



Sensitivities-Based Method Workflow

1. Read input files
   a) Bucket assignment
   b) Identification of risk factors
2. Compute sensitivities with respect to risk factors
   a) Net Sensitivities
   b) Weighted Sensitivities
3. Aggregation
   a) Intra-bucket $\rightarrow$ medium, high, low correlation
   b) Inter-bucket $\rightarrow$ medium, high, low correlation
4. Final capital requirement calculation

1. a) Bucket Assignment

Input: sample_trades.csv and underlying_bucket_mapping.csv
Classes: `CSVParser`, `Trade`, `Main`

Each record in sample_trades.csv is assigned a bucket based on the underlying. For example:

$$\text{Trade: } 4 \rightarrow \text{Underlying: AMZN} \rightarrow \text{Bucket: } 5$$

Underlying	Bucket	Capitalizzazione	Economia	Settore
AAPL	8	Grande	Economia avanzata	Tecnologia
AMZN	5	Grande	Economia avanzata	Beni di consumo e servizi
BABA	1	Grande	Economia emergente	Beni di consumo e servizi

Input table: excerpt from `underlying_bucket_mapping.csv`

1. b) Identification of Risk Factors

Input: sample_trades.csv

Classes: `CSVParser`, `VolatilityInterpolator`, `Trade`, `Main`

- **For delta and curvature** → Spot price of the underlying
- **For vega** → Implied volatility mapped to regulatory tenors (0.5Y, 1Y, 3Y, 5Y, 10Y)

```
DealNumber| Underlying | Bucket | RiskFactorDelta | RiskFactorVega
---------------------------------------------------------------------------
1         | WMT        | 5      | Spot-WMT        | null
2         | JPM        | 8      | Spot-JPM        | null
3         | AAPL       | 8      | Spot-AAPL       | ImpliedVol-AAPL-1.0Y
4         | AMZN       | 5      | Spot-AMZN       | ImpliedVol-AMZN-3.0Y
5         | MSFT       | 8      | Spot-MSFT       | ImpliedVol-MSFT-3.0Y
6         | NFLX       | 5      | Spot-NFLX       | ImpliedVol-NFLX-3.0Y
```

Focus: Automatic Differentiation (AD)

Automatic differentiation $\leftrightarrow$ chain rule:

$$\frac{\partial y}{\partial x_m} = \sum_l \frac{\partial y}{\partial x_l} \cdot \frac{\partial x_l}{\partial x_m},$$

Forward

$$\frac{\partial x_m}{\partial x_i} = \sum_{l=1}^{k(m)} \frac{\partial x_{i_l^{(m)}}}{\partial x_i} \cdot \frac{\partial f_m}{\partial x_{i_l^{(m)}}}\left(x_{i_1^{(m)}}, \ldots, x_{i_{k(l)}^{(m)}}\right)$$

Forward mode applies the chain rule starting from the inputs: for each node x_m (from n to N) and for each independent variable x_i, $\frac{\partial x_m}{\partial x_i}$ is computed propagating the information forward along the computational graph.

Backward (AAD)

$$\frac{\partial y}{\partial x_m} = \sum_{l \in I} \frac{\partial y}{\partial x_l} \cdot \frac{\partial f_l}{\partial x_m}\left(x_{i_1^{(l)}}, \ldots, x_{i_{k(l)}^{(l)}}\right)$$

Backward mode applies the chain rule backwards: for each node x_m, derivatives from dependent nodes are aggregated, propagating $\frac{\partial y}{\partial x_m}$ backward. Initial condition: $\frac{\partial y}{\partial y} = 1$

2. Computation of Sensitivities with Respect to Risk Factors

Classes: `AADPricer`, `CurvatureRiskCalculator`, `Main`

Delta

$$s_{i,k}^{FD} = \frac{V_i(1.01 \cdot EQ_k) - V_i(EQ_k)}{0.01}$$

$$\downarrow$$

$$\boldsymbol{s_{i,k}^{AAD} = \frac{\partial V_i}{\partial EQ_k} \cdot EQ_k}$$

Vega

$$s_{i,k}^{FD} = \frac{V_i(1.01 \cdot \sigma_k, x, y) - V_i(\sigma_k, x, y)}{0.01}$$

$$\downarrow$$

$$\boldsymbol{s_{i,k}^{AAD} = \frac{\partial V_i}{\partial \sigma_k} \cdot \sigma_k}$$

Curvature

$$CVR_k^{\pm} = -\sum_i \left\{ V_i \left(x_k^{RW^{\text{Curvature}\pm}} \right) - V_i(x_k) \mp RW_k^{\text{Curvature}} \cdot s_{ik} \right\}$$

2. Computation of Sensitivities – Example Output

Here is an excerpt of the **output**:

```
=== SENSITIVITY CALCULATION =======================================
Calculating trade: 4
Underlying AMZN | Bucket: 5 | AssetType: Option | OptionStyle: European
Value AAD: 7,301334 | Value FD: 7,301334 | Value Analytic: 7,261207
Delta AAD: 35,392896 | Delta FD: 35,857758 | Analytic Delta: 35,157002
Vega AAD: 12,281135 | Vega FD: 12,281502 | Analytic Vega: 12,214215
Time AAD: 594,264 ms | Time FD: 897,606 ms | Time Analytic: 0,000 ms
```

```
=== CURVATURE RISK CALCULATION ========================================
RiskFactor: Spot-AAPL |Bucket: 8 | CVR +: -8,885463 | CVR -: -14,019368
RiskFactor: Spot-NFLX |Bucket: 5 | CVR +: -2,724541 | CVR -: -4,186483
RiskFactor: Spot-MSFT |Bucket: 8 | CVR +: -9,869637 | CVR -: -15,638343
RiskFactor: Spot-AMZN |Bucket: 5 | CVR +: -4,085770 | CVR -: -4,227744
```

2. a) Net Sensitivities

Classes: `Main`, `SensitivityAggregator`

For delta and vega: Net Sensitivity is calculated for each risk factor:

$$s_k = \sum_i s_{i,k}$$

```
=== NET SENSITIVITIES DELTA ==============================
RiskFactor: Spot-JPM  | Bucket: 8 | Net Delta: 100,000000
RiskFactor: Spot-AAPL | Bucket: 8 | Net Delta: 60,985109
RiskFactor: Spot-NFLX | Bucket: 5 | Net Delta: 71,012533
RiskFactor: Spot-MSFT | Bucket: 8 | Net Delta: 59,138392
RiskFactor: Spot-AMZN | Bucket: 5 | Net Delta: 35,392896
RiskFactor: Spot-WMT  | Bucket: 5 | Net Delta: 90,000000

=== NET SENSITIVITIES VEGA ========================================
RiskFactor: ImpliedVol-AAPL-1.0Y | Bucket: 8 | Net Vega: 15,226626
RiskFactor: ImpliedVol-MSFT-3.0Y | Bucket: 8 | Net Vega: 14,659538
RiskFactor: ImpliedVol-AMZN-3.0Y | Bucket: 5 | Net Vega: 12,281135
RiskFactor: ImpliedVol-NFLX-3.0Y | Bucket: 5 | Net Vega: 15,626644
```

2. b) Weighted Sensitivities

Classes: `Main`, `SensitivityAggregator`

For delta and vega: Weighted Sensitivity is calculated for each risk factor using regulatory weights:

$$WS_k = RW_k \cdot s_k$$

```
=== WEIGHTED SENSITIVITIES DELTA ==============================
RiskFactor: Spot-JPM  | Bucket: 8 | Weighted Delta: 50,000000
RiskFactor: Spot-AAPL | Bucket: 8 | Weighted Delta: 30,492555
RiskFactor: Spot-NFLX | Bucket: 5 | Weighted Delta: 21,303760
RiskFactor: Spot-MSFT | Bucket: 8 | Weighted Delta: 29,569196
RiskFactor: Spot-AMZN | Bucket: 5 | Weighted Delta: 10,617869
RiskFactor: Spot-WMT  | Bucket: 5 | Weighted Delta: 27,000000

=== WEIGHTED SENSITIVITIES VEGA ========================================
RiskFactor: ImpliedVol-AAPL-1.0Y | Bucket: 8 | Weighted Vega: 11,843270
RiskFactor: ImpliedVol-MSFT-3.0Y | Bucket: 8 | Weighted Vega: 11,402189
RiskFactor: ImpliedVol-AMZN-3.0Y | Bucket: 5 | Weighted Vega: 9,552267
RiskFactor: ImpliedVol-NFLX-3.0Y | Bucket: 5 | Weighted Vega: 12,154403
```

3. a) Intra-Bucket Aggregation

Classes: `Main`, `SensitivityAggregator`

- **For delta and vega** $\rightarrow$
  $K_b = \sqrt{max(0, \sum_k WS_k^2 + \sum_k \sum_{l \neq k} (\rho_{kl} WS_k WS_l))}$
- **For curvature** $\rightarrow$ $K_b = \max\left(K_b^+, K_b^-\right)$, where:

$$K_b^{\pm} = \sqrt{\max\left(0,\ \sum_k \max(CVR_k^{\pm}, 0)^2 + \sum_k \sum_{l \neq k} \rho_{kl}\, CVR_k^{\pm}\, CVR_l^{\pm}\, \psi(CVR_k^{\pm}, CVR_l^{\pm})\right)}$$

```
=== INTRA-BUCKET AGGREGATION ===============================
--- Delta ------------------------------------------------
Bucket: 5 | Delta (M: 39,593061, H: 40,442746, L: 38,724736)
Bucket: 8 | Delta (M: 72,665777, H: 74,326043, L: 70,966680)
--- Vega -------------------------------------------------
Bucket: 5 | Vega (M: 16,370731, H: 16,590878, L: 16,147584)
Bucket: 8 | Vega (M: 17,433313, H: 17,672925, L: 17,190362)
--- Curvature --------------------------------------------
Bucket: 5 | Curvature (M: 0,000000, H: 0,000000, L: 0,000000)
Bucket: 8 | Curvature (M: 0,000000, H: 0,000000, L: 0,000000)
```

3. b) Inter-Bucket Aggregation

Classes: `Main`, `SensitivityAggregator`

- **For delta and vega** $\rightarrow$ $K_{\text{class}} = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c}$
- **For curvature** $\rightarrow$ $k_{\text{class}} = \sqrt{\max\left(0, \sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)\right)}$

```
=== INTER-BUCKET AGGREGATION ============
--- Delta --------------------------------
(M: 90,181645, H: 91,520037, L: 88,823088)
--- Vega ---------------------------------
(M: 25,754159, H: 26,118930, L: 25,384147)
--- Curvature ----------------------------
(M: 0,000000, H: 0,000000, L: 0,000000)
```

4. Final Capital Requirement Calculation

Classes: `Main`, `SensitivityAggregator`

The final capital requirement is given by:

$$K_{final} = \max\{K_{medium}, K_{high}, K_{low}\}$$

```
=== FINAL RESULTS ========================
Final Capital Requirement Delta: 91,520037
Final Capital Requirement Vega: 26,118930
Final Capital Requirement Curvature: 0,000000
```

Conclusions

The implementation of the project described above led to the following conclusions:

- AAD timings are comparable to FD (the efficiency of the former method becomes more evident as input size increases).
- Greater accuracy of AAD in derivative calculation (taking into account Monte Carlo simulation errors).