A Note about Notation

Mathematical notation is the language that we use to communicate mathematics. Thus, if you want to be able to understand written mathematics or have others understand what you are writing, it is important to use notation correctly. As you progress through this course and others that follow, you will see how important it is. Imagine trying to navigate through a foreign city using directions written in a language you do not know. This is what reading a mathematics textbook will be like if you do not understand how to correctly use notation. Since understanding notation is so important to learning and understanding mathematics, I will grade your work and notation on tests and quizzes as well as checking that your answer is correct. Making an effort to use notation correctly on homework will make the process more automatic when it comes time for a test. If you have questions about using notation correctly, please ask. It is important!

The following are some tips based on common errors.

- Use equal signs correctly. The expressions on either side of the equal sign should be just that – equal. Do not use an equal sign to indicate that you are progressing from one step to the next if the expressions on either side of the equal sign are not equivalent. In this case you use an arrow or just start a new separate line.
- Use parentheses correctly. Parentheses are used to group expressions together. For example,

  $x \cdot y + 2$ and $x(y+2)$

  have different meanings. In the first expression x is only being multiplied by y. In the second expression x is being multiplied by the entire expression $y+2$. In other words,

  $x \cdot y + 2 = xy + 2$ while $x(y+2) = xy + 2x$.

  Additionally,

  $x - \sin x$ is a difference while
  $x(-\sin x)$ is a product.

  When differentiating a sum or difference, parentheses must be used. For example, to differentiate the sum $x^2 + 5x - 2$ you would write

  $\frac{d}{dx}\left(x^2 + 5x - 2\right)$ not $\frac{d}{dx}x^2 + 5x - 2$ (This would just be the derivative of x^2 plus the expression $5x - 2$. You would not be differentiating $5x - 2$ in this case.).

  Parentheses are also important when working with trigonometric functions. The expressions

  $\sin x + 3$ and $\sin(x+3)$

are not equivalent. In the expression $\sin x + 3$ you are taking the sine of *x* and then adding 3 to the result while in the expression $\sin(x+3)$ you are taking the sine of the entire sum $x+3$.

- Use function notation correctly. For example, the notation $f(x) = x^2 + 3$ indicates that *f* is the name of the function and the variable *x* in parentheses is the independent variable. *f(x)* is not the product of *f* and *x*! Additionally, the notation $f(5)$ represents the output of *f* when $x = 5$. Thus, *f(5)* is the *y*-value 28. This also means that $f(x+h)$ represents the expression you get when you replace each *x* in the rule for *f(x)* with the expression *x+h*. In other words,

$$f(x+h) = (x+h)^2 + 3 = x^2 + 2xh + h^2 + 3$$

- Use interval notation correctly. To express the interval $1 \le x \le 4$ using interval notation (notice that the endpoints are included) write $[1, 4]$. To express the interval $1 < x < 4$ (notice that the endpoints are not included) write $(1, 4)$. Using a square bracket means that you are including the endpoint while using a parenthesis means you are not including the endpoint. The smaller value is always written to the left of the larger value, and they are separated by a comma. A negative infinity symbol is always preceded by a parenthesis while a positive infinity symbol is always followed by a parenthesis (a square bracket is never used in either case).
- Use function and operator notation correctly. This is particularly important when working with limits and derivatives. Examples of operators are $\frac{d}{dx}$, $\lim\limits_{x \to a}$, +, and -. Each tells you to perform some type of action. Examples of functions are $f(x)$, $f'(x)$, $\sin x$, and $\frac{d}{dx}(2x+5)$. Functions are rules for assigning output values to input values where each input has only one output. An operator is an action while a function is a thing (think of it as a verb versus a noun). Thus, an operator cannot equal a function (an apple cannot equal an orange). Just as it would not make sense to write

$$+ = 5,$$

it would not make sense to write

$$\frac{d}{dx} = 2x + 6.$$

Instead, we would write $\frac{d}{dx}\left(x^2 + 6x\right) = 2x + 6$. Likewise, we would write

$\lim\limits_{x \to 2}(x+3) = 5$ not $\lim\limits_{x \to 2} = 5$.