1.4 exponential Functions

- the function of $f(x)=2^x$ is an exponential function
  - because x is the exponent

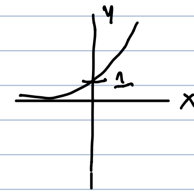
$y=a^x$, $a>1$, $0<a<1$

Ex: $y=2^x$ is an exp func

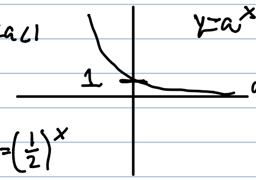
($y=x^2$ is a power function, not an exp function)

Domain is the set of all numbers $(-\infty, \infty)$
range is positive numbers $(0, \infty)$
y intercept is $(0,1)$
Horizontal asymptote $y=0$ (x-axis)

If $y=a^x$
$a>1$
Ex: $y=2^x$



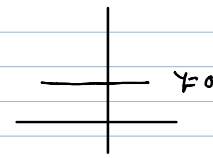
If $0<a<1$



Ex = $y=\left(\frac{1}{2}\right)^x$

Similarities: any positive real number with zero put in will get 1

Laws of exponents - If a and b are positive numbers and x and y are any real numbers then,



1. $a^{x+y} = a^x a^y$
2. $a^{x-y} = \frac{a^x}{a^y}$
3. $(ab)^x = a^x b^x$

Ex: Find $2^2 2^3 = (2\cdot2)\cdot(2\cdot2\cdot2) = 2^5$

$\cdot(2^2)^3 = 2^2\cdot2^2\cdot2^2$
$= (2\cdot2)\cdot(2\cdot2)\cdot(2\cdot2) = 2^6$

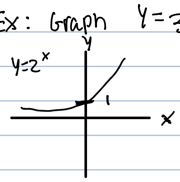
(b) $\frac{x^{2n}\cdot x^{3n-1}}{x^{n+2}} = \frac{x^{2n+(3n-1)}}{x^{n+2}} = \frac{x^{5n-1}}{x^{n+2}}$

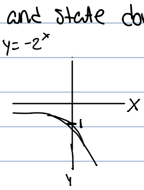
$= x^{5n-1-(n+2)} = x^{4n-3}$

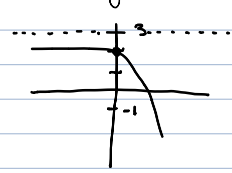
(c) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \frac{\sqrt{ab^{\frac{1}{2}}}}{(ab)^{\frac{1}{3}}} = \frac{(ab^{\frac{1}{2}})^{\frac{1}{2}}}{(ab)^{\frac{1}{3}}} = a^{\frac{1}{2}}(b^{\frac{1}{2}})^{\frac{1}{2}} = \frac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = a^{\frac{1}{2}-\frac{1}{3}}\cdot b^{\frac{1}{4}-\frac{1}{3}}$

$= a^{\frac{3}{6}-\frac{2}{6}}\, b^{\frac{3}{12}-\frac{4}{12}} = a^{\frac{1}{6}}\, b^{-\frac{1}{12}} = \boxed{\frac{a^{\frac{1}{6}}}{b^{\frac{1}{12}}}} = \frac{\sqrt[6]{a}}{\sqrt[12]{b}}$

Ex: Graph $y=3-2^x$ and state domain and range





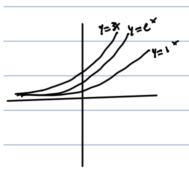


Domain = $(-\infty, \infty)$
range = $(-\infty, 3)$

The number e

- e is an irrational number and occurs frequently in applications

$e \approx 2.71828$



Ex: If $f(x)=5^x$ show that $\frac{f(x+h)-f(x)}{h} = 5^x\left(\frac{5^h-1}{h}\right)$

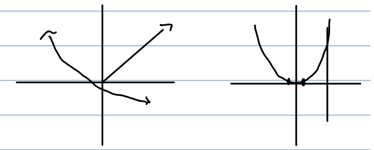
#23 in book

$\frac{f(x+h)-f(x)}{h} = \frac{5^{x+h}-5^x}{h} = \frac{5^x5^h-5^x}{h} = \frac{5^x(5^h-1)}{h} =$

$5^x\left(\frac{5^h-1}{h}\right)$

Section 1.5 Inverse Functions & logarithms

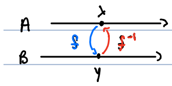
one to one function | a non one to one



a Function f is called a one-to one function if it never takes on the same value twice

$f(x_1) \neq f(x_2)$, whenever $x_1 \neq x_2$

a Function is one to one if the graph has only one value for x

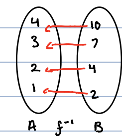


Domain of f^{-1} = range of f

range of f^{-1} = domain of f

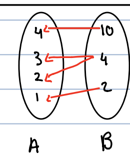
In order to have an inverse a function f must be one to one.

Ex:



f^{-1} inverse is a function

f was one to one



The inverse relation is not a function for g which was not one to one

• Let f be a one to one function with domain A and range B. Then the inverse function f^{-1} has a domain B and a range A.

$f^{-1}(x) = y$ $\quad f(y) = x$

$f^{-1}(f(x)) = x$ for every x in A

$f(f^{-1}(x)) = x$ for every x in B

How to find the inverse of a one to one function

1. write $y = f(x)$
2. Solve that equation for x in terms of y
3. Express f^{-1} as a function of x interchange x and y
   - the resulting equation is $y = f^{-1}(x)$

Ex: Find f^{-1} given the one to one function

$f(x) = \frac{4x-1}{2x+3}$

1. replace $f(x)$ with y

$y = \frac{4x-1}{2x+3}$

2. Solve eqn for x in terms of y

1. $y(2x+3) = 4x-1$
2. $2xy + 3y = 4x - 1$
3. $2xy - 4x = -1 - 3y$
4. $x(2y-4) = -1-3y$
5. $x = \frac{-1-3y}{2y-4}$
6. $y = \frac{-1-3x}{2x-4}$

$f^{-1}(x) = \frac{-1-3y}{2x-4}$

Check to make sure the function is one-to-one if its not restrict its domain so that its equal to the range of f

Ex: find g^{-1} for one to one function $g(x) = \sqrt{9-2x}$

$y = \sqrt{9-2x}$

$y^2 = 9 - 2x$

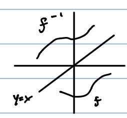
$x = \frac{y^2-9}{-2} = \frac{9-y^2}{2} = \frac{9}{2} - \frac{1}{2}y^2$

$\boxed{y = \frac{9}{2} - \frac{1}{2}x^2}$ - restrict the Domain

range of $g(x)$ is $[0, \infty)$

Domain of $g^{-1}(x)$ is $[0, \infty)$

$g^{-1}(x) = \frac{9}{2} - \frac{1}{2}x^2$, $x \geq 0$



Logarithmic Functions

$f^{-1}(x) = y \quad f(x) = y$

$\log_a x = y \iff a^y = x$

$\log_a(a^x) = x$ for every $x \in R$

$a^{\log_a x} = x$ for every $x > 0$

• The log function $\log_a$ has a domain of $(0, \infty)$ and range R its graph is the reflection of the graph $y = a^x$ about the line $y = x$

* Most important log func have a base $a > 1$.

$y = a^x$ Fast — if the base is larger it rises more quickly

$y = \log_a x$ Slow

Laws of logs — if x are positive #'s

1. $\log_a(xy) = \log_a x + \log_a y$ — sum of quotients
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ — difference of quotients
3. $\log_a(x^r) = r \log_a x$ (where r is a real number)

Ex: find the exact value

$\log_3 100 - \log_3 18 - \log_3 50$

$= \log_3 100 - (\log_3 18 + \log_3 50)$

$= \log_3 100 - (\log_3 [(18)(50)])$

$= \log_3 \frac{100}{(18)(50)} = \log_3 \frac{2}{18} = \log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = \boxed{-2}$

Natural Logs:

• The log with a base e is called the natural log

$\log_e x = \ln$

$\ln(e^x) = x \quad x \in R$

$e^{\ln x} = x \quad x > 0$

$\ln e = 1$

Ex: Find exact value $e^{-2\ln 5}$

$e^{-2\ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{25}$

Change of Base Formula: $\log_a x = \frac{\ln x}{\ln a}$

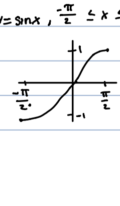
Ex $\log_8 5 = \log_8 5 = \frac{\ln 5}{\ln 8}$

Inverse Trig Functions:

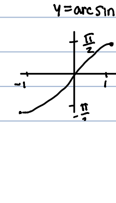
$\sin^{-1} x \neq \frac{1}{\sin x} \qquad \frac{1}{\sin x} = \csc x$

Inverse Sine: $y = \arcsin x$ or $y = \sin^{-1} x$

$y = \sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



$y = \arcsin x$



Domain: $[-1, 1]$

range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Ex: arcsin $(\frac{1}{2})$ = (what is the sin value $(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2})(\frac{1}{2})$?

A) $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

B) $\sin^{-1}(1) = \frac{\pi}{2}$ $\quad -1 \le x \le 1$

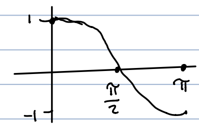
Ex: A) $\sin(\sin^{-1}(\frac{1}{2})) = \frac{1}{2}$

B) $\sin^{-1}(\sin(\frac{\pi}{3})) = \frac{\pi}{3}$

C) $\sin^{-1}(\sin\frac{3\pi}{4}) = \frac{\pi}{4}$

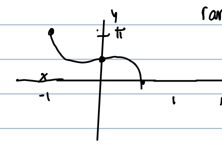
Inverse Cosine

$y = \cos x, \; 0 \le x \le \pi$



$y = \arccos = \cos^{-1} x$ Domain $[-1, 1]$

range: $[0, \pi]$



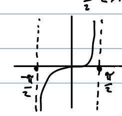
- $\cos^{-1}(\cos x) = x \quad 0 \le x \le \pi$
- $\cos(\cos^{-1} x) = x \quad -1 \le x \le 1$

Inverse Tangent (not one to one)

$y = \arctan x$

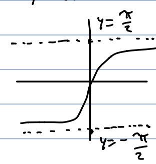
$-\frac{\pi}{2} < x < \frac{\pi}{2}$

y=tanx



$y = \tan^{-1} x$ Domain: $(-\infty, \infty)$

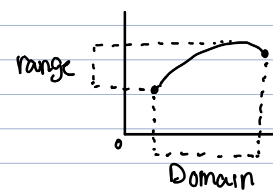
range: $(-\frac{\pi}{2}, \frac{\pi}{2})$



1.1 → 1.3 Main Topics

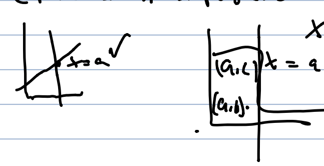
1.1:

- Domain → independent variable: one # of x exactly in set D & D & E are real numbers
- Range → dependent variable: all possible values of f(x)



- $\frac{f(a+h) - f(a)}{h}$] difference quotient
  ↳ avg rate of change of f(x) between x=a & x=a+h

- A graph is a function if it pass the vertical line test



- square root of a negative # is not defined as a real #

piecewise functions

$|a| = a$ if $a \ge 0$

$|a| = -a$ if $a < 0$

- point slope: $y - y_1 = m(x - x_1)$

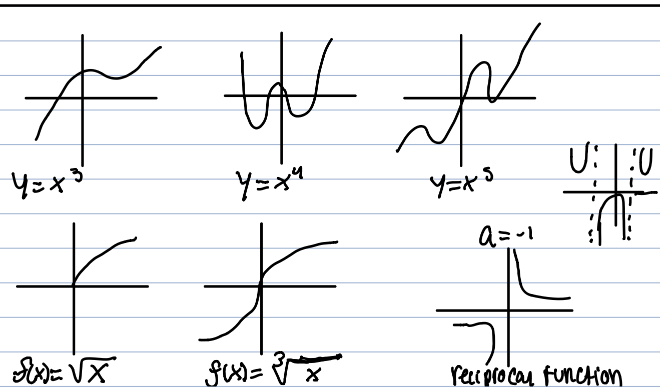
$f(-x) = (-x)^2 = x^2 = f(x)$] symmetry

$f(-x) = (-x)^3 = x^3 = -f(x)$] odd function

Increasing func: $f(x_1) < f(x_2)$

decreasing func $f(x_1) > f(x_2)$

1.2



$f(x) = x^a$, a is a constant called power function

a = n where n is a positive integer

rational functions $f(x) = \frac{P(x)}{Q(x)}$ ratio of 2 polynomials

- domain consists of all values of x $Q(x) \neq 0$

Trigonometric Functions

Domain of cosine & Sin = $(-\infty, \infty)$ $|\sin x| \leq 1$ $|\cos x| \leq 1$

$\sin x = 0$ when $x = n\pi$ n an integer

$\sin(x+2\pi) = \sin x$ | $\cos(x+2\pi) = \cos x$

$\tan x = \frac{\sin x}{\cos x}$ $\tan(x+\pi) = \tan x$ for all x

Logarithmic Functions:

$f(x) = \log_b x$] base b is a positive constant] inverse of exponential

-domain = $(0, \infty)$ and range is $(-\infty, \infty)$ increase slowly when $x > 1$