Algebra Reference Sheet

Multiplying Fractions

When multiplying fractions, multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Examples: 1) $\frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35}$

2) $\frac{2}{x} \cdot \frac{8}{x+1} = \frac{16}{x(x+1)}$

Adding Fractions

To add fractions first find a common denominator (you can always use the product of the denominators of the terms you are adding) and rewrite each term as an equivalent fraction over the common denominator. Then add the numerators only. Write the sum of the numerators over the common denominator.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Examples: 1) $\frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$

2) $\frac{2}{x} + \frac{8}{x+1} = \frac{2(x+1)}{x(x+1)} + \frac{8x}{x(x+1)} = \frac{2(x+1)+8x}{x(x+1)} = \frac{10x+2}{x(x+1)}$

Cancelling Common Factors (Reducing a Fraction to Lowest Terms)

When you reduce a fraction to lowest terms, you cancel any *factor* that is common to the numerator and denominator. We are allowed to do this because any factor (except zero) divided by itself is 1.

Examples: 1) $\frac{a}{a} = 1$ if $a \neq 0$ so $\frac{3a}{5a} = \left(\frac{3}{5}\right)\left(\frac{a}{a}\right) = \frac{3}{5} \cdot 1 = \frac{3}{5}$

2) $\frac{x + x^2}{1 - x^2} = \frac{x(1+x)}{(1-x)(1+x)} = \left[\frac{x}{1-x}\right]\left[\frac{1+x}{1+x}\right] = \left[\frac{x}{1-x}\right] \cdot [1] = \frac{x}{1-x}$

We can cancel a factor of $1 + x$ from the numerator and denominator in example (2) because $\frac{1+x}{1+x} = 1$.

Avoiding a Common Error When Reducing Fractions: You cannot cancel the x^2 in the numerator and denominator of $\frac{x + x^2}{1 - x^2}$ because the x^2 is not a *factor* in the numerator and denominator. The x^2 is a term in a sum in the numerator and a term in a difference in the denominator. You can only cancel a *factor* common to the numerator and denominator.

Solving a Quadratic Equation by Factoring

Before factoring a quadratic equation be sure that *all* terms are on *one* side of the equation so that you have a quadratic polynomial that equals zero.

$x^2 + 2x = 24$ Subtract 24 from each side of the equation.

$x^2 + 2x - 24 = 0$ Equation must be in the form $ax^2 + bx + c = 0$ *before* you factor.

Find binomial factors whose product (when FOILed) is the quadratic polynomial which is equal to zero in your equation.

$(x+6)(x-4) = x^2 + 6x - 4x - 24 = x^2 + 2x - 24$ so

$(x+6)(x-4) = 0$

Notice that you have a product of two factors that is zero (this is called the Zero Product Property). This implies that at least one of the factors must be zero. Thus,

$x + 6 = 0$ or $x - 4 = 0$

Solve each of the linear equations: $x = -6$ or $x = 4$

Avoiding a Common Error When Solving a Quadratic Equation by Factoring: Remember to move all terms to one side of the equation *before* you factor. The following is a common error:

$x^2 + 2x = 24$

$x(x+2) = 24$

$x = 24$ or $x + 2 = 24$

Why is this wrong? There are countless combinations of factors whose product is 24: 6 and 4, 2 and 12, $\frac{1}{2}$ and 48, $\frac{1}{3}$ and 72, etc. Thus, you cannot assume that if $x(x+2) = 24$, then $x = 24$ or $x + 2 = 24$. Notice that $x = 24$ does not satisfy the original equation nor does $x = 22$. You can assume that either $x + 6 = 0$ or $x - 4 = 0$ when $(x+6)(x-4) = 0$ because of the Zero Product Property: if a product of factors is zero, then at least one of the factors must be zero. Understanding the reasoning behind the technique of solving a quadratic equation by factoring makes it easier to remember why we move all terms to one side of the equation *before* factoring.

The Argument of a Trigonometric Function

The argument of a trigonometric function is an angle. The argument is the input to the trigonometric function. For example $\sin x$ means find the sine of the angle *x*. It does not mean sin times *x*. Consequently, you cannot pull factors of the argument through the trigonometric function. In other words $\sin(4x) \neq 4\sin(x)$. You also must remember to write an argument for each trigonometric function (ie write "$\sin x$" not just "sin").

Order of Operations

The order of operations is important for evaluating expressions or algebraic manipulation. PEMDAS (parentheses, exponents, multiplication, division, addition, subtraction) or "Please Excuse My Dear Aunt Sally" can help you remember the order of operations. Please note that multiplication and division really have the same rank in the order of operations as do addition and subtraction.

Example: $2+3(x+x^3) = 2+3x+3x^3$ because you distribute the 3 over $x+x^3$ first. Note that $2+3(x+x^3) \neq 5\left(x+x^3\right)$ because distributing the 3 is done first.

The Square Root of a Sum or Difference

The square root of a sum or difference is not equal to the sum or difference of the square roots. In notation

$\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$ and $\sqrt{a-b} \neq \sqrt{a}-\sqrt{b}$.

Try substituting some values in

$\sqrt{9+4} = \sqrt{13}$ but $\sqrt{13} = \sqrt{9+4} \neq \sqrt{9}+\sqrt{4} = 3+2 = 5$

The Square of a Sum or Difference

The square of a sum or difference is not the sum or difference of the squares. In notation

$(a+b)^2 \neq a^2+b^2$ and $(a-b)^2 \neq a^2-b^2$

Remember that you must multiply out the factors using FOIL:

$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$ and

$(a-b)^2 = (a-b)(a-b) = a^2-2ab+b^2$