MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) The government issuing bonds to control the nation's money supply through the central bank, also to manage inflation, and raise capital for infrastructure and fundraising.
   (b) The long-term part of yield curve might flatten if the interest rate is increased by the central bank due to the prediction of not growing economy in the future.
   (c) Quantitative easing means bank purchasing longer-term financial assets, US Fed employed this to stabilize financial markets and helped keep low borrowing costs which during a recession.

2. 10 (or 11) bonds are: CAN 2.25 MAR 01, CAN 1.5 SEP 01, CAN 1.25 MAR 01, CAN 0.5 SEP 01, CAN 0.25 MAR 01, CAN 1 SEP 01, CAN 1.25 MAR 01, CAN 2.75 SEP 01, CAN 3.5 MAR 01, CAN 3.25 SEP 01, CAN 4 MAR 01. These bonds are selected because differences of their maturity date are 6 months, which can be helpful in bootstrapping. We further assume the most recent coupon payment was made on 2023-09-01 for all 10 bonds, and having all maturity dates the same can improve accuracy of calculation.

3. The eigenvalue associated with the covariance matrix tells us the amount of variance for each eigenvectors or in other words, principle component. The eigenvectors tells us the direction of variance, if you rank them from high to low, it determines the most significant principle component. Combining them, they tell you which is the significant or the largest direction of largest variance.

Empirical Questions - 75 points

4.
   (a) We calculate accrued interest using the product of n/365 and the coupon payment where n is defined by difference between the last payment and the current date. We then calculate the dirty price using the summation of accrued interest and close price, and let

$$dirtyprice = \frac{facevalue}{(1+YTM)^n} + \sum_{t=0}^{n} \frac{coupon_t}{(1+YTM)^t}$$

   be the equation of YTM. We assume the last payment was made in 2023/9/1 and the face value is equal to 100 for all 10 bonds. The algorithm used in Python was an approximation of the true yield rate: if the yield rate is true, then the equation would be balanced with an error of plus or minus 0.005. Otherwise, the algorithm compares the difference between the estimated value and the true value, if the estimation is larger than the true, it decrease the estimated yield by 0.00001 and conversely for the smaller case.

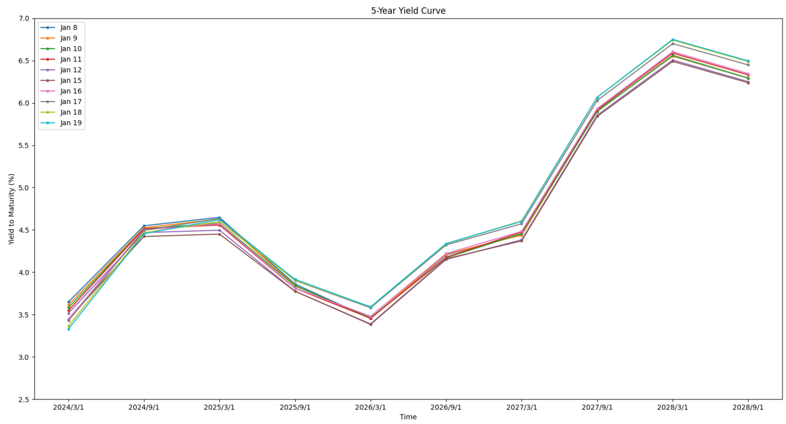

Figure 1: 5 Year Yield Curve

(b) We use bootstrapping to find the spot rate. For maturities less than 6 months, let

$$r(T) = -\frac{log(dirtyprice/notional)}{t}$$

be the equation of spot rate. For maturities between six months and a year, let

$$P = p1 * e^{-r(t1)t1} + p2 * e^{-r(t2)t2}$$

be the equation of spot rate, where r(t2) is the only unknown variable. Note that t1 and t2 represent the current time and the next time, and they usually have a difference of 0.5 year.

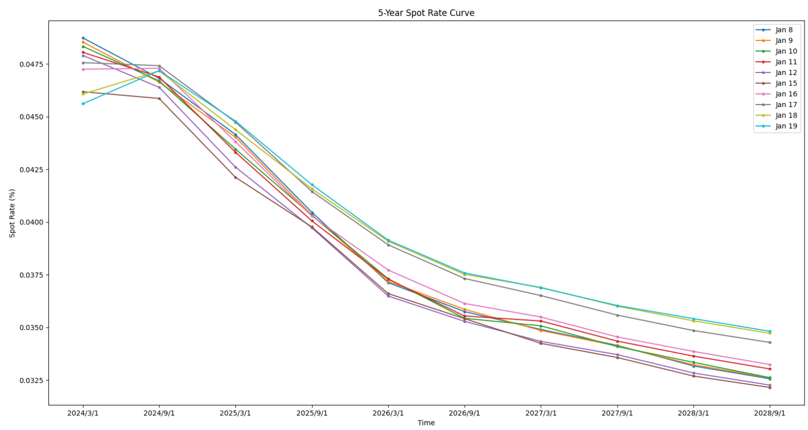

Figure 2: 5 Year Spot Rate Curve

(c) Given spot rate of all periods, for periods between time t and t + n, let

$$F_{t,t+n} = \frac{S_{t+n} * (t+n) - S_t * t}{n}$$

be the formula of forward rate F, where t equals to 1, t+n equals to 2, 3, 4.

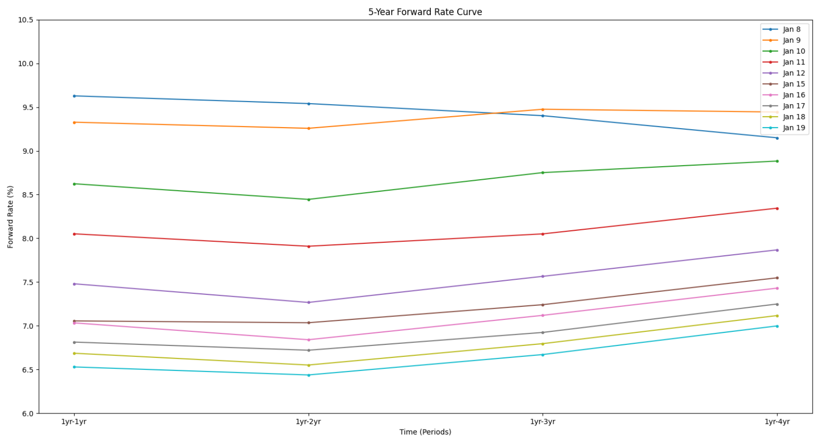

Figure 3: 5 Year Forward Rate Curve

5. The covariance matrix for the time series of daily log-returns of yield is:

$$\begin{bmatrix} 0.000040 & 0.000041 & 0.000061 & 0.000025 & 0.000046 \\ 0.000041 & 0.000069 & 0.000096 & 0.000036 & 0.000088 \\ 0.000061 & 0.000096 & 0.000182 & 0.000099 & 0.000171 \\ 0.000025 & 0.000036 & 0.000099 & 0.000098 & 0.000134 \\ 0.000046 & 0.000088 & 0.000171 & 0.000134 & 0.000230 \end{bmatrix}$$

The covariance matrix for the forward rates is:

$$\begin{bmatrix} 0.000736 & 0.000671 & 0.000722 & 0.000607 \\ 0.000671 & 0.000842 & 0.000750 & 0.000629 \\ 0.000722 & 0.000750 & 0.000976 & 0.000900 \\ 0.000607 & 0.000629 & 0.000900 & 0.000918 \end{bmatrix}$$

6. The eigenvalues of the yield covariance matrix is:

$$\begin{bmatrix} 5.12770156 \times 10^{-4} & 7.14011224 \times 10^{-5} & 1.91439113 \times 10^{-6} & 1.28630532 \times 10^{-5} & 1.97856244 \times 10^{-5} \end{bmatrix}^T$$

The eigenvectors of the yield covariance matrix is:

$$\begin{bmatrix} -0.18045335 & -0.43207108 & 0.26505871 & -0.76661662 & 0.35041977 \\ -0.29681037 & -0.51286863 & -0.66786236 & -0.07498022 & -0.44408019 \\ -0.56571149 & -0.38163505 & 0.24512173 & 0.59302848 & 0.3500825 \\ -0.37990727 & 0.49337881 & -0.55249878 & -0.13391749 & 0.53764209 \\ -0.64419032 & 0.40151262 & 0.34404097 & -0.19251042 & -0.51805598 \end{bmatrix}$$

The eigenvalues of the forward rate covariance matrix is:

$$\begin{bmatrix} 3.02583952 \times 10^{-3} & 3.07402663 \times 10^{-4} & 1.14265235 \times 10^{-4} & 2.40364143 \times 10^{-5} \end{bmatrix}^T$$

The eigenvectors of the yield covariance matrix is:

$$\begin{bmatrix} -0.45044151 & -0.4200376 & -0.7636623 & 0.19362529 \\ -0.47641232 & -0.59332211 & 0.63770652 & 0.11971031 \\ -0.55700039 & 0.26884951 & -0.01851335 & -0.7855748 \\ -0.50978849 & 0.63186804 & 0.09903304 & 0.57537025 \end{bmatrix}$$

The first eigenvalue and its associated eigenvector implies the direction of the largest variance.

References and GitHub Link to Code

1. Quantitative Easing: https://www.un.org/development/desa/dpad/publication/un-desa-policy-brief-no-129-the-monetary-policy-response-to-covid-19-the-role-of-asset-purchase-programmes/

2. PCA: https://builtin.com/data-science/step-step-explanation-principal-component-analysis

3. Github Link: https://github.com/Michelle1488/APM466