Stan Lab

CS 109b Staff
2/21/18 – 2/22/18

Motivation

- How do we normalize posteriors?

Normalization Constant Example:

Given a dataset $D = [x_1, x_2, \dots . x_n]$

Bayes Rule tells us that:

$P(\theta \mid D) = \frac{P(D|\theta)P(\theta)}{P(D)}$, where

$P(\theta)$ is our prior distribution,
$P(D \mid \theta)$ is our likelihood function,
$P(\theta \mid D)$ is our posterior distribution

Problem: How do we calculate $P(D)$?

$P(D) = \int P(D, \theta) d\theta$,
which can be very high-dimensional and difficult to compute.

Motivation

- How do we normalize posteriors?
  - Conjugate priors Beta, Normal
  - Metropolis-Hastings
  - Other forms of MCMC
- Strong developments in both sampling algorithms and computational power have made Bayesian models more feasible

Background

- Stan was developed by Andrew Gelman (PhD from Harvard in 1990) and his lab at Columbia University
  - BDA
- Many predecessors to Stan such as Bugs, Jags, etc.
- Landmark paper: *The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo*
  - Inspired by ideas from physics

Concepts of momentum in spacing the steps

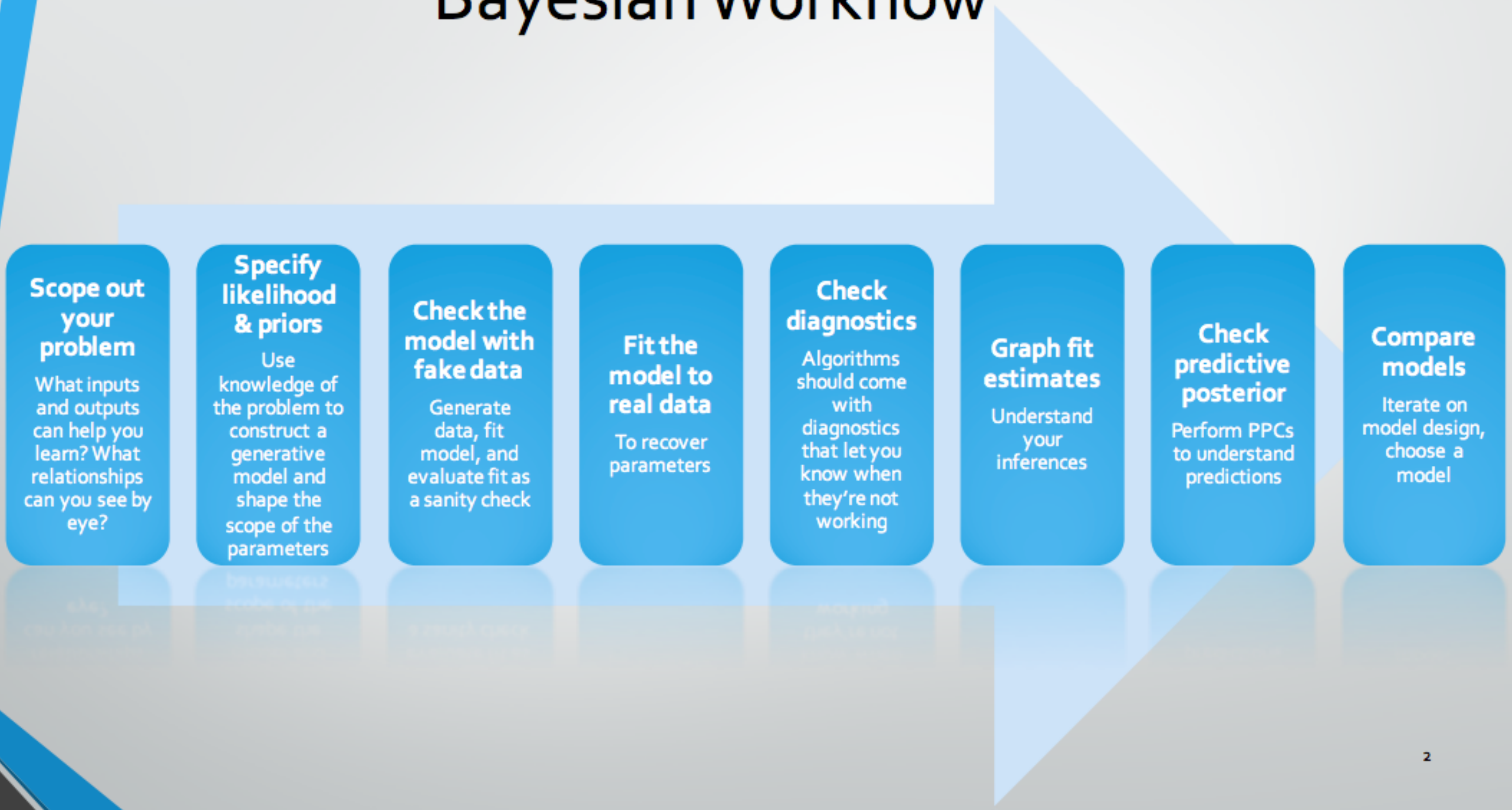
Bayesian Workflow

"How to structure the process of your analysis to maximise [sic] the odds that you build useful models."

-Jim Savage

Sean Talts
Core Stan Developer

Bayesian Workflow



Scope out your problem
What inputs and outputs can help you learn? What relationships can you see by eye?

Specify likelihood & priors
Use knowledge of the problem to construct a generative model and shape the scope of the parameters

Check the model with fake data
Generate data, fit model, and evaluate fit as a sanity check

Fit the model to real data
To recover parameters

Check diagnostics
Algorithms should come with diagnostics that let you know when they're not working

Graph fit estimates
Understand your inferences

Check predictive posterior
Perform PPCs to understand predictions

Compare models
Iterate on model design, choose a model

Modeling Heights and Weights

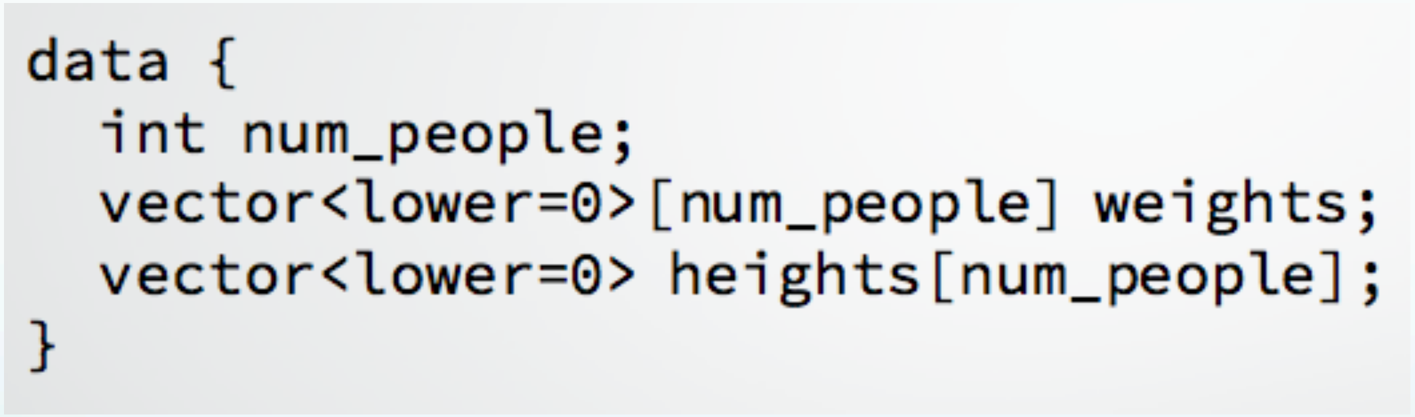
$$height \sim N(\alpha + \beta * weight, \sigma^2)$$

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - Data
  - Transformed Data (optional)
  - Parameters
  - Transformed Parameters (optional)
  - Model
  - Generated Quantities (optional)

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - **Data**



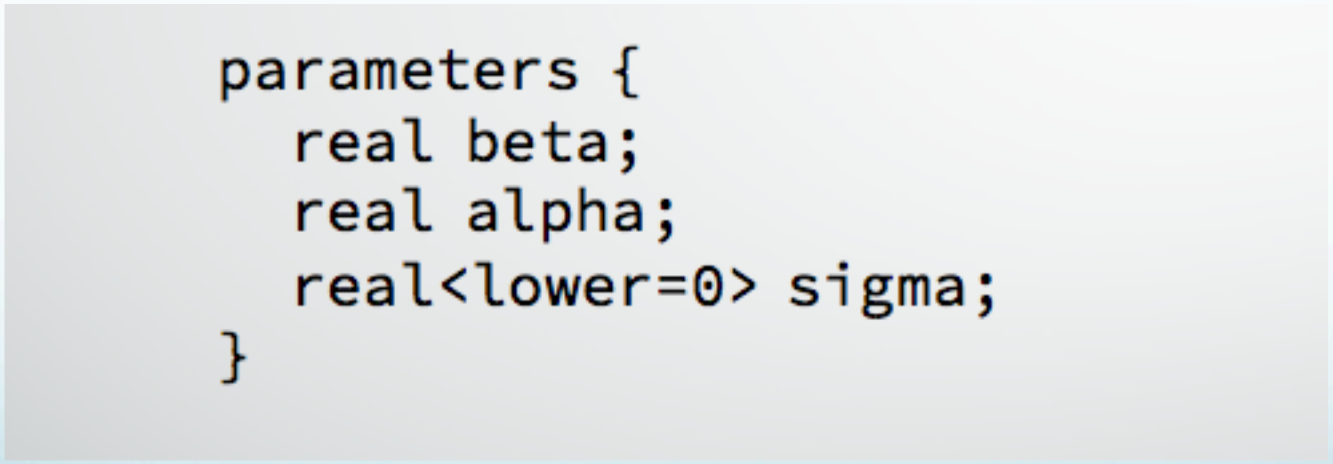
```
data {
   int num_people;
   vector<lower=0>[num_people] weights;
   vector<lower=0> heights[num_people];
}
```

Notice how variables are statically-typed, rather than the dynamically-typed format you've used in R and in Python. You'll also have to specify whenever you want to end a line with a semicolon.

Stan also allows you to bound both data and parameters.

Modeling Heights and Weights

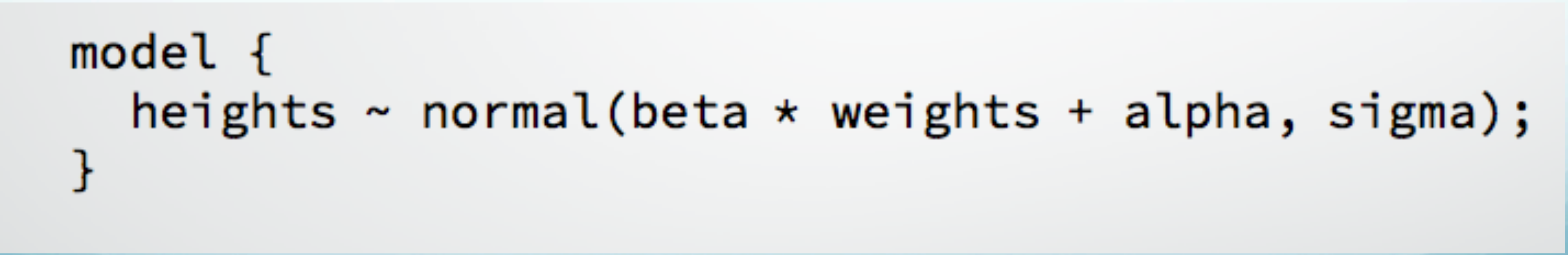
- In Stan, code is structured in blocks
  - Functions
  - Data
  - **Parameters**



```
parameters {
  real beta;
  real alpha;
  real<lower=0> sigma;
}
```

Modeling Heights and Weights

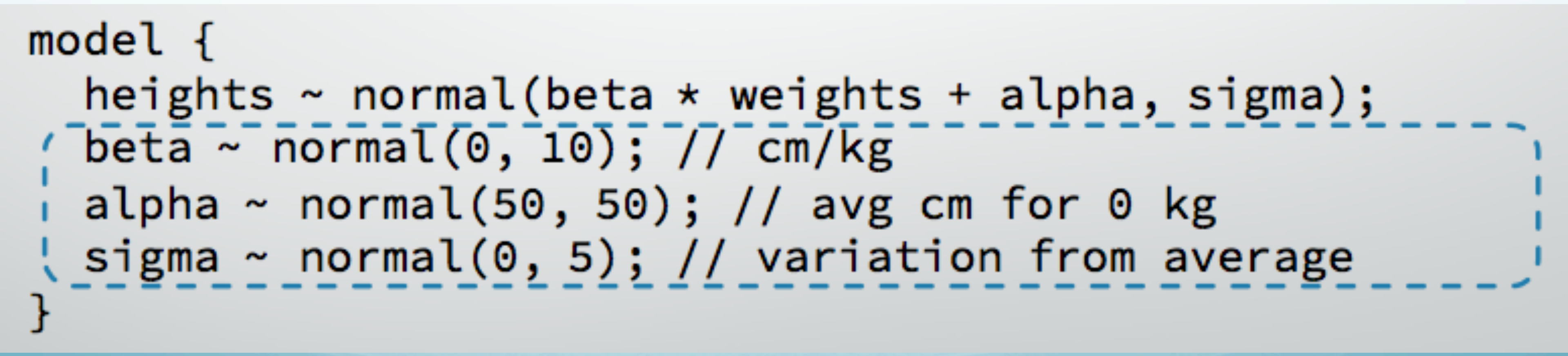
- In Stan, code is structured in blocks
  - Functions
  - Data
  - Parameters
  - **Model**



```
model {
  heights ~ normal(beta * weights + alpha, sigma);
}
```

Modeling Heights and Weights

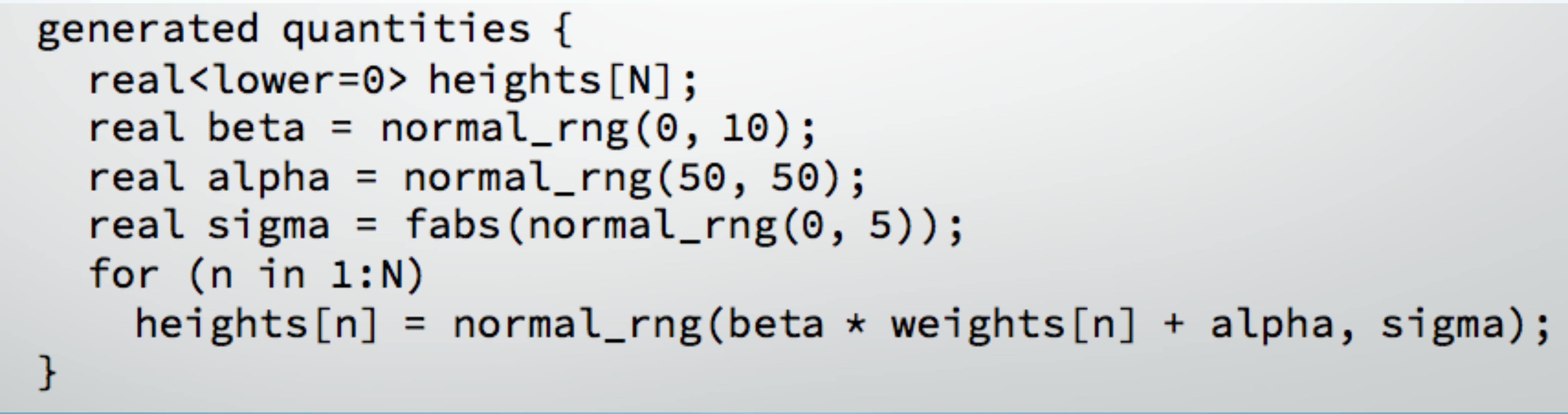
- In Stan, code is structured in blocks
  - Functions
  - Data
  - Parameters
  - **Model**



```
model {
  heights ~ normal(beta * weights + alpha, sigma);
  beta ~ normal(0, 10); // cm/kg
  alpha ~ normal(50, 50); // avg cm for 0 kg
  sigma ~ normal(0, 5); // variation from average
}
```

Modeling Heights and Weights

- After we fit our model, we can perform a sanity check
  - 1) Draw parameter values from the posterior
  - 2) Generate data based on those parameter values
  - 3) Fit model to generated data
  - 4) Check if fit is reasonable



```
generated quantities {
  real<lower=0> heights[N];
  real beta = normal_rng(0, 10);
  real alpha = normal_rng(50, 50);
  real sigma = fabs(normal_rng(0, 5));
  for (n in 1:N)
    heights[n] = normal_rng(beta * weights[n] + alpha, sigma);
}
```

Your Turn!

- Open a blank R script and source the file “count_data.R”, which is provided in the Lab materials
- Make sure you have the “rstan”, “ggplot2”, and “bayesplot” packages installed
- Try your best to fit the model described by the instructor