MEASURING SIMILARITIES ACROSS MUSICAL COMPOSITIONS: AN APPROACH BASED ON THE *RAGA* PARADIGM

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Abstract

We take a preliminary look at computational methods of identifying similarity in musical compositions. We have chosen the "Raga" paradigm of Indian classical music as the basis of our formal model since it is well understood and semi-formal in nature. We address the issues of why we need a computational basis of the judgment whether two compositions are similar or not, and why traditional distance metrics fail. The Raga paradigm is discussed and the computational attributes of a raga are defined. Machine learning techniques for assimilating the tuple that formally defines a raga are explored, and matching algorithms for identifying genres of music that have been learnt are proposed. This is essentially a positional paper and we have just started testing some of our models, but current research shows a lot of promise.

1. Introduction

Scientific research in music has many dimensions and several applications. Presently, many papers are addressing the issue of standardizing various musical techniques and parameters such as pitch, timbres, 'shruti's, and different types of alankar and styles of music. Acoustic qualities of different instruments, singers and genres have also been explored [1,2,3,4]. Work on Swara and Shruti detection from audio input is also going on [5]. Here, we take a preliminary look at computational methods of identifying similarity in musical compositions. We have chosen the Raga paradigm of Indian classical music as the basis of our formal model since it is well-understood and semi-formal in nature.

An insight into the incentive for such work may be as follows. We would ultimately like to be able to pinpoint the features that enable us to identify that a particular music sets a sad mood, or to identify that a particular piece was composed by Beethoven. Which features to use and how to use them are the cruxes of the problem. A music piece is a multidimensional work. Conventional compositions consist primarily of the beat sequence(taal), the note sequence, the lyrics, vocal and accompanying instrumental rendition; and all these taken together in the form of audio input. We propose to work in the space of note sequences, because it seems to be the most important and defining feature of a musical piece. The other facets may be looked at separately. Work is underway on converting an audio input to a note sequence though there are some open problems like that of "sa" detection [5].

The paper is organized as follows. In Sections 2, we discuss the motivation underlying our work. Section 3 proposes possible similarity metrics for our purpose. In Section 4, the *Raga* paradigm is discussed and the computational attributes of a *raga* are looked into. In Sections 5 & 6, machine-learning techniques for assimilating the tuple that formally defines a *raga* are explored, and matching models for identifying genres of music that have been learnt are proposed. This is essentially a positional paper in the sense that much of the proposed models are yet to be experimentally validated, but current research shows a lot of promise.

2. Why Search For Similarity

Historically, different forms of music developed independently in different parts of the globe with their own cultural and temporal biases. The fact that all of them share the same basic set of notes can be explained using the theory of harmonics from acoustics. Even then, not any combination of notes are musical. Some compositions are aesthetically more beautiful than others. What are the properties of a sequence of notes which make it musical or soothing to the ears? Why do we like some type of music more than some other types? Why do some compositions arouse a feeling of melancholy while some other compositions are embodiment of serenity or divinity? How can we associate a music piece to Beethoven and another piece to R. D. Burman just by listening? Our aim is to address all these questions and find out the characteristic features of a composition (if there are any), which are responsible in making that composition what it is.

At the very beginning, it should be mentioned that most of the qualities of music like mood, aesthetics etc. are subjective and the same composition which gives someone the feeling of joy might be sorrowful for someone else. A person's reaction to a particular composition is affected by his/her cultural background, past experiences and many other psycho-social factors and not just the composition. Detailed study of such factors is not the objective of this paper. Rather, we are interested in identifying the objective features behind these subjective qualities, if there are any. Empirically we find that certain traits of compositions are uniquely identified by majority of the listeners. Experiments might be carried out to find which features are identified uniquely by the listeners in a statistically significant way. However, for our purpose we assume that "compositions with similar patterns have similar impact on people with similar psycho-social background." Exactly what features of compositions will be considered and what is meant by a pattern will be discussed at length in rest of the paper. In this section, we shall try to place our hypothesis on the theory of Indian Classical Music and discuss some of the practical applications of measuring similarity.

2.1 Theory of Raga in Indian Classical Music

The basic unit of Indian Classical Music is a *Raga*. 'Raga' literally means "colour". Thus a piece of music or *raga* is supposed to paint different moods and thoughts. Raga *Desh* is famous for its feeling of 'belongingness' while raga *Lalit* is known to impart a sense of 'beginning'. Every raga is associated with a time of the day when it is supposed to be rendered. Thus *Bhairav* is an early morning raga whereas *Yaman* is suitable for the evening and Malkauns is to be rendered late in the midnight. Some ragas are associated with seasons too. *Miyan ka Malhar* is ideally sung in the rainy season while *Vasant* is the coveted raga of spring.

The association of ragas with *rasa* (mood) or time of the day or a particular season is indicative of "**similar pattern** – **similar impact**" hypothesis described above. Although the theory of raga has never been scientifically tested, for ages the theory has been accepted and its roots are well established in Indian society. One aim of this work is also to see to what extent the raga theory can be scientifically established.

2.2 Applications

Besides searching for the answers to basic questions of musicality and effect of music on mood, the concept of similarity between two compositions has several practical applications. Some of them are as follows:

• Artificial Composer: Given a particular situation, the mood of the song, the lyrics and other related information, an artificial composer comes up with a newly composed tune aptly fitted to the lyrics. The first step to such a system is to pinpoint the features which would generate a particular mood and other situation specific impacts. Pattern studies across compositions can lead us to the goal.

- Database Query: Often people want to listen to a particular kind of songs but they are unable to specify formally what type they exactly want. This problem can be neatly solved using similarity searching measures. Suppose we have a huge database of compositions and user says (s)he would like to listen to the songs which are similar to songs A, B, C etc. The system can automatically learn the features common to all the pieces and searches for that features in other songs of the database and appropriately returns the desired songs.
- Studying the Chronological and Geographical Development of Music: Different classes and genres of music influence each other and all of them evolve in course of time. Similarity measures among genres of music, among different *gharanas* or among the music of different composers can be a good indicator of how one form has been influenced by the other. By coupling this measure with other historical and socio-cultural facts, we might be able to trace the history and evolution of genres of music.

3. Similarity Metrics

Musical pieces, as mentioned before, are multidimensional in nature. Ideally we should choose the audio input as the space in which to define our metric. However, we are interested in the note sequence of the composition only – precisely due to the following reasons:

- The lyrics pertaining to a music piece has its own pragmatics, and hence biases the user towards a specific mood. Since we do not want to use such explicit linguistic information, which have already been studied in detail, we will avoid using this.
- Vocal quality of the renderer and the specific instruments used in a piece are often sufficient to produce some bias, which might be based on cultural conventions. In the Indian context, *Shehnai* is associated with festivities and *Jaltarang* or *Santoor* have a lively tonal effect. Such prejudices should be avoided.
- Rhythm or 'taal' is an integral part of any composition. It is a measure of the pace and the periodicity of the composition, and hence can be an important metric. This may be studied separately.

The sequence of notes in a composition plays an important role in imparting the characteristics of that music. Mood, genre, temporal and spatial settings are largely governed by the note sequence. This concept has been the heart of the Raga paradigm of Indian Classical Music. It seems worthwhile to search for the basic characteristics of a composition in its note sequences following the Raga formalism. In the next section we look at the Raga formalism in depth. However, even before that we need to find out when we say two sequences of notes are similar and if we do, what is the degree of their similarity.

3.1 Formal Model of a Composition

Any note sequence is a string of alphabets belonging to a finite set \mathcal{M} , where \mathcal{M} is the union of three sets \mathcal{M}_{tar} , \mathcal{M}_{madhya} , \mathcal{M}_{mandra} , each of them consisting of the 12 basic notes or *swaras*. The subscripts tar, madhya and mandra are indicative of the higher, medium and lower octaves respectively.

The sets have been enumerated as follows;

$$\mathcal{M} = \mathcal{M}_{tar} \cup \mathcal{M}_{madhya} \cup \mathcal{M}_{mandra}$$

$$\mathcal{M}_{madhya} = \{ S, r, R, g, G, m, M, P, d, D, n, N \}$$

$$\mathcal{M}_{mandra} = \{ S<, r<, R<, g<, G<, m<, M<, P<, d<, D<, n<, N< \}$$

$$\mathcal{M}_{tar} = \{ S>, r>, R>, g>, G>, m>, M>, P>, d>, D>, n>, N> \}$$

Where,

Our Notation	Indian Notation
S	sa
r	komal re
R	suddha re
g	komal ga
G	suddha ga
m	suddha ma
M	tivra ma
P	pa
d	komal dha
D	suddha dha
n	komal ni
N	suddha ni
X>	Note <i>X</i> in higher octave (tar saptak)
X<	Note <i>X</i> in lower octave (mandra saptak)

Table 1: Notation Index

The concepts of *grace notes*, *gamak* and *meend* have not been taken care of by the proposed model. Since we are not interested in complete modeling of the raga system, formalizing the basic note sequence suffices, though we may augment the notation later to fit in the above concepts. Having mapped the composition to the form of a "string", we may try and exploit the well-studied properties of strings, which form the basis of Formal Language Theory [6].

3.2 Traditional Approaches

There has been a plethora of work on string matching algorithms. The exact string matching algorithms like the Rabin-Karp or Shift Or algorithm tries to find exact match among the substrings of two given strings [7]. However for most of the practical problems, approximate string matching and maximal alignment algorithms are more useful. Just to cite an example, in the field of *bioinformatics* the problems of nearest *gene* matching or *protein folding* are often solved using approximate string matching algorithms [8,9]. These algorithms define several metrics for calculating distance or in other words the similarity between two strings. Suppose we run such an algorithm on a database of note sequences (compositions) to find out compositions similar to a given one. Possibly we will end up with a handful of nearly identical pieces rendered in slightly different ways. However such an algorithm would fail to point out compositions which are of, say, similar mood. Since the traits we are trying to identify do not reside in the entire string but in certain parts of the string and the repetition patterns, domain specific pattern matching algorithms are likely to perform better.

However, in order to construct a domain specific pattern matching algorithm, we need to identify the features of a note sequence which are directly responsible for imparting certain cognitively discernible properties of the composition like mood and genre. Which features to choose is itself an open question and hence we would like to learn them by taking a large set of compositions sharing a certain quality and identifying some similar patterns commonly present. We would like to ensure that this is a two way correlation, i.e. the identified pattern(s) are unique to those compositions having the quality in question. Since we require a formal model to start with, we have chosen the Raga formalism.

4. The Raga Formalism

"The Raga system is unique for Hindustani Classical Music. A Raga has a specific melodic structure with arrangement of notes. Certain essential features are extremely necessary to establish a Raga. In the Hindustani Classical Music sphere, Ragas are many and each has its distinctive qualities. Besides there is a broad time cycle which is followed while rendering a Raga. By the definition which is

normally used to define a Raga, the most prominent feature which stands out is that a Raga should 'colour' or please the minds of the listeners."[10]

The motivation behind choosing the Raga formalism as the basis of our work is threefold.

- It is semiformal and hence many aspects of it are computable.
- The *Raga* "grammar" is multifaceted in the sense that very different paradigms of features are used like a defining note set (constraint based feature), most prominent notes (frequency based features), characteristic note substrings (sting based feature) etc.
- Each *Raga* is associated with certain *rasa*(s)(mood), time of the day, season of the year and a specific overall "personality". Since our goal is to search for similarities and distinctive traits in compositions, the *Raga* formalism seems to be a good platform to start with.

We discuss the defining features of a *Raga*, as given in the literature and discuss its computational feasibility in the following subsections.

4.1 Thaat

That is a way of classification of Ragas. Each That is a set of seven notes in an octave. Classification of Raga into a That is empirical rather than strictly formal, because a raga might use more than 7 notes. For example, that Khamaj contains the notes $\{S, R, G, m, P, D, n\}$ but the raga desh belonging to this that uses both n(komal ni) and N(suddha ni). Therefore, that is not strictly computable – though it may be computable for some ragas.

However, the set of notes used by a raga is computable. We call this the cover set $C_{\mathcal{R}}$ for the raga \mathcal{R} . If the cover set consists of seven notes, we can identify the *thaat*. If it contains less than seven notes, it is mappable to a set of possible thaats, whereas if it contains more than seven notes, it is not directly mappable to any *thaat*. As a matter of fact there are 10 *thaat*s defined in Hindustani classical music out of a possible 32, while Carnatic classical music has all 32 *thaats*.

4.2 Arohana & Avarohana

Arohana is the ascending sequence of notes which the raga follows. Any ascending sequence in improvised portions of the raga follows the pattern defined in *Arohana* strictly. Similarly, *Avarohana* is the corresponding descending sequence.

To define *Arohana* and *Avarohana* formally, instead of treating M as a set, we have to define a full linear ordering on its elements. This is already present in form of the frequency of the notes. In \mathcal{M}_{tar} , \mathcal{M}_{madhya} and \mathcal{M}_{mandra} , the ordering of the notes is from left to right (supremum being at the right) as presented in 3.1. Further any element of \mathcal{M}_{mandra} is less than any element of \mathcal{M}_{tar} . Arohana is thus a strictly increasing sequence specific to a raga. However, this is not always the case, and there may be occasional decreasing steps in the *Arohana*. For example, the *Arohana* for raga *Puriya Dhanashri* is "N < r G M (P M) (d P M) d N S>" where the parenthesized substrings are clearly descending sequences. The case is exactly similar for *Avarohana*.

While strictly increasing and decreasing substrings are easily computable, "crooked" *Arohanas* and *Avarohanas* pose a bigger challenge.

4.3 Vadi, Samvadi and Vivadi

Vadi is the most prominent (sonant) note in the composition, and the most highly emphasized note during improvisations. *Samvadi* is the second most prominent note in the composition. *Vivadi* is the set of the notes strictly not used in the Raga. *Durbala* are notes used very infrequently in the Raga. The set of remaining notes are called *Anuvadi*.

If "prominence" means significantly changing the amplitude or using techniques like *gamaka* (melodic embellishment giving special vibratory effects) while rendering the note, our formal model

would fail to capture the concept of prominence and in that light *Vadi* and *Samvadi* will remain incomputable. However if the concept of prominence is based on the note's frequency in the composition, then we may obtain the *Vadi* and *Samvadi* by obtaining the frequency distribution of notes over the composition. It should be noted here that these concepts may be ambiguous. For a particular raga, the notes which the literature defines as *Vadi* and *Samvadi* are often found to be inconsistent from the point of view of maximum emphasis [11].

4.4 Pakada

'Pakada' literally means a grip. In musical parlance, *pakada* is the signature phrase for the raga. Given two very similar ragas, they differ at least in their *pakada*s.

Formally, the *pakada* is any string which conforms to the *Arohana* and *Avarohana*. It is important from a computational point of view since it is unique for the raga. The problem, however, lies in the facts that the length of the *pakada* is unbounded, and the *pakada* might not be present in its entirety in the composition, but parts of it may be distributed over the composition as the underlying "theme" of the composition. Such hurdles force a nondeterministic way of identifying the *pakada*. To take a case in point, raga *Puriya Dhanashri* has the *pakada* N < r G M P M d P M d N S > r > N d P M P G M R G M G R S of length 26, while raga*Yaman*has the*pakada*G M D N S > N D P M R G R N < R S of length 15.

4.5 Other Features

Gambhirya or Chanchalata of a Raga may be measured by the relative positions of consecutive notes in the composition. A chanchal raga frequently jumps between low and high frequency notes whereas a gambhira raga is more sluggish. For example, Malkauns is a gambhira raga, while Miyan Ka Malhar is a chanchala raga. The notion of gambhirya is computable by measuring the extent of jumps between consecutive notes.

Ragas are also classified based on the range of notes they use. Ragas which heavily use notes from *Sa* to *Ma* are known as *purvangavadi* whereas those which use notes from *Ma* to *Ni* are called *uttarangavadi*. Interestingly, *purvangavadi* ragas are sung from sunrise to sunset, and *uttarangavadi* ragas are sung at night. We can computationally classify a raga as *uttarangavadi* or *purvangavadi* depending on the relative frequencies of different notes used.

'Chalana' literally means the course or path. *Chalanas* are strings larger than the *pakada*, and are generally followed while improvising a raga. They tend to be *gharana* specific and are not formally defined but handed down from *guru* to *shishya*. Computationally, *chalanas* can be identified like *pakadas*, but are usually large and not unique to a raga.

5. Computing Features

There are two distinctive phases of computation: the learning phase – where we extract features common to a set of compositions, and the matching phase – where we decide, given a new composition, whether it is like the set of compositions on which we applied pattern extraction techniques. The learning phase may be computationally intensive, but since it is usually performed offline, the time complexity is not a big factor. We may place the features to be learnt in three different categories:

5.1 The Cover Set and Frequency-based Features

These are all features computable from a 1-gram frequency distribution. The frequency distribution

 $F = \{ count_a \}$ for all alphabets $a \in \mathcal{M}$ where $count_a$ is the number of occurrences of alphabet a in the set of compositions P.

The Cover Set *C* of a set of compositions may be obtained by :

 $C = \bigcup C_{\mathcal{R}}$ for each composition R belonging to that set.

= \bigcup **a** for each alphabet **a** with *count_a*>= 1 in \mathcal{F}

We may further refine the cover set by only keeping those members which occur above a threshold $t_l > 1$. The significance of this threshold will be seen shortly.

The Vivadi Set \mathcal{V} is the set of notes which are not used in any composition \mathcal{R} belonging to the set of compositions. However, experts may use them a few times in improvised sequences. Thus, the Vivadi set \mathcal{V} is the set of notes with frequency below a certain threshold. We may thus say:

$$V = \mathcal{M} - C$$

The vadi and samvadi on the other hand, are notes used most often in any composition. We can define the high frequency set \mathcal{H} as

 $\mathcal{H}= \bigcup \{\mathbf{a}\}$ for each alphabet \mathbf{a} with $count_a >= t_h$ in \mathcal{F} where t_h is a high threshold.

The t_h and t_l may be fixed, or learnt based on sharp gradient changes in the frequency distribution \mathcal{F} .

In the matching phase, we find out the frequency distribution \mathcal{F} for the composition R which we want to match. We then apply the same technique to find out the $C_{\mathcal{R}}$ $\mathcal{V}_{\mathcal{R}}$ and $\mathcal{H}_{\mathcal{R}}$ of a composition R. We then try and find the corresponding distances between the C, \mathcal{V} and \mathcal{H} of the composition in question and the set of compositions learnt. We may take the number of elements in the intersection of the corresponding sets, and may weight them by the relative frequencies of the elements of the intersection set in \mathcal{R} .

We may not learn or match the formal vadi, samvadi and vivadi of a raga this way, but as we are interested in matching paradigms of compositions and not learning ragas, this method suffices.

5.2 Arohana - Avarohana

The Arohana and Avarohana are substrings which increase and decrease respectively with a certain tolerance. The increase/decrease of note sequences based on the linear order within an octave is given by the Arohana and Avarohana, and how to cross an octave. It is thus not necessary to distinguish between similar notes of tar, madhya and mandra.

We may approach the problem from 3 angles:

- Learning Arohana and Avarohana within a given tolerance
- Learning Arohana and Avarohana as strictly increasing or decreasing subsequences
- Purely statistical approaches

To learn Arohana and Avarohana with a given tolerance, we need to define tolerance. Tolerance can be taken to be the number of elements of the sequence which violate the monotonicity of the Arohana or Avarohana.

Arohana can then be learnt from a set of compositions by marking out increasing sequences within the given tolerances. We can then go for maximal overlap of the most frequently marked strings to generate new strings, as long as the monotonicity aberrations do not cross the tolerance value. We can apply the same technique for Avarohana.

If, however we are merely interested in the modus operandi of increase and decrease of notes in the sequences, we need not look at "crooked" sequences. We can simply mark out all increasing substrings. We can then compute the longest increasing subsequences which occur most frequently.

We may also take another approach which starts off with the smallest note (based on the total linear order) above a cutoff frequency t_l in marked increasing substrings. This is the first element of the Arohana. We choose the next element based on which next smallest alphabet (based on the total linear order) occurs in the marked substrings with a frequency above the cutoff frequency t_l . We go on appending alphabets in this fashion until we cannot add any further. This is the learnt Arohana. We can similarly grow the Avarohana.

We may also look at purely statistical methods. We find the frequency distributions of 3-grams and 4-grams. We can overlap the most frequent 3-grams and 4-grams to obtain the Arohana and Avarohana as long as the monotonicity aberrations do not overshoot the tolerance.

In the matching phase, we simply see whether the ascending and descending sequences match the constraints imposed by the Arohana and Avarohana. Ideally, they should clearly follow the constraints. But we may count the number of exceptions and give a measure of belief that one composition follows a particular Arohana/Avarohana based on this.

5.3 Pakada and Chalana

The "pakada" is the signature string of the composition. Snatches of it are omnipresent throughout the composition and it is unique to the composition. Hence it is crucial that we are able to identify the pakada closely.

We may use some evolution strategies for identifying the pakada. Genetic algorithms may be used in this case[12]. For learning, we may first split up the set of compositions into strings of various lengths less than standard pakada lengths. The fitness function may be defined on the basis of the degree of match found with other strings in the gene pool. The crossover operators may be looked on as maximal overlap of the selected genes. Ways of introducing variation (mutation) without landing up at wrong results need to be identified also.

We may look at simpler statistical models like splitting up the compositions into 4-grams or 5-grams, and then taking overlaps of the most frequent 4-grams or 5-grams. In this way, we generate strings upto a threshold length.

We need not necessarily arrive at a single pakada. If a composition or set of compositions has/have many signature phrases, we would ideally like to identify all of them.

In the matching phase, we may generate the pakadas as described above. We can then apply maximal substring matching algorithms, and the length of the maximally matched substring gives us a measure of belief that one composition belongs to a particular genre of compositions based on this. Other than string matching, more generalized pattern matching algorithms may also be tried out.

We can also try matching frequent 3-grams or 4-grams in the composition to be matched with the pakada of the genre.

Chalanas are larger strings which are more specific to the "gharana" of a composition. We may obtain chalanas in a similar fashion as pakadas. Chalana identification is only required if we want to

match music even more closely than over paradigms. It is intuitive that the chalanas should be larger than the pakadas in this respect.

5.4 Other Metrics

The chanchalata or gambhirya may be another computable metric. The chanchalata may be easily calculated by defining a distance function between elements of $\mathcal M$. The distance is intuitive from the distance between the elements of $\mathcal M$ in the linear ordering defined. The average distance "jump" between successive notes in a set of compositions gives us the chanchalata or gambhirya. The chanchalata of a set of compositions can be taken to be a mean value or a range. In the matching phase, we simply calculate the chanchalata of the composition to be matched and check its difference with the chanchalata of the set of the set of compositions we have learnt.

Uttarangvadi and Purvangvadi compositions may also be identified using simple statistical counts. The predominant range in the frequency distributions of the learnt set and the matching composition may be calculated, and their overlap can give us a measure of closeness.

These two metrics can be used as elimination metrics as in eliminating those candidate paradigms where these parameters match poorly.

In conclusion, we point out that a yes – no answer in rarely wanted to the question of whether one composition is of a particular kind. We should ultimately output a belief based on the studied parameters, depending on which parameters give significant results.

6. Conclusion

In this paper we have discussed how similarity measures between note sequences of two or more compositions can be used to answer some of the basic questions of aesthetics, moods and impacts of music. We have also discussed some practical applications of similarity measures. For its basic notion of mood-music relations and semiformal nature we have used the raga theory of Indian classical music as our starting ground. We are experimenting on the proposed method. The frequency distribution of notes and its relation to thaat, vadi, vivadi and samvadi notes has been studied and the results are significant. Unfortunately, we could not test our methods on sufficiently large datasets as note sequence of detailed improvisations by the experts are never written down in Indian tradition. Improvisation is impromptu. Neither are there good software for extracting the note sequence from the audio inputs. Currently we are working on the Arohana and Avarohana metrics.

Although a major portion of the paper deals with the theory of raga and its different formalisms, it should be noted that our objective is not raga learning or detection. We intend to measure similarities across different genres of music, be it oriental, western or folk. In the process, we might have to redefine the metrics, design entirely new metrics and leave many of the conventional raga metrics. It will be interesting to study how the other dimensions of a composition like the rhythm, choice of instruments, and emphasis on specific notes in terms of the amplitude affect the quality of the music.

Future endeavours in this field might include modeling of note sequences as a fractal space and defining new distance measures between sequences; modeling compositions as sequence of relative frequency jumps between adjacent notes rather then just sequences of notes and developing the applications stated in this paper.

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