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Reformulation of parameters of the logistic function applied to power curves of wind turbines

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ABSTRACT

The current procedure for obtaining the parameters of the logistic function, used as a model for the power curve of wind turbines, provides meaningless values. These values are different for each wind turbine and obtaining them requires an optimization process. This paper proposes a procedure to obtain the parameters of the 4-parameter logistic function based on the features of the power curve, providing a model that is a function of the power curve parameters supplied by the manufacturer. Furthermore, that model can be used to derive another 4-parameter model and a 3-parameter model is proposed for certain conditions. The three models consist of a continuous function which simplifies the implementation of the curve in a computer program compared to piecewise models. In addition, the probability density function of the output power of a wind turbine is derived by using each model.

1. Introduction

The power curve of a wind turbine (WT) reveals the relationship between the wind speed at hub height and the electric power supplied, and so it is widely used when analyzing or studying a WT or a wind farm. Here, it is understood that the power curve of a WT is the one provided in the manufacturer's documentation. This paper will focus on this curve although measurements in WTs in a wind farm will eventually reveal operating conditions which can slightly differ from those described by the manufacturer. Of course, circumstances such as turbulence, wind shear, wake effects, icing, gusty winds or even component fatigue in the WT will affect the operating condition. As time goes by, other problems linked to ageing may also appear and these can interfere with optimum output of the WT blades. Power curves given by the manufacturer can nonetheless be interpreted as good approximations to a standard WT behaviour, so there will be no more discussion here about operating conditions themselves.

As defined in [1], WT manufacturers obtain the power curve from tests that provide pairs of points for wind speed-power every 0.5 m/s. Hence, the power curve will be the function, $P = f(u)$, which minimizes the distance to these points. Manufacturers [2–6] usually provide a graph establishing this relationship, which can be very helpful when obtaining the output power of a WT from the wind speed at hub height. However, when dealing with a large amount of data and in order to implement the relationship in a software program, the use of such a graph can be cumbersome, and this is why mathematical expressions are used for power curves. The expressions can substitute the pairs of points for piecewise continuous or continuous functions.

Among the applications of the models of the power curve it can be pointed out to indicate anomalies in the WT working, to forecast the power supplied by a WT, to simulate potential scenarios of wind power production and to compare the performance of different WT.

Currently, several models are preferred for establishing the relationship because they have a low value for errors and are easy to manage. In most cases, models consist of a continuous or piecewise function defined for all values of wind speed, extending the values obtained in the manufacturer's tests [7] to all possible values. Furthermore, given that the output power of a WT equals zero for values of wind speed lower than the cut-in wind speed, u_{ci}, (2–5 m/s) and higher than the cut-out one, u_{co}, (20–30 m/s), this will be the interval to define a model for the power curve.

The most commonly used models are those that utilize polynomial functions to approximate the relationship mentioned previously. Linear [8], quadratic [9–12], cubic [13–15], least-square [16,17] and spline [16,18–20] models are well known, but there are others [16,21–24]. In all cases the function is piecewise due to the specific shape of the power curve, which can be seen in Fig. 1. However, there are some other models that are based on a single continuous function [25–27] and provide good results. Among

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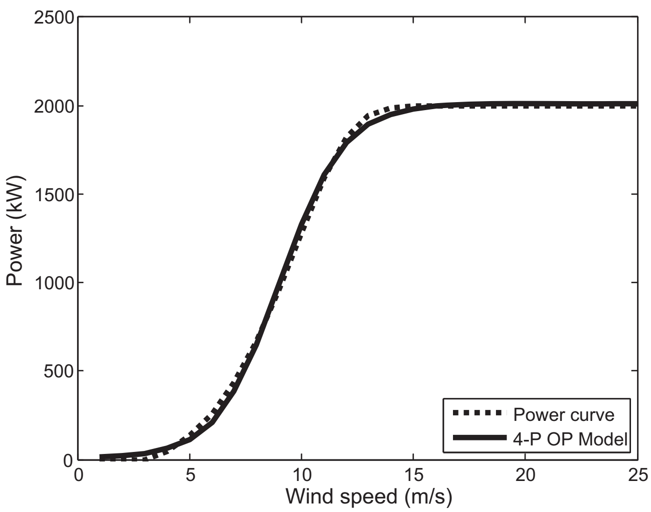

Fig. 1. 4-Parameter logistic model for the power curve of Vestas V80.

them, the 4-parameter logistic function and the 5-parameter logistic function are most commonly used.

Among the piecewise models, linear, quadratic and cubic ones appear most frequently in the literature. However, when compared with those based on a single continuous function, they provide errors around the rated wind speed of the WT that are almost completely avoided when using the logistic functions. In fact, in piecewise models, there is a lack of continuity of the slope around the rated wind speed value, which does not represent the real working of the WT, while in the case of the logistic functions this does not happen because they consist of a single function and it is derivable on the domain. This is the main reason of the inaccuracy of the piecewise models.

Moreover, when performing electrical system analysis it is very helpful to make available continuous models in order to derive analytical expressions that can be used as inputs of the analysis itself. For example, an analytical method to solve the Probabilistic Load Flow requires the Probability Density Function (PDF) of the power supplied by the generators as input, which can be easily obtained from the logistic functions and can hardly be derived in a single expression from piecewise models.

As all these models need the use of the aforementioned parameters, some procedure must be described for obtaining them. In fact, for the 4-parameter logistic function the procedure is already established [25] and the results of the comparison between the manufacturer's power curve and the model can be seen in Fig. 1.

The error values when using the 4-parameter (4-P) logistic model using an optimization process (OP) to obtain the parameters (4-P OP Model) for Vestas V80 and other WT are shown in Table 1 (Mean Absolute Percentage Error, MAPE), Table 2 (Mean Absolute Error, MAE) and Table 3 (Root Mean Squared Error, RMSE).

The procedure suggested in [25] may not always be the best option as this will depend on the kind of objective being pursued. In this paper, an alternative procedure to obtain the parameters of the 4-parameter logistic function is provided in order to improve the model. Additionally, some simplifications of the model are presented.

This paper is organized as follows: in Section 2, the 4-parameter logistic function is introduced and the suggested procedure for obtaining its parameters is given. In Section 3, the PDF of the output power of a WT is derived using the models presented here, taking advantage of its continuity. Section 4 includes results of the application of the models for several WTs and the conclusions are finally given in Section 5.

Table 1
MAPE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	0.0096	0.0100	0.0079	0.0078	0.0118	0.0087	0.0087
4P-DP	0.0162	0.0133	0.0151	0.0119	0.0169	0.0143	0.0129
4P-DS	0.0185	0.0103	0.0163	0.0113	0.0181	0.0148	0.0111
3P-DP	0.0198	0.0109	0.0176	0.0124	0.0194	0.0158	0.0117
Linear	0.0492	0.0474	0.0770	0.0394	0.0200	0.0694	0.0443
Quadratic	0.0902	0.0307	0.1324	0.0763	0.0299	0.1194	0.0763
Cubic	0.1276	0.0563	0.1775	0.1099	0.0599	0.1607	0.1099

Table 2
MAE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	19.2696	20.4170	18.1581	16.0462	27.1972	31.4132	60.5502
4P-DP	32.4614	27.2336	34.7689	24.3311	38.8846	51.3634	89.9470
4P-DS	36.9029	21.2036	37.5170	23.2441	41.6216	53.3197	77.3338
3P-DP	39.5474	22.3069	40.5822	25.3636	44.6043	57.077	81.6301
Linear	98.4040	97.2145	177.0171	80.8145	46.1067	249.6831	310.0873
Quadratic	180.3773	62.9964	304.6291	156.3958	68.7169	429.8800	533.5663
Cubic	255.2821	115.5071	408.2103	225.3624	128.5431	578.3840	768.8938

Table 3
RMSE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	25.3319	26.2691	24.5692	21.2895	39.1501	42.6797	95.6218
4P-DP	56.2597	39.7432	59.9937	38.1073	68.3914	85.3121	146.1193
4P-DS	55.4600	35.1524	54.6059	35.5069	65.3848	80.6301	129.8385
3P-DP	60.1208	38.5687	60.3170	40.6524	72.2248	90.3829	147.0264
Linear	184.6801	172.6738	303.5940	156.1934	86.0137	449.0788	449.0788
Quadratic	313.4841	128.0031	486.2839	278.4891	143.2952	714.1565	714.1565
Cubic	431.7377	230.4151	639.4723	392.4452	247.3919	941.3501	941.3501

Table 4
Values of parameter '*a*' for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	2011.1	2065.2	2308.1	2062.9	2310.4	3620.0	7010.9
4P-DP	2000	2050	2300	2050	2300	3600	6995
4P-DS	2000	2050	2300	2050	2300	3600	6995
3P-DP	2000	2050	2300	2050	2300	3600	6995

Table 5
Values of the parameter '*m*' for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	2.6650	−0.959	0.3980	−2.6650	1.7140	0.0270	−6.1620
4P-DP	58.8241	13.2559	40.3625	14.5302	55.4994	37.9601	20.8823
4P-DS	0	0	0	0	0	0	0
3P-DP	0	0	0	0	0	0	0

Table 6
Values of the parameter '*n*' for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	622.922	461.212	519.045	343.993	667.839	556.459	532.502
4P-DP	645.192	458.014	516.351	341.187	650.616	560.327	548.440
4P-DS	645.192	458.014	516.351	341.187	650.616	560.327	548.440
3P-DP	357.702	383.590	316.867	266.146	374.403	364.959	431.355

2. Proposed models

The models proposed in this section are based on the 4-parameter logistic function model, in which the relationship between wind speed and generated power is a continuous curve described in Eq. (1). Notice that this function can only be applied to values of wind speed between u_{ci} and u_{co}, the power is 0 for wind speeds outside this range.

$$P(u) = a\frac{1 + me^{-u/\tau}}{1 + ne^{-u/\tau}} \tag{1}$$

where, a, m, n, τ are parameters of the model and considering that $m \neq n$. The values of the parameters can be obtained by using optimization techniques as has been proven in [25], providing a good solution considering error as the sole objective. However, these parameters have no technical meaning. If the intention is to know the influence of the WT's features and behaviour, another type of model is needed.

The main reason for using parameters with technical meaning is in order to assess the power curve from a theoretical point of view and obtain the weight of each parameter not just in the power curve but in derived expressions such as, for instance, the PDF of the output power. This assessment cannot be done if the parameters are obtained from an optimization process. Furthermore, as the parameters with technical meaning are obtained from a deterministic process, they can be calculated easily and used as starting points in the optimization process to obtain the parameters of the power curve as in [25] and so reduce computing time.

The values of these parameters when using the 4-P OP Model for Vestas V80 and other WT are shown in Table 4 (a), Table 5 (m), Table 6 (n) and Table 7 (τ).

2.1. 4-Parameter model using a deterministic process

The approach in this section consists of obtaining the parameters of the model from the features of a WT power curve. Every feature assessed imposes a constraint that must be satisfied by the model, and this contributes to the configuration of the final result.

The first step consists of considering a number of features equal to the number of parameters in the model, in this case four, which are:

(i) $\lim\limits_{u\to\infty} P(u) = P_r$

When the wind speed tends to infinity, the limit of the function has to be the rated power of the WT (P_r). In fact, the value of the function is equal to P_r for values of wind speed between u_r and u_{co}.

(ii) $\left.\frac{dP(u)}{du}\right]_{u=u_{ip}} = s$

At the inflection point (u_{ip}), the first derivative of the function with respect to the wind speed equals the slope of the curve. This constraint gives the model the same slope as the power curve at its inflection point.

The inflection point and the slope of the curve can easily be obtained from manufacturer data if they are provided as pairs of points, as defined in [1]. The vector of differences (difference between each value and the previous one) is obtained from the vector of power values: its maximum value provides the slope and the wind speed corresponding to this value, the abscissa of the inflection point.

(iii) $\left.\frac{d^2P(u)}{du^2}\right]_{u=u_{ip}} = 0$

At the inflection point (u_{ip}), the second derivative of the power with respect to the wind speed is 0. With this constraint,

Table 7
Values of the parameter 'τ' for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	1.4090	1.3790	1.4780	1.4870	1.3320	1.4150	1.3810
4P-DP	1.4483	1.3988	1.5256	1.5208	1.3801	1.4506	1.4048
4P-DS	1.4483	1.3988	1.5256	1.5208	1.3801	1.4506	1.4048
3P-DP	1.5936	1.4404	1.6549	1.5885	1.5088	1.5560	1.4604

the inflection points of the power curve and the model will coincide.

(iv) $P\left(u_{ip}\right) = a\frac{1+me^{-u_{ip}/\tau}}{1+ne^{-u_{ip}/\tau}}$

Eq. (1) at the inflection point (u_{ip}) of the power curve equals the corresponding value of the power. This constraint is just to make the model pass by the inflection point of the power curve.

Therefore, these features also have to be satisfied by the model in Eq. (1). Clearing the parameters, the values obtained are:

(i) $a = P_r$
(ii) $n = e^{2su_{ip}/(P_r - P_{ip})}$
(iii) $m = \left(\frac{2P_{ip}}{P_r} - 1\right)n$
(iv) $\tau = \frac{P_r - P_{ip}}{2s}$

where, P_{ip} is the value of $P(u_{ip})$.

The model has been called 4-P DP (4-parameter logistic model using a deterministic process to obtain the parameters), and its parameters are obtained directly from the features of the power curve (P_r, s, u_{ip}, P_{ip}). Notice that these four values, P_r, s, u_{ip}, P_{ip} are obtained directly from the power curve instead of using an optimization process that requires an algorithm and a proper initialization.

These four values characterize the power curve of the WT because from them it can be established a good approximation to it. The rated power (P_r) is given by the manufacturers and indicates the maximum power that the WT may provide and this power is supplied for wind speeds between rated wind speed and cut-out wind speed. Two other values are the coordinates of the inflection point (u_{ip}, P_{ip}), which is a characteristic point of the power curve because it is where the gradient of power reaches its maximum. Finally, the remaining value is the slope of the curve (s) in the inflection point which is the value of the gradient in that point, and, as it has been said, it is the maximum value that the slope can reach.

Furthermore, the expressions of the parameters allow us to make some simplifications and derivations considering circumstances related to the features of the power curve, as will be seen later in this paper.

Consideration of the parameters obtained by the process described above allows us to convert Eq. (1) into Eq. (2).

$$P(u) = P_r \frac{1 + \left(\frac{2P_{ip}}{P_r} - 1\right) e^{2s(u_{ip}-u)/(P_r - P_{ip})}}{1 + e^{2s(u_{ip}-u)/(P_r - P_{ip})}} \tag{2}$$

The comparison of this model with the power curve is shown in Fig. 2.

The error values when using the 4-P DP Model for Vestas V80 and other WTs are shown in Table 1 (MAPE), Table 2 (MAE) and Table 3 (RMSE). The parameter values in these cases are expressed in Table 4 (a), Table 5 (m), Table 6 (n) and Table 7 (τ).

In this case, it can be seen that for low values of wind speed (<5 m/s) the approximation is not very good. However, it is true that u_{ci} is usually between 2 and 5 m/s. The rest of the model is more or less like the better one using the 4-parameter logistic function.

Alternatively, another model may be obtained by simply arranging the parameters, as can be seen in Eq. (3):

$$P(u) = a\frac{1 + e^{k_1 - u/\tau}}{1 + e^{k_2 - u/\tau}} \tag{3}$$

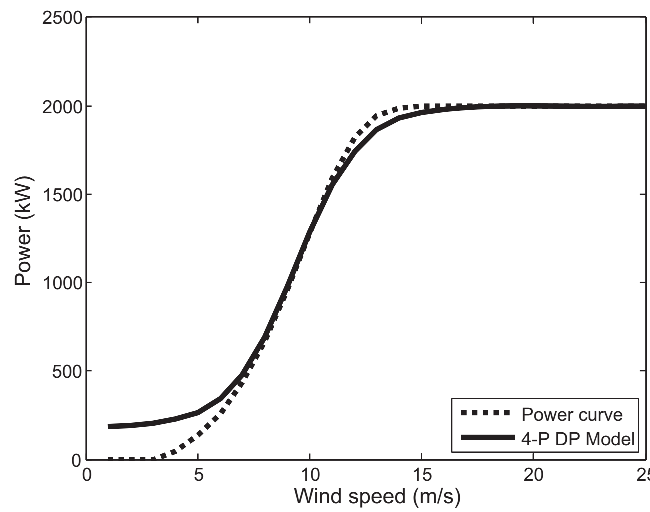

Fig. 2. Model 4-P DP for the power curve of Vestas V80 2MW.

where,

(i) $a = P_r$
(ii) $k_2 = \frac{2su_{ip}}{P_r - P_{ip}}$
(iii) $k_1 = \log\left(\frac{2P_{ip}}{P_r} - 1\right) + k_2$
(iv) $\tau = \frac{P_r - P_{ip}}{2s}$

These expressions are simpler than the previous ones except for k_1. From this point on no attention will be paid to k_1, but the others will be very helpful.

2.2. 4-Parameter model using a simplified deterministic process

Models (1) and (3), as $m \ll n$, or $k_1 < k_2$, can be simplified by considering $m = 0$, or $k_1 \to +\infty$, as in Eqs. (4) and (5).

$$P(u) = a\frac{1}{1 + ne^{-u/\tau}} \tag{4}$$

$$P(u) = a\frac{1}{1 + e^{k_2 - u/\tau}} \tag{5}$$

If the values of the remaining parameters are substituted in the model, (6) is obtained.

$$P(u) = \frac{P_r}{1 + e^{2s(u_{ip}-u)/(P_r - P_{ip})}} \tag{6}$$

This is also a 4-parameter model (P_r, s, u_{ip}, P_{ip}) named 4-P DS (4-parameter model using a deterministic but simplified process to obtain the parameters). The comparison of this model with the power curve can be seen in Fig. 3.

The error values obtained for the 4-P DS Model for Vestas V80 and other WTs are available in Tables 1–3. The values of the parameters are listed in Tables 4–7.

2.3. 3-Parameter model using a deterministic process

Another interesting case is where the approximation $2P_{ip} = P$ can be made. In this case the model is reduced to Eq. (7).

$$P(u) = \frac{P_r}{1 + e^{4s(u_{ip}-u)/P_r}} \tag{7}$$

This model, named 3-P DP (3-parameter model using a deterministic process to obtain the parameters), depends on only three parameters of the real power curve (P_r, s, u_{ip}). In fact, these are

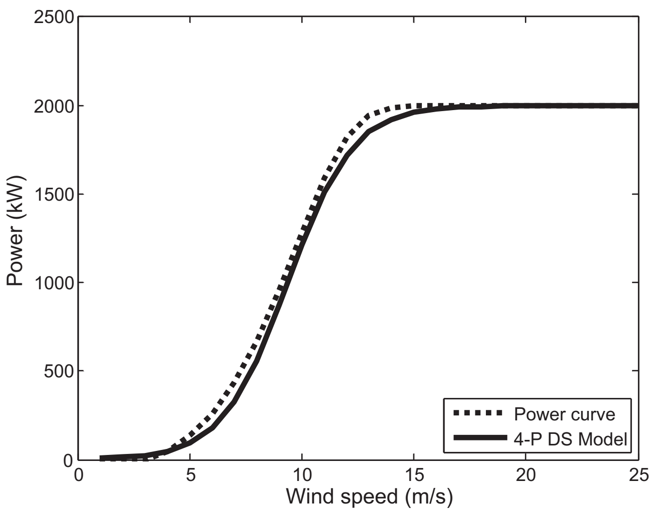

Fig. 3. Model 4-P DS for the power curve of Vestas V80 2MW.

parameters that may be obtained easily from the WT manufacturer. Notice that, from this equation, the influence of the slope, inflection point and rated power is easy to derive. Eq. (7) was already proposed in [28] with a slightly different configuration of parameters. However, the parameters in that case do not have a technical interpretation.

The comparison of this model with the power curve gives Fig. 4.

The error values and parameters for the 3-P DP Model for Vestas V80 and other WTs are shown in Tables 1–3 and Tables 4–7, respectively.

3. Derived expressions

One of the main applications of a WT power curve model is to obtain an expression for the PDF of the output power [29]. The PDF provides the cumulative behaviour of the power and can be used as input data to solve more complex problems such as the probabilistic load flow, where the probabilistic nature of the power is taken into account.

The PDF of the power supplied by a WT can easily be derived using Eqs. (1), (6) and (7). The Weibull distributed nature of wind speed must be considered [30–32], defined in Eq. (8) for values higher than zero, and a change of variables must be made as in Eq. (9) [33].

$$f(u) = \frac{k}{C}\left(\frac{u}{C}\right)^{k-1} e^{-\left(\frac{u}{C}\right)^k} \tag{8}$$

$$f_P(P) = f_u(u(P))\left|\frac{\mathrm{d}u}{\mathrm{d}P}\right| \tag{9}$$

So, the wind speed as a function of the output power and its derivative are needed.

In the case of the 4-parameter model, the expression of the wind speed as a function of the power is shown in Eq. (10) and its derivative in Eq. (11). The parameters a, m, n, τ are used because of the complexity that arises by using P_r, s, u_{ip}, P_{ip}.

$$u = -T\log\left(\frac{a-P}{Pn-am}\right) \tag{10}$$

$$\frac{\mathrm{d}u}{\mathrm{d}P} = \frac{\tau a(n-m)}{(a-P)(Pn-am)} \tag{11}$$

In the case of the 4-parameter model, considering $m = 0$, or $k_1 \to +\infty$, the expression of the wind speed as a function of the power is expressed in Eq. (12) and its derivative in Eq. (13).

$$u = u_{ip} - \frac{P_r - P_{ip}}{2s}\log\left(\frac{P_r}{P} - 1\right) \tag{12}$$

$$\frac{\mathrm{d}u}{\mathrm{d}P} = \frac{P_r\left(P_r - P_{ip}\right)}{2sP(P_r - P)} \tag{13}$$

Finally, in the case of the 3-parameter model, the expression of the wind speed as a function of the power is shown in Eq. (14) and its derivative in Eq. (15).

$$u = u_{ip} - \frac{P_r}{4s}\log\left(\frac{P_r}{P} - 1\right) \tag{14}$$

$$\frac{\mathrm{d}u}{\mathrm{d}P} = \frac{{P_r}^2}{4sP(P_r - P)} \tag{15}$$

The PDF of the output power of a WT using the 4-parameter model is shown in Eq. (16).

$$f(P) = \frac{k}{C}\left(\frac{-\tau\log\left(\frac{a-P}{Pn-am}\right)}{C}\right)^{k-1} e^{-\left(\frac{-\tau\log\left(\frac{a-P}{Pn-am}\right)}{C}\right)^{k}} \times \left|\frac{\tau a(n-m)}{(a-P)(Pn-am)}\right| \tag{16}$$

If the parameters were obtained using an optimization process, the shape of the PDF would be as in Fig. 5.

$C = 8$, $k = 2$.

However, if the parameters were obtained using a deterministic process, the shape of the PDF would be as in Fig. 6.

$C = 8$, $k = 2$.

Fig. 6 shows the influence of the error provided by the 4P-DP model. The PDF of the output power is different from Fig. 5 due to higher output power values for low wind speed values. The consequence is that, using this model, the PDF has non-zero

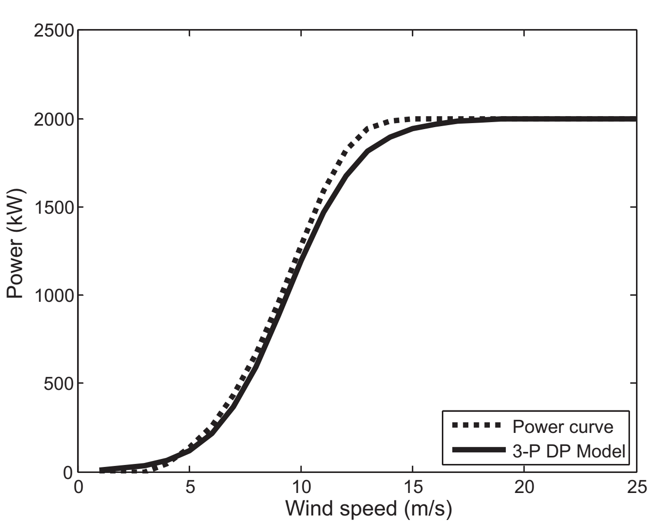

Fig. 4. Model 3-P DP for the power curve of Vestas V80 2MW.

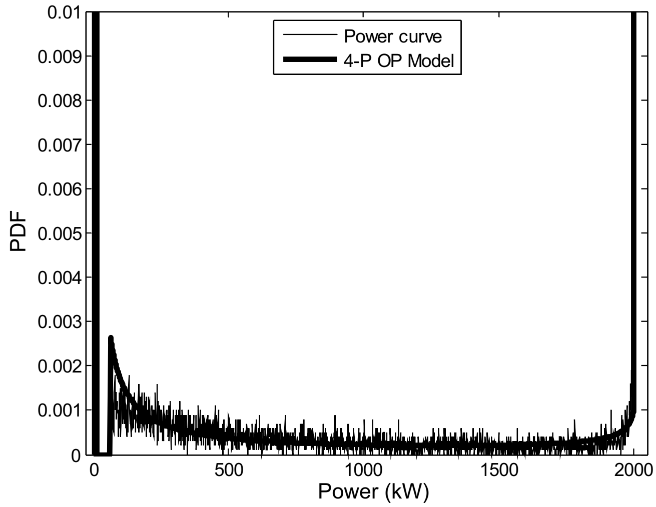

Fig. 5. PDF of the output power of a Vestas V80 2MW using model 4-P OP,

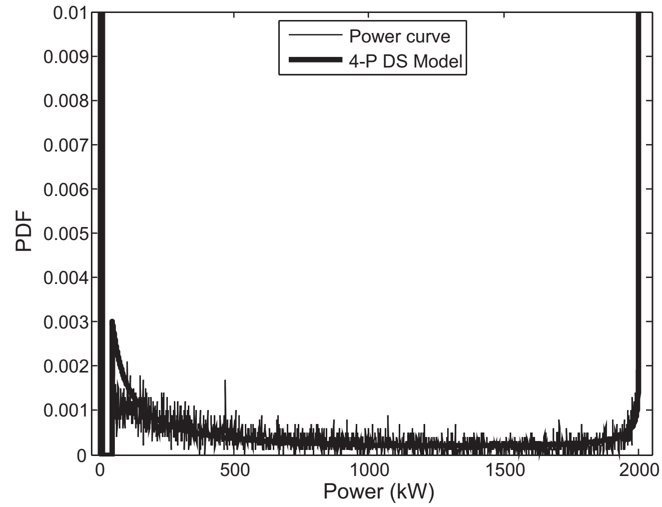

Fig. 7. PDF of the output power of a Vestas V80 2MW using model 4-P DS, $C = 8, \quad k = 2$.

values for higher values of power than in the 4P-OP case, and the probabilities of these values are higher at first. This difference also appears when comparing the model with the 4P-DS and the 3P-DP models.

The PDF of the output power of a WT using the 4-parameter model considering $m = 0$, or $k_1 \to +\infty$, is expressed in Eq. (17).

$$f(P) = \frac{k}{C}\left(\frac{u_{\mathrm{ip}} - \frac{P_\mathrm{r}-P_\mathrm{ip}}{2s}\log\left(\frac{P_\mathrm{r}}{P}-1\right)}{C}\right)^{k-1} e^{-\left(\frac{u_{\mathrm{ip}} - \frac{P_\mathrm{r}-P_\mathrm{ip}}{2s}\log\left(\frac{P_\mathrm{r}}{P}-1\right)u}{C}\right)^{k}} \times \left|\frac{P_\mathrm{r}\left(P_\mathrm{r}-P_\mathrm{ip}\right)}{2sP\left(P_\mathrm{r}-P\right)}\right| \tag{17}$$

The PDF obtained in this case is shown in Fig. 7.

$C = 8, \quad k = 2.$

The PDF of the output power of a WT using the 3-parameter model is expressed in Eq. (18).

$$f(P) = \frac{k}{C}\left(\frac{u_{\mathrm{ip}} - \frac{P_\mathrm{r}}{4s}\log\left(\frac{P_\mathrm{r}}{P}-1\right)}{C}\right)^{k-1} e^{-\left(\frac{u_{\mathrm{ip}} - \frac{P_\mathrm{r}}{4s}\log\left(\frac{P_\mathrm{r}}{P}-1\right)}{C}\right)^{k}} \times \left|\frac{{P_\mathrm{r}}^2}{4sP\left(P_\mathrm{r}-P\right)}\right| \tag{18}$$

The PDF obtained in this case is shown in Fig. 8.

$C = 8, \quad k = 2.$

These PDFs have a gap between 0 and $P(u_{\mathrm{ci}})$. The reason is that the probability of the power being inside this interval is zero. However, in each case, this gap is different due to the value of the model at u_{ci}.

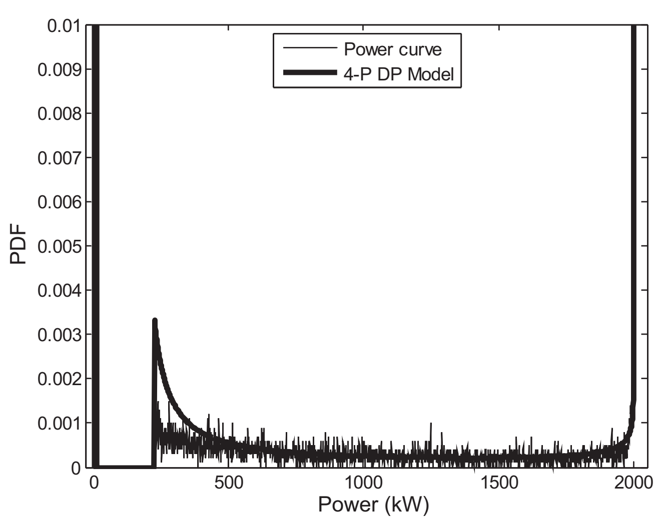

Fig. 6. PDF of the output power of a Vestas V80 2MW using model 4-P DP, $C = 8, \quad k = 2$.

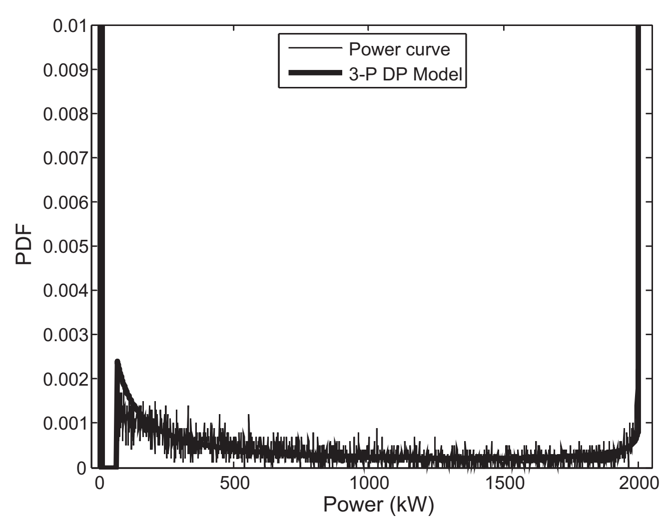

Fig. 8. PDF of the output power of a Vestas V80 2MW using model 3-P DP, $C = 8, \quad k = 2$.

4. Results

In order to check the models proposed in Section 2, several WTs are taken into account. The WTs being checked are the following:

(1) Vestas V80 (2000 kW).
(2) Enercon E82 (2050 kW).
(3) Siemens S82 SWT-2.3 82 (2300 kW).
(4) Repower MM82 (2050 kW).
(5) Nordex N90 (2300 kW).
(6) Siemens SWT-3.6 107 (3600 kW).
(7) Vestas V164 (6995 kW).

The reasons for choosing these WTs are, firstly, to make a detailed comparison of WTs of the same rated power from different manufacturers (WT 1–5), and, secondly, to check the models for a wide range of rated powers (WT 6 and 7).

First of all, the error when using each of the models for all of the WTs is obtained. Three types of errors are assessed: MAPE in Eq. (19), MAE in Eq. (20) and RMSE, in Eq. (21).

$$\mathrm{MAPE} = \frac{1}{P_r N}\sum_{i=1}^{N}\left|P(u_i) - P_m(i)\right| \tag{19}$$

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}\left|P(u_i) - P_m(i)\right| \tag{20}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(P(u_i) - P_m(i)\right)^2} \tag{21}$$

where, P_r is the rated power, N is the number of pairs of points of the power curve, $P(u_i)$ is the power obtained using the model at a wind speed u_i and $P_m(i)$ is the power value provided by the manufacturer corresponding to wind speed u_i. In Table 1, MAPE errors are shown.

The models checked are those proposed here (4P-DP, 4P-DS and 3P-DP) and the 4P-OP. In addition, the most common piecewise continuous models are checked for comparison purposes (Linear, Quadratic and Cubic).

Notice that MAPE relates the error with the rated power, so this value is more or less the same regardless of the rated power of the WT. When using the optimization procedure (4P-OP) to obtain the parameters, MAPE is lower than in the other cases as expected. However, when using the other models, MAPE values ascend to no more than the double the 4P-OP values. It can also be observed that, in some cases, such as E82, R82 and V164, MAPE is lower when using 4P-DS than when using 4P-DP. Furthermore, for E82 and V164, MAPE is lower when using the 3-parameter 3P-DP than when using the 4-parameter 4P-DP. When assessing MAPE errors for piecewise continuous models, there are big differences between them and those based on logistic functions. The reason is that these models are a good approximation for a wide range of values but they are very bad when they approach the rated wind speed. MAE errors are expressed in Table 2.

In this case, the Mean Absolute Error is measured, so it is higher if the rated power is also higher. Considering this type of error, in all cases the 4P-OP model is the best option. However, the values obtained when using the other models are not very high. For piecewise continuous models, as before, errors are higher than in the other models due to the approximation not being so good around the rated wind speed. Table 3 shows RMSE values.

When considering the RMSE, a greater difference appears because it is a squared error, and so the higher values are penalised more. However, it can be seen that in some cases, such as E82, R82, N90 and V164, RMSE is lower than the double of using model 4P-OP for the models proposed here. For piecewise continuous models, due to the squared error, the differences with those based on logistic functions are greater than in the previous cases. The values of parameter 'a' for all WTs using the different models are represented in Table 4.

All models proposed here consider the rated power of the WT to be the best option for parameter 'a'. When using the 4P-OP model the parameter is not higher than 101% of the rated power. Parameter 'm' is given in Table 5.

This is the parameter showing more differences when comparing both the 4P-OP and the 4P-DP models. The fact is that this is the least important parameter, as it is usually lower than 'n' and the approximations proposed in Section 2 can be used. On the other hand, for models 4P-DS and 3P-DP, 'm' equals zero by definition. Parameter 'n' is shown in Table 6.

Here, it can be seen for each case that for the first two models, 4P-OP and 4P-DS, the value of the parameter is similar and for the last model it is different. The reason may be that 'n' is highly influenced by the difference $P_r - P_{ip}$, which is approximated when using model 3P-DP. Obviously, when using model 4P-DS, the value is the same as when using model 4P-DP. Finally, in Table 7, the values of parameter 'τ' are shown.

Parameter 'τ' is similar to 'n' in that the main difference is with model 3P-DP. The reason may be the same as in the previous case, as 'τ' also depends on the difference $P_r - P_{ip}$.

Moreover, in order to avoid consuming too much time when obtaining the parameters through an optimisation process, 4P-OP, the values of the parameters obtained with model 4P-DP were used as a starting point. This means that the parameters obtained for model 4P-OP are, at least, as good as those obtained for model 4P-DP and that the time needed to obtain them is dramatically reduced.

5. Conclusion

Three WT power curve models based on a reformulation of the parameters of the logistic function are presented in this paper. The first one, called 4P-DP, is similar to the known 4-parameter logistic function but considers a deterministic procedure to obtain its parameters, which makes the model meaningful, i.e., it provides information on the WT behaviour, and has no appreciable errors. A second model, called 4P-DS, is obtained from the previous one, by means of the simplification $m = 0$, with the result that some issues are improved, such as the approximation to the power curve for lower values of wind speed and the ease with which other expressions like the PDF can be derived. The third model, called 3P-DP, based on considering the approximation $2P_{ip} = P_r$, improves all the features of the other models and reduces the number of parameters to three, except for the induced error, which is greater, as expected, but also acceptable. Moreover, the three models consist of a continuous function which simplifies the implementation of the curve in a computer program compared to piecewise models.

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