Fractal Measures in Paper Marbling

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Introduction

Paper marbling is an old art form with traditions in Europe, Turkey (Ebru), and Japan (Suminagashi). Here we view the formation of patterns created by paper marbling through the lens of nonlinear dynamics. Classic work[1] showed that repeated application of a chaotic area-preserving map concentrates the gradient of a scalar field on a set that is generically fractal and results in specific restrictions on the fractal dimension spectrum. Through a collaboration with professional marblers[2] we have numerically tested these ideas on scans of marbled paper created by the repeated action of a combing process. The beautiful patterns that emerge clearly show the fractal measures inherent in the art of paper marbling.

Background

Marbling:

- Creating marbled paper involves floating paints on the surface of a liquid bath, potentially using tools like combs or a stylus to stir the fluid, and transferring the resulting pattern to paper.
- The liquid bath is a mixture of water and carrageenan.
- The relative concentration of carrageenan controls the viscosity. Our experiments are in the Stokes regime.
- We apply acrylic paints, mixed with additives that modify the surface tension and ensure even spreading.
- Once deposited on the bath's surface, the paints are stirred using a stylus or a comb to create the desired pattern.
- Once complete, the image is transferred to paper.

Fractals:

•A scalar function (representing pigment concentration) is deformed by the repeated action of a combing operation.
•The gradient of this scalar concentrates into a fractal.

$$\mu = \frac{\int_A |\nabla\phi|^\gamma d^2\mathbf{x}}{\int_{A_0} |\nabla\phi|^\gamma d^2\mathbf{x}}$$

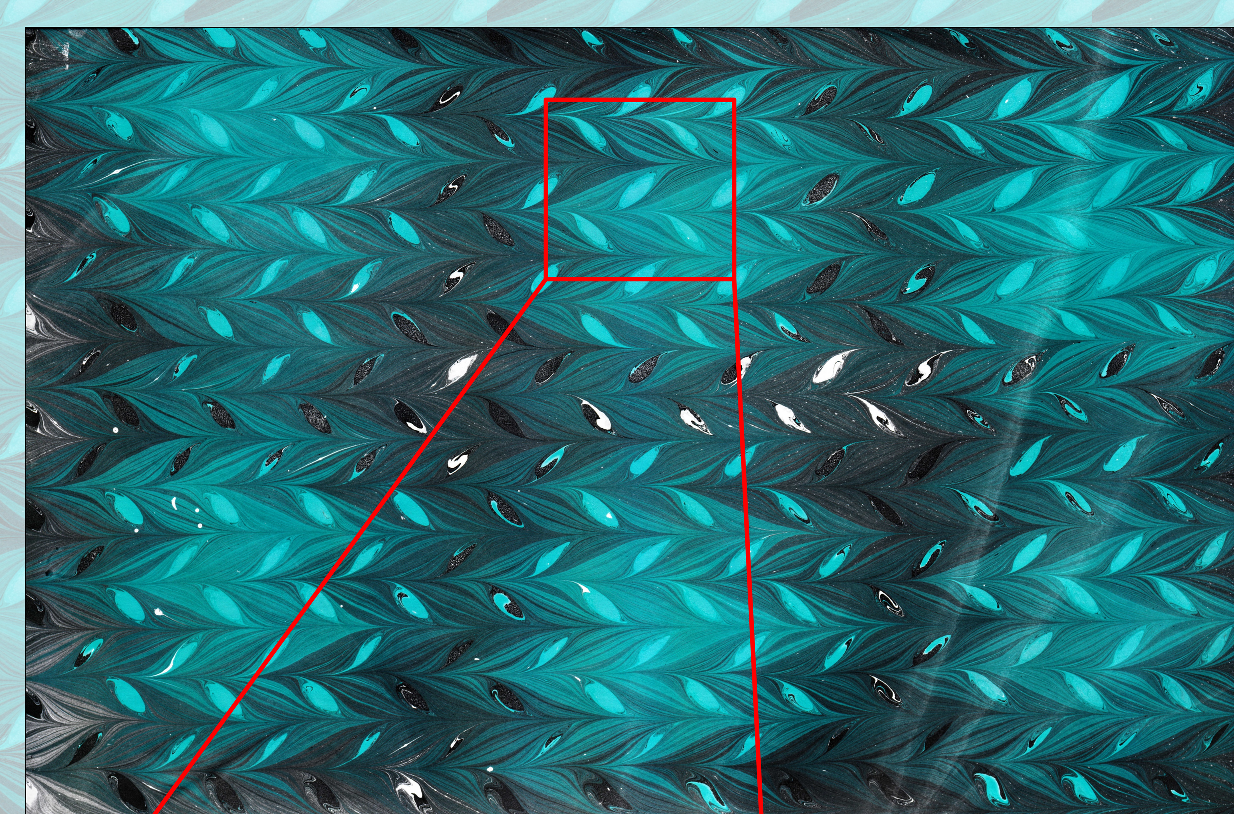
•This measure (mu) is indicates where (area A) the gradient of the scalar is large. It has one free parameter, γ.

$$D_q = \frac{1}{q-1}\lim_{\epsilon\to 0}\left[\frac{\ln \Sigma_i \mu_i^q}{\ln \epsilon}\right]$$

•D_q is the spectrum of dimensions for the measure. Different values of q meaure dimension in different ways. D_0 is the box-counting dimension, D_1 is the "information dimension." Non-integer values indicate a fractal dimension.

Theory[1]

- We consider the passive advection of a scalar field (pigment concentration) due to a given 2D incompressible flow (generated by the combing motion).
- Repeated application of the combing operation will concentrate the gradient of the scalar on a fractal set
- Flow is chaotic: close points diverge exponentially in time.
- If the stretching rate is spatially nonuniform, then we can conclude the following about the spectrum of dimensions, D_q:
  - $D_0 = 2$ for any γ. The usual box counting dimension is always 2, a consequence of incompressibility.
  - When $\gamma \to 0$, $D_q = 2$ for any value of q.
  - When $q \to 1$ (the information dimension), $D_1 < 2$, which makes this a fractal dimension.



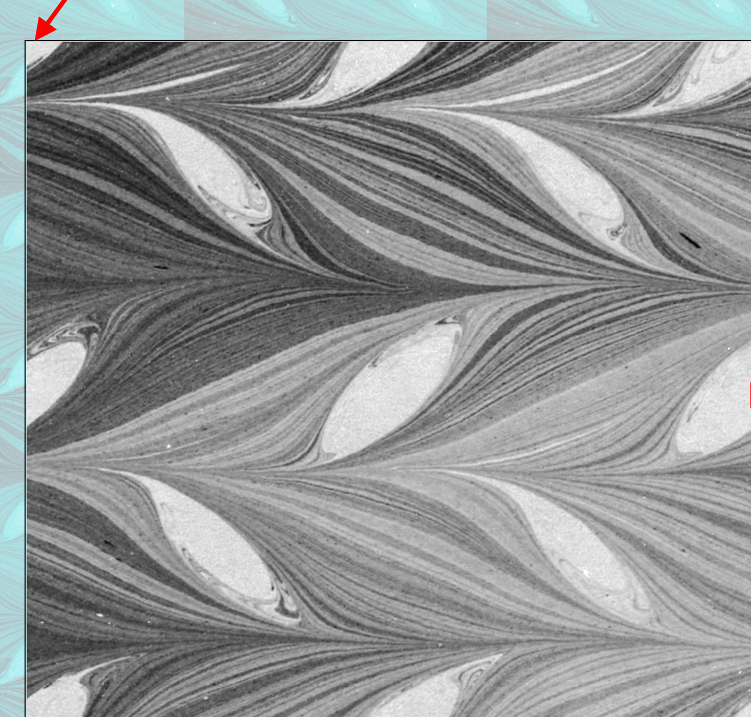
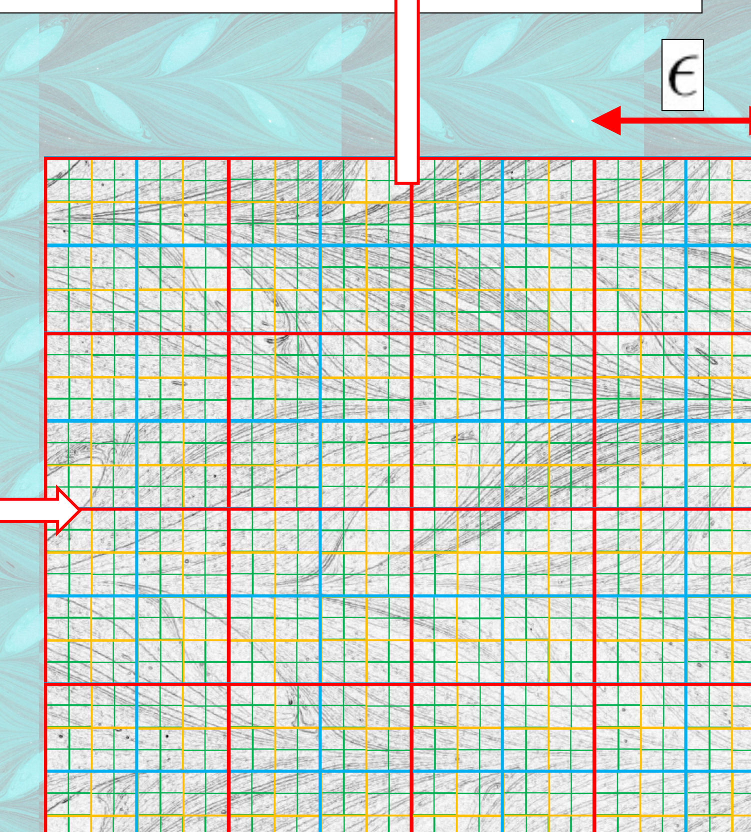


Image Subset & Grayscale Conversion

Gradient of the Pigment Concentration



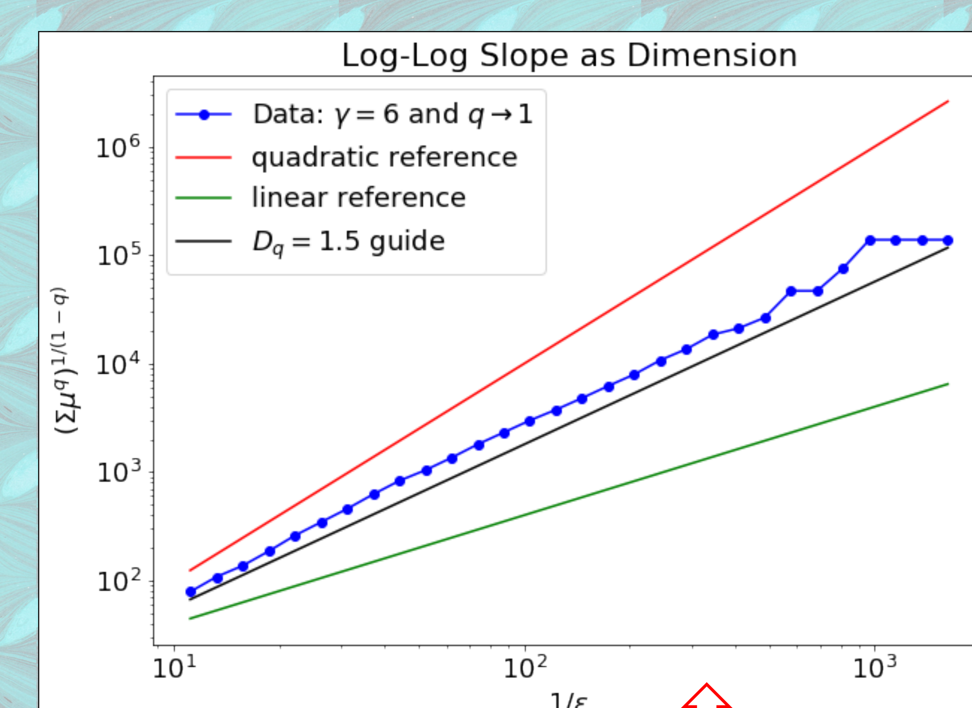
Grids of Different Sizes

Methods

Obtaining the Images:
•Using a pre-determined, consistent set of initial conditions, we dripped paint onto the surface of the bath.
•Perform 1 operation on the bath: horizontal then vertical Getgel patterns (Getgel – rake across, offset comb by ½ a tine spacing, then rake back).
•Repeat the process (2, 5, and 10 times), lifting a print after each set of operations. Fractal emerges for higer iterates.

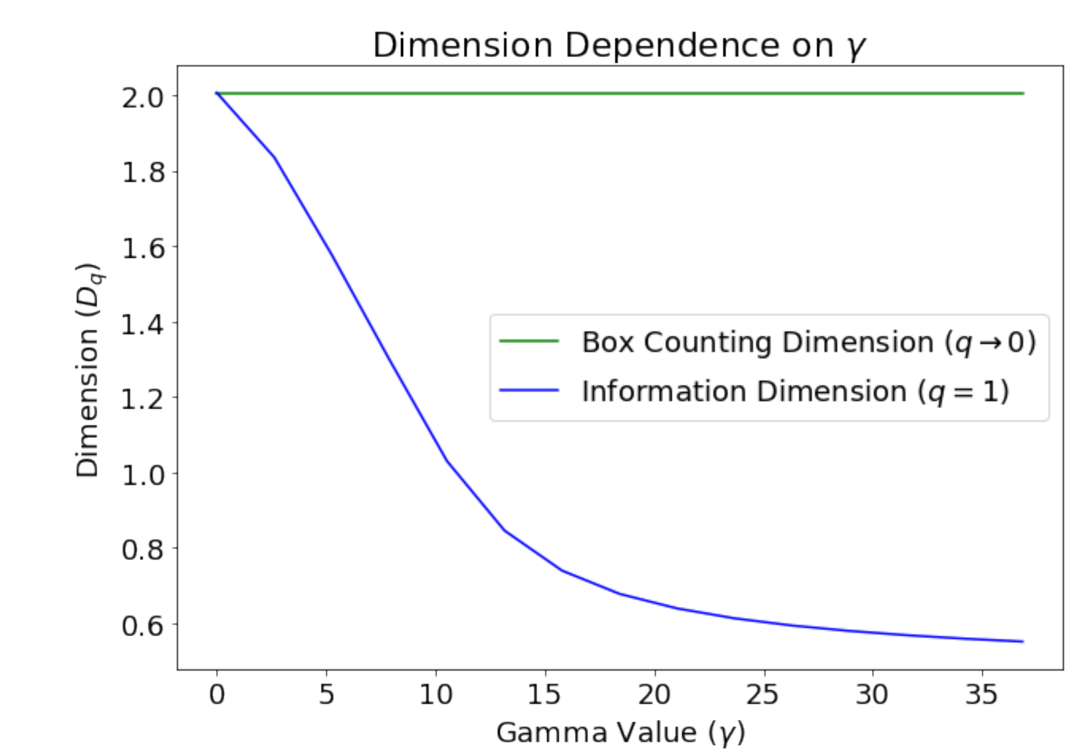
Processing the Images:
•Created Python code for image and data analysis.
•First choose subset of full image and convert to grayscale.
•Calculate the magnitude of the gradient (black is large).
•Divide using increasingly fine grids of size ϵ
•For a given grid, calculate the measure of each square.
•To get the dimension D_q, take the slope of the log-log plot:

$$(\Sigma\mu^q)^{1/(1-q)} \quad \text{vs.} \quad 1/\epsilon$$



Results

- Since the combing process stretches and folds the paint in a spatially nonuniform manner, we see the three predictions from the theory section in the following graph:
  - The box counting dimension, D_0, is 2 for all γ.
  - When $\gamma = 0$, we see that both the box counting dimension and the information dimension, D_1, are 2.
  - Most interestingly, the information dimension is fractal for non-zero γ values. This shows that there are measures which capture how the gradient of the scalar field concentrates on a fractal set.



Future Work

- There are several other experiments that can be conducted with paper marbling and using similar analysis methods as in this project.
- The viscosity of the liquid bath can be altered by changing the ratio of water and carrageenan used.
- The same comb, and the same motions with the comb were used throughout this project, however, we can experiment with the comb in a few ways:
  - Use a comb with different spacing, or different sized tines.
  - Change the motion of the comb (different pattern).
  - Change the speed of the motion of the comb.

References

1. Fractal measures of passively convected vector fields and scalar gradients in chaotic fluid flows, Ott, Edward and Antonsen, Thomas M., Phys. Rev. A, 39, 7, 1989 10.1103/PhysRevA.39.3660

2. Chena River Marblers, Amherst, MA