

Tutorial of ST5215

AY2020/2021 Semester 1

15 Sep 2020

Exercise 1 Let X_1, \dots, X_n be i.i.d. from a uniform distribution on $(-\theta, \theta)$, where $\theta > 0$ is an unknown parameter.

(a). Find a minimal sufficient statistic T .

(b). Define

$$V = \frac{\bar{X}}{\max_i X_i - \min_i X_i}$$

where \bar{X} is the sample mean. Are T and V are independent?

Exercise 2 An object with weight θ is weighed on scales with different precision. The data X_1, \dots, X_n are independent, with $X_i \sim N(\theta, \sigma^2)$, $i = 1, \dots, n$ with the standard deviation σ known. Consider the absolute deviation loss $L(\theta, a) = |\theta - a|$.

(a). What is the risk of the naive estimator X_1 ?

(b). Use Rao-Blackwell theorem to find a better estimator.

Exercise 3 Consider an estimation problem with a parametric family $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ and the squared error loss. If $\theta_0 \in \Theta$ satisfies that $P_\theta \ll P_{\theta_0}$ for any $\theta \in \Theta$, show that the estimator $T \equiv \theta_0$ is admissible.