Tutorial of ST5215

AY2020/2021 Semester 1

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Exercise 1 Suppose X has an exponential family distribution with density

$$p_\theta(x) = h(x)e^{\eta(\theta)T(x)-A(\theta)}$$

Derive the mean and variance formulas

$$\mathrm{E}_\theta[T(X)] = \frac{A'(\theta)}{\eta'(\theta)}, \quad \mathrm{V}_\theta[T(X)] = \frac{A''(\theta)}{[\eta'(\theta)]^2} - \frac{\eta''(\theta)A'(\theta)}{[\eta'(\theta)]^3}$$

Exercise 2 Let X and Y be two random variables such that Y has the binomial distribution $Bi(\pi, N)$ and, given $Y = y, X$ has the binomial distribution $Bi(p, y)$

(a) Suppose that $p \in (0,1)$ and $\pi \in (0,1)$ are unknown and N is known. Show that (X, Y) is minimal sufficient for (p, π).

(b) Suppose that π and N are known and $p \in (0,1)$ is unknown. Show whether X is sufficient for p and whether Y is sufficient for p

Exercise 3 Let $X_1, \ldots, X_n$ be i.i.d. random variables having the Lebesgue p.d.f.

$$f_\theta(x) = \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^4 - \xi(\theta)\right\}$$

where $\theta = (\mu, \sigma) \in \Theta = \mathcal{R} \times (0, \infty)$.

Show that $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ is an exponential family, where P_θ is the joint distribution of $X_1, \ldots, X_n$ and that the statistic

$$T = \left(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2, \sum_{i=1}^n X_i^3, \sum_{i=1}^n X_i^4\right)$$

is minimal sufficient for $\theta \in \Theta$.