Tutorial of ST5215

AY2020/2021 Semester 1

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Exercise 1 Let $\mathcal{C}$ be a collection of subsets of Ω and $\Gamma = \{\mathcal{F} : \mathcal{F}$ is a σ-field on Ω and $\mathcal{C} \subset \mathcal{F}\}$.
Show that $\Gamma \neq \emptyset$ and $\sigma(\mathcal{C}) = \bigcap_{\mathcal{F}\in\Gamma} \mathcal{F}$.

Exercise 2 Let $(\Omega, \mathcal{F})$ be a measurable space. $f : \Omega \mapsto \mathcal{R}$ is Borel if and only if $f^{-1}(a, \infty) \in \mathcal{F}$ for all $a \in \mathcal{R}$.

Exercise 3 Show that a monotone function from $\mathcal{R}$ to $\mathcal{R}$ is Borel

Exercise 4 Let f be a Borel function on $\mathcal{R}^2$. Define a function g from $\mathcal{R}$ to $\mathcal{R}$ as $g(x) = f(x, y_0)$, where y_0 is a fixed point in $\mathcal{R}$. Show that g is Borel.