Homework 3, ST5215

AY2020/2021 Semester 1

Due: 27 Oct 2020

Instruction

- Only PDF format and one PDF file will be accepted
- Name your PDF file in the format: StudentID.pdf, where "StudentID" is replaced with your ID number
- Go to LumiNus system and upload your PDF file to the folder *Submissions/HW3*. Before the deadline, you can update your previous submission by first deleting the old submission and then submitting the updated version.
- There are 6 questions in this assignment.

Problem set

JS = Mathematical Statistics, 2nd Ed, Jun Shao, 2003

Problem 1. Exercise 3.6.34 in JS

Problem 2. Exercise 3.6.37 in JS

Problem 3. Prove the second Borel-Cantelli lemma: For a sequence of pairwisely independent events $\{A_n\}_{n=1}^{\infty}$, if $\sum_{n=1}^{\infty} P(A_n) = \infty$, then $P(A_n \quad i.o.) = 1$.

Problem 4. Suppose X and $\{X_n\}$ are r.v.s. defined on a common probability space. Prove that if for any sub-sequence $\{X_{n_k}\}_{k=1}^{\infty}$ there exists a sub-sub-sequence $\{X_{n_{k_i}}\}_{i=1}^{\infty}$ such that $X_{n_{k_i}} \overset{a.s.}{\to} X$, then $X_n \overset{\mathcal{P}}{\to} X$.

Problem 5. Exercise 1.6.146 in JS

Problem 6. Exercise 1.6.155 in JS