Tutorial of ST5215

AY2020/2021 Semester 1

13 Oct 2020

Exercise 1. Suppose X is a nonnegative random variable. Show that

$$EX \leq \sum_{i=0}^{\infty} P(X \geq n) \leq 1 + EX.$$

Therefore, X is integrable if and only if $\sum_{i=0}^{\infty} P(X \geq n) < \infty$

Exercise 2. If $X_n \stackrel{D}{\to} c$ for a constant c, then $X_n \stackrel{P}{\to} c$.

Exercise 3. Suppose that $X_n \stackrel{\mathrm{D}}{\to} X$. Then, for any $r > 0$

$$\lim_{n\to\infty} E|X_n|^r = E|X|^r < \infty$$

if and only if $\{|X_n|^r\}$ is uniformly integrable in the sense that

$$\lim_{t\to\infty} \sup_n E\left(|X_n|^r I_{\{|X_n|>t\}}\right) = 0$$

Exercise 4. Let $X_1, X_2, \ldots$ be a sequence of identically distributed random variables with $E|X_1| < \infty$ and let $Y_n = n^{-1} \max_{1\leq i\leq n} |X_i|$. Show that $\lim_n E(Y_n) = 0$ and $\lim_n Y_n = 0$ a.s.