Tutorial of ST5215

AY2020/2021 Semester 1

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Exercise 1. Let $(Y_1, \ldots, Y_n)$ be a random sample such that Y_i is distributed as $N(\theta, \theta)$ with an unknown $\theta > 0$.

Show that one of the solutions of the likelihood equation is the unique MLE of θ. Obtain the asymptotic distribution of the MLE of θ.

Exercise 2. Let $(X_1, \ldots, X_n)$ be a random sample from the exponential distribution on (a, ∞) with scale parameter θ, where $a \in \mathcal{R}$ and $\theta > 0$ are unknown. Obtain the asymptotic relative efficiency of the MLE of a with respect to the UMVUE of a.

Exercise 3. Consider a linear model in matrix form $X_{n\times 1} = Z_{n\times p}\boldsymbol{\beta}_{p\times 1} + \epsilon_{n\times 1}$. Under the assumption that $\epsilon \sim N(\mathbf{0}_n, \sigma^2 \boldsymbol{I}_n)$ where σ is known, compute the Fisher information $I(\boldsymbol{\beta})$. When is $I(\boldsymbol{\beta})$ positive definite?