Tutorial of ST5215

AY2020/2021 Semester 1

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Exercise 1. Let $X_1, \ldots, X_n$ be i.i.d. from P with $EX_1^4 < \infty$ and unknown mean $\mu \in \mathcal{R}$ and variance $\sigma^2 > 0$. Consider the estimation of $\vartheta = \mu^2$ and the following three estimators: $T_{1n} = \bar{X}^2, T_{2n} = \bar{X}^2 - S^2/n, T_{3n} = \max\{0, T_{2n}\}$, where $\bar{X}$ and S^2 are the sample mean and variance.
Show that the amse's of $T_{jn}, j = 1, 2, 3$, are the same when $\mu \neq 0$ but may be different when $\mu = 0$.
Which estimator is the best in terms of the asymptotic relative efficiency when $\mu = 0$?

Exercise 2. Let $X_1, \ldots, X_n$ be a random sample of random variables with $EX_i = \mu, \mathrm{Var}(X_i) = 1$, and $EX_i^4 < \infty$. Let $T_{1n} = n^{-1}\sum_{i=1}^n X_i^2 - 1$ and $T_{2n} = \bar{X}^2 - n^{-1}$ be estimators of μ^2, where $\bar{X}$ is the sample mean.

Find the asymptotic relative efficiency of T_{1n} with respect to T_{2n}.