Tutorial of ST5215

AY2020/2021 Semester 1

20 Oct 2020

Exercise 1. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with Lebesgue p.d.f.

$$f(x) = \frac{1}{2c} \frac{1}{x^2 \log x} I_{|x|>3} \tag{1}$$

where $c = \int_{x=3}^{\infty} 1/(x^2 \log x)\, \mathrm{d}x$.

Show that $E\,|X_1| = \infty$ but $n^{-1} \sum_{i=1}^{n} X_i \to_p 0$

Exercise 2. Suppose that X_n is a random variable having the binomial distribution with size n and probability $\theta \in (0,1), n = 1, 2, \ldots$.

Define $Y_n = \log(X_n/n)$ when $X_n \geq 1$ and $Y_n = 1$ when $X_n = 0$.

Show that $\lim_n Y_n = \log\theta$ a.s. and $\sqrt{n}\,(Y_n - \log\theta) \to_d N\left(0, \frac{1-\theta}{\theta}\right)$

Exercise 3. Let $X_1, X_2, \ldots$ be independent random variables such that X_j has the uniform distribution on $[-j, j], j = 1, 2, \ldots$ Show that

$$\frac{\sum_{j=1}^{n} X_j}{n^2} \xrightarrow{\mathcal{D}} N(0, 1/6) \tag{2}$$