# Tutorial of ST5215

AY2020/2021 Semester 1

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Assume the conditions in Cramér-Rao lower bound hold and $\Theta \subset \mathcal{R}$.

Exercise 1. Suppose $T$ is an estimator of $g(\theta)$ with bias $b(\theta)$ and $b$ is differentiable. Prove

$$\mathrm{Var}(T) \geq \frac{\left(g'(\theta) + b'(\theta)\right)^2}{I(\theta)} \tag{1}$$

Exercise 2. Show that for any fixed $\theta$, there exists a random variable $T$ such that $ET = g(\theta)$ and $\mathrm{Var}(T)$ attains the Cramér-Rao lower bound if and only if

$$T = \left[\frac{g'(\theta)}{I(\theta)}\right] \frac{\partial}{\partial\theta} \log f_\theta(X) + g(\theta) \tag{2}$$

Exercise 3. Show that there exists an unbiased estimator $T(X)$ of $g(\theta)$ such that $\mathrm{Var}(T)$ attains the Cramér-Rao lower bound if and only if

$$f_\theta(X) = \exp[\eta(\theta)T(x) - \xi(\theta)]h(x), \tag{3}$$

where $\xi(\theta)$ and $\eta(\theta)$ are differentiable functions such that $\xi'(\theta) = g(\theta)\eta'(\theta)$ and $I(\theta) = \eta'(\theta)g'(\theta)$