Tutorial of ST5215

AY2020/2021 Semester 1

15 Oct 2020

Exercise 1. Suppose that $X_n \xrightarrow{\mathrm{D}} X$. Then, for any $r > 0$

$$\lim_{n\to\infty} E|X_n|^r = E|X|^r < \infty$$

if and only if $\{|X_n|^r\}$ is uniformly integrable in the sense that

$$\lim_{t\to\infty} \sup_n E\left(|X_n|^r I_{\{|X_n|>t\}}\right) = 0$$

Exercise 2. Let $X, X_1, X_2, \ldots$ be random variables. Show that if $\lim_n X_n = X$ a.s., then $Y_n := \sup_{m>n} |X_m|$ is bounded in probability.