Homework 1, ST5215

AY2020/2021 Semester 1

Due: 1 Sep 2020

Instruction

- Only PDF format and one PDF file will be accepted
- Name your PDF file in the following format: [StudentID].pdf, where [StudentID] is replaced with your ID number
- Go to LumiNus system and upload your PDF file to the folder *Submissions/HW1*. Before the deadline, you can update your previous submission by first deleting the old submission and then submitting the updated version.
- There are 10 problems in this assignment. Grades are given based on correct understanding. You can use any other result in any reference, as long as it is not one of the problems you are asked to prove and you cite it properly.

Problem set

JS = Mathematical Statistics, 2nd Ed, Jun Shao, 2003

Problem 1. If $f : \mathcal{R} \mapsto \mathcal{R}$ is a continuous function, then it is Borel measurable

Problem 2. Ex 1.6.12 in JS

Problem 3. Ex 1.6.23 in JS

Problem 4. Ex 1.6.35 in JS

Problem 5. Ex 1.6.40 in JS

Problem 6. Ex 1.6.46 in JS

Problem 7. Ex 1.6.47 in JS

Problem 8. Ex 1.6.58 (b,c) in JS

Problem 9. Ex 1.6.86 in JS. Please note that the condition in the question does not assume any of XY, $YE(X \mid \mathcal{A})$, or $XE(Y \mid \mathcal{A})$ is integrable.

Problem 10. Which of the following parametrizations are identifiable? (Prove or disprove.)

(a). $X_1, \ldots, X_p$ are independent with $X_i \sim \mathcal{N}\left(\alpha_i + \nu, \sigma^2\right)$

$$\theta = \left(\alpha_1, \alpha_2, \ldots, \alpha_p, \nu, \sigma^2\right) \tag{1}$$

and P_θ is the distribution of $\mathbf{X} = (X_1, \ldots, X_p)$

(b). Same as (a) with $\alpha = (\alpha_1, \ldots, \alpha_p)$ restricted to

$$\left\{(a_1, \ldots, a_p) : \sum_{i=1}^{p} a_i = 0\right\} \tag{2}$$

(c). X and Y are independent $\mathcal{N}\left(\mu_1, \sigma^2\right)$ and $\mathcal{N}\left(\mu_2, \sigma^2\right), \theta = (\mu_1, \mu_2)$ and we observe $Y - X$

(d). $X_{ij}, i = 1, \ldots, p; j = 1, \ldots, b$ are independent with $X_{ij} \sim \mathcal{N}\left(\mu_{ij}, \sigma^2\right)$ where $\mu_{ij} = \nu + \alpha_i + \lambda_j, \theta = \left(\alpha_1, \ldots, \alpha_p, \lambda_1, \ldots, \lambda_b, \nu, \sigma^2\right)$ and P_θ is the distribution of $X_{11}, \ldots, X_{pb}$

(e). Same as (d) with $(\alpha_1, \ldots, \alpha_p)$ and $(\lambda_1, \ldots, \lambda_b)$ restricted to the sets where $\sum_{i=1}^{p} \alpha_i = 0$ and $\sum_{j=1}^{b} \lambda_j = 0$