Homework 2, ST5215

AY2020/2021 Semester 1

Due: 29 Sep 2020

Instruction

- Only PDF format and one PDF file will be accepted
- Name your PDF file in the following format: [StudentID].pdf, where [StudentID] is replaced with your ID number
- Go to LumiNus system and upload your PDF file to the folder *Submissions/HW2*. Before the deadline, you can update your previous submission by first deleting the old submission and then submitting the updated version.
- There are 6 questions in this assignment.

Problem set

JS = Mathematical Statistics, 2nd Ed, Jun Shao, 2003

Problem 1. Exercise 2.6.35 in JS

Problem 2. Exercise 2.6.53 in JS

Problem 3. Let $(X_1, \ldots, X_n)$ be a random sample from $E(0, 100)$ (See Table 1.2). Use Basu's theorem to show that $X_n^4 / \sum_{j=1}^n X_j^4$ and $\sum_{j=1}^n X_j$ are independent, $i = 1, \ldots, n$

Problem 4. Let $Y_1, \ldots, Y_n$ be independent with $Y_i \sim N\left(\alpha + \beta x_i, \sigma^2\right), i = 1, \ldots, n$ where $x_1, \ldots, x_n$ and σ^2 are known constants, and α and β are unknown parameters. We assume x_i's are not equal.

(a) Use the idea behind the method of moments to find an estimator of (α, β) (Hint: consider $\sum_i EY_i$ and $\sum_i E[Y_i x_i]$

(b) Find the maximum likelihood estimators $\hat{\theta} = (\hat{\alpha}, \hat{\beta})$ of $\theta = (\alpha, \beta)$

(c) Is the $\hat{\beta}$ you found in (b) unbiased? What is its MSE?

Problem 5. Exercise 2.6.63 in JS

Problem 6. Consider estimating success probability $\theta \in [0,1]$ from data $X \sim \text{Binomial}(n, \theta)$ under squared error loss. Define $\delta_{a,b}$ by

$$\delta_{a,b}(X) = a\frac{X}{n} + (1-a)b.$$

which might be called a linear estimator, because it is a linear function of X

(a) Find the variance and bias of $\delta_{a,b}$.

(b) If $a > 1$, show that $\delta_{a,b}$ is inadmissible by finding a competing linear estimator with better risk. Hint: Find an unbiased estimator with smaller variance.

(c) If $b > 1$ or $b < 0$, and $a \in [0, 1)$, show that $\delta_{a,b}$ is inadmissible by finding a competing linear estimator with better risk. Hint: Find an estimator with the same variance but better bias.

(d) If $a < 0$, find a linear estimator with better risk than $\delta_{a,b}$