Tutorial of ST5215

AY2020/2021 Semester 1

20 Aug 2020

Exercise 1 (Generalization of Hölder's inequality).
For $0 < p < 1$ and $q = -p/(1-p)$

$$E|XY| \geqslant (E|X|^p)^{1/p} (E|Y|^q)^{1/q}$$

Exercise 2 (Generalization of Minkowski's inequality).
$\left(E\left(\sum_{j=1}^n |X_j|\right)^r\right)^{1/r} > \sum_{j=1}^n (E|X_j|^r)^{1/r} \quad \text{for } 0 < r < 1$

Exercise 3 Let Y be measurable from $(\Omega, \mathcal{F})$ to $(\Lambda, \mathcal{G})$ and Z a function from $(\Omega, \mathcal{F})$ to $\mathcal{R}^k$. If Z is Borel on $(\Omega, \sigma(Y))$, then there is a Borel function h from $(\Lambda, \mathcal{G})$ such that $Z = h \circ Y$

Exercise 4 Let ϕ_X be a ch.f. of X. Show that $|\phi_X| \leq 1$, and uniformly continuous.

Exercise 5 Find the ch.f. and m.g.f. for the Cauchy distribution (i.e., P_X has p.d.f. $f(x) = (\pi(1+x^2))^{-1}$

Exercise 6 If X_i has the Cauchy distribution $C(0,1), i = 1, \ldots, k$, then Y/k has the same distribution as X_1.