Tutorial of ST5215

AY2020/2021 Semester 1

10 Sep 2020

Exercise 1 Let $X_1, \ldots, X_n$ be i.i.d. random variables having the Lebesgue p.d.f. $\theta^{-1} e^{-(x-\theta)/\theta} I_{(\theta,\infty)}(x)$, where $\theta > 0$ is an unknown parameter.
(a) Find a statistic that is minimal sufficient for θ.
(b) Show whether the minimal sufficient statistic in (a) is complete.

Exercise 2 Let $X_1, \ldots, X_n$ be i.i.d. from the $N\left(\theta, \theta^2\right)$ distribution, where $\theta > 0$ is a parameter. Find a minimal sufficient statistic for θ and show whether it is complete.

Exercise 3 Suppose that $(X_1, Y_1), \ldots, (X_n, Y_n)$ are i.i.d. random 2–vectors having the normal distribution with $EX_1 = EY_1 = 0, \operatorname{Var}(X_1) = \operatorname{Var}(Y_1) = 1$, and $\operatorname{Cov}(X_1, Y_1) = \theta \in (-1, 1)$
(a) Find a minimal sufficient statistic for θ.
(b) Show whether the minimal sufficient statistic in (a) is complete or not.
(c) Prove that $T_1 = \sum_{i=1}^n X_i^2$ and $T_2 = \sum_{i=1}^n Y_i^2$ are both ancillary but (T_1, T_2) is not ancillary.