Tutorial of ST5215

AY2020/2021 Semester 1

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Exercise 1. [Neyman and Scott (1948)] Suppose we have a sample of size d from each of n normal populations with common unknown variance but possibly different unknown means

$$X_{ij} \in \mathcal{N}\left(\mu_i, \sigma^2\right), \quad I = 1, \ldots, n, \quad j = 1, \ldots, d$$

where all the X_{ij} are independent.
(a) Find the maximum-likelihood estimate of σ^2.
(b) Show that for d fixed, the MLE of σ^2 is not consistent as $n \to \infty$. Why doesn't Theorem 17 apply?
(c) Find a consistent estimate of σ^2.

Exercise 2. Let $X = (X_1, \ldots, X_n)$ be a random sample of random variables with probability density f_θ. Find an MLE of θ and its asymptotic distribution in each of the following cases
(i) $f_\theta(x) = e^{-(x-\theta)}I_{(\theta,\infty)}(x), \theta > 0$
(ii) $f_\theta(x) = \theta(1-x)^{\theta-1}I_{(0,1)}(x), \theta > 1$