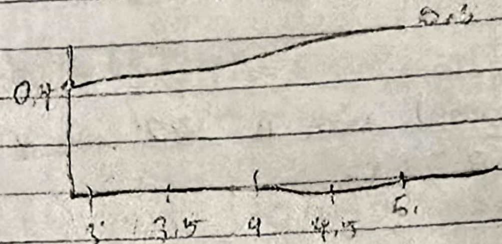
HW 9

1. a. $\int_3^5 (0.075x + 0.2)\,dx = 0.0375x^2 + 0.2x\Big]_3^5 = (0.0375(5)^2 + 0.2(5)) - (0.0375(3)^2 + 0.2(3))$

$= 1.93 - 0.93 = 1$



b. $P(X \le 4) = \int_3^4 (0.075x + 0.2)\,dx = 0.0375x^2 + 0.2x\Big]_3^4$

$= 1.4 - 0.9375 = 0.4625$

c. $P(3.5 \le X \le 4.5) = \int_{3.5}^{4.5} (0.075x + 0.2)\,dx = 0.0375x^2 + 0.2x\Big]_{3.5}^{4.5} = 0.5$

$P(4.5 \le X) = P(4.5 \le X) = \int_{4.5}^{5} (0.075x + 0.2)\,dx = 0.2781$

2. $f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10}\,dx = 0.5$

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10}\,dx = 0.5$

c. $P(-2 \le X \le 3) = \int_{-2}^{3} 1/10\,dx = 0.5$

d. $P(k < X < k+4) = \int_{k}^{k+4} 1/10\,dx = 1/10\,x\Big]_{k}^{k+4} = 0.4$

5.

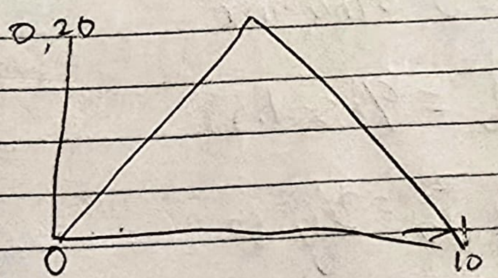
a. $1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 kx^2\,dx = \frac{kx^3}{3}\Big]_0^2 = \frac{8k}{3} \Rightarrow k = \frac{3}{8}$

b. $P(0 \le x \le 1) = \int_0^1 3/8\, x^2\,dx = 1/8\, x^3\Big]_0^1 = 1/8$

c. $P(1 \le x \le 1.5) = \int_1^{1.5} 3/8\, x^2\,dx = 1/8\, x^3\Big]_1^{1.5} = 1/8(3/2)^3 - 1/8(1)^3 = +9/64$

d. $P(x \ge 1.5) = 1 - \int_{1.5}^2 3/8\,dx = 1/8\,x^3\Big]_{1.5}^2 = 1/8(2)^3 - 1/8(1.5)^3 = 0.578$

8. a.



b. $\int_{-\infty}^{\infty} f(y)\,dy = \int_0^5 \frac{1}{25} y\,dy + \int_5^{10} (2/5 - 1/25\,y)\,dy = y^2/50\Big]_0^5 + (2/5\,y - 1/50\,y^2)\Big]_5^{10}$

$= 25/50 + [(4-2)-(2-1/2)] = 1/2 + 1/2 = 1$

c. $P(y \le 3) = \int_0^3 1/25\, y\,dy = y^2/50\Big]_0^3 = 9/50 = 0.18$

d. $P(y \le 8) = \int_0^5 1/25\,y\,dy + \int_5^8 (2/5 - 1/25\,y)\,dy = 23/25 = 0.92$

e. $P(3 \le y \le 8) = P(y \le 8) - P(y < 3) = 0.92 - 0.18 = 0.74$

f. $P(y < 2 \text{ or } y > 6) = \int_0^2 1/25\,y\,dy + \int_6^{10} (2/5 - 1/25\,y)\,dy = 0.4$

11, a, $P(x \le 1) = F(1) = 1^2/4 = 0.25$

b, $P(0.5 \le x \le 1) = F(1) - F(0.5) = 1^2/4 - 0.5^2/4 = 0.187$

c, $P(x > 1.5) = 1 - P(x \le 1.5) = 1 - F(1.5) = 1 - 1.5^2/4 = 0.437$

d, $0.5 = F(\tilde{\mu}) = \frac{\tilde{\mu}^2}{4} = \tilde{\mu}^2 = 2 = \sqrt{2}$

e, $f(x) = F'(x) = x/2$ for $0 \le x < 2$. and $= 0$ otherwise

f, $E(x) = \int_0^2 x \cdot x/2\, dx = 1/2 \int_0^2 x^2\, dx = \frac{x^3}{6}\Big]_0^2 = 8/6 \approx 1.33$

g, $E(x^2) = \int_0^2 x^2 \frac{x}{2}\, dx = 1/2 \int_0^2 x^3\, dx = x^4/8 \Big]_0^2 = 2$, so

$V(x) = E(x^2) - [E(x)]^2 = 2 - (8/6)^2 = 8/36$

h, $E(x)^2 = 2$

12, a, $P(x < 0) = F(0) = 0.5$

b, $P(-1 \le x \le 1) = F(1) - F(-1) = 0.6875$

c, $P(x > 0.5) = 1 - P(x \le 0.5) = 1 - F(0.5) = 1 - 0.6836 = 0.3164$

d, $f(x) = F'(x) = \frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right) = 0 + \frac{3}{32}\left(4 - \frac{3x^2}{3}\right) = 0.093(4 - x^2)$

e, $F(\tilde{\mu}) = 0.5$, $F(0) = 0.5$,

17. $\frac{X-A}{B-A} = P \rightarrow X = A + (B-A)P$

b. $E(X) = \int_A^B x \cdot 1/B-A \, dx = A+B/2$

$E(X^2) = (A^2+AB+B^2)/(3) \rightarrow V(X) = E(X^2) - [E(X)]^2 = (B-A)^2/12$

$\sigma_X = \sqrt{V(X)} = B-A/\sqrt{12}$

c. $E(X^n) = \int_A^B x^n \cdot 1/B-A \, dx = (1/B-A)(x^{n+1}/n+1)\Big]_A^B = \frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}$