Topology optimization of heat conduction problems

Workshop on industrial design optimization for fluid flow
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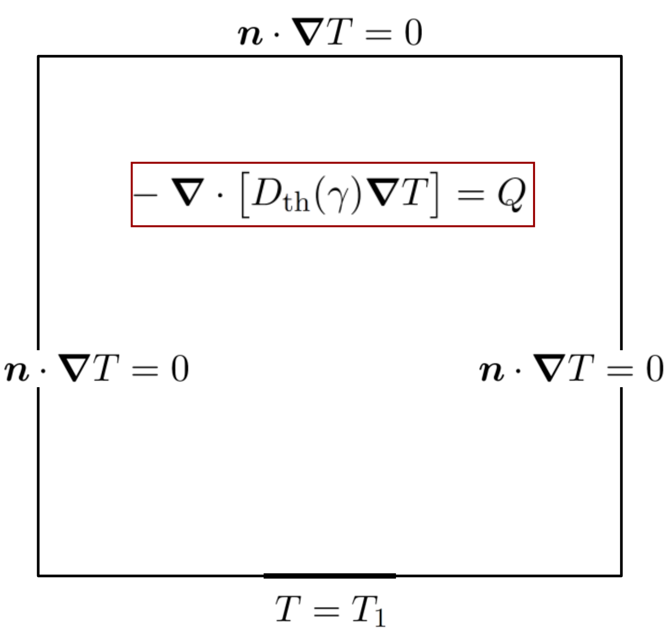
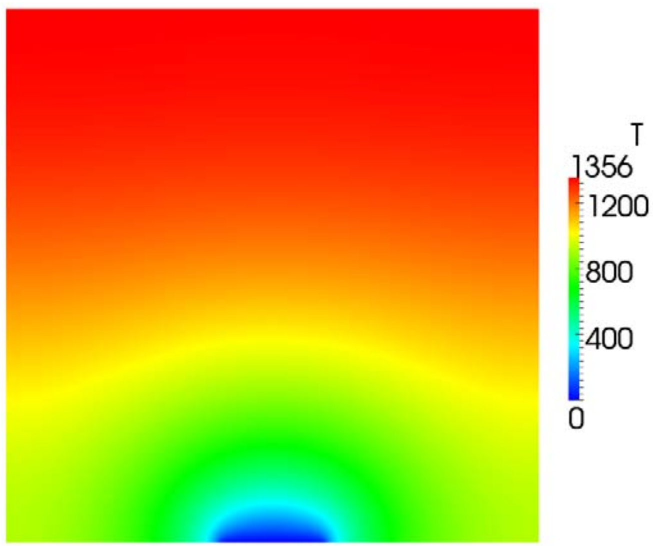
Motivation and outline

- **Challenges**
  - Topology optimization of coupled heat conduction and fluid flow
  - Large scale systems: OpenFOAM
  - FVM + Topology optimization + diffusion problems
- **Outline**
  - Optimization of simple conduction problem
  - FVM on diffusion problems
  - Constant convection added
  - Preliminary tests of unstructered mesh and 3D

Simple heat conduction problem

- **Pure heat conduction problem**
  - **Constant and homogeneous heat source** Q
  - **Insulating walls and heat sink at bottom** $T_1 = 0$
  - **For topology optimization: Design dependent thermal diffusivity** $D_{th}(\gamma)$





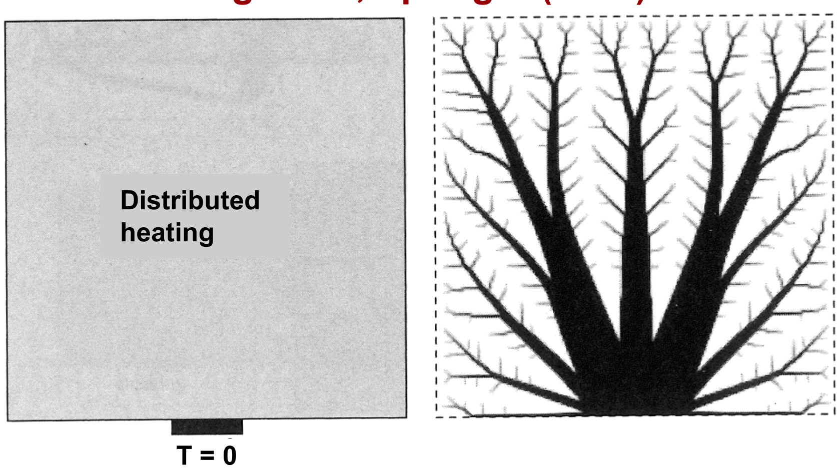
Topology optimization of conduction problem

- **Topology optimization**
  - Minimize the weighted average temperature
  - Distribution of two materials with different conductivity
  - High conductivity D_{th2} : black (Max. volume 40%)
  - Low conductivity D_{th1} : White
  - $D_{\mathrm{th}}(\gamma) = D_{\mathrm{th1}} + (D_{\mathrm{th2}} - D_{\mathrm{th1}})\,\gamma^3$, $D_{\mathrm{th2}}/D_{\mathrm{th1}} = 1000$

- **IPOPT for the optimization routine**
  - Gradent based optimization
  - Approximations of the hessian

- **Benchmark example**
  - FEM calculation
  - MMA for the optimization loop

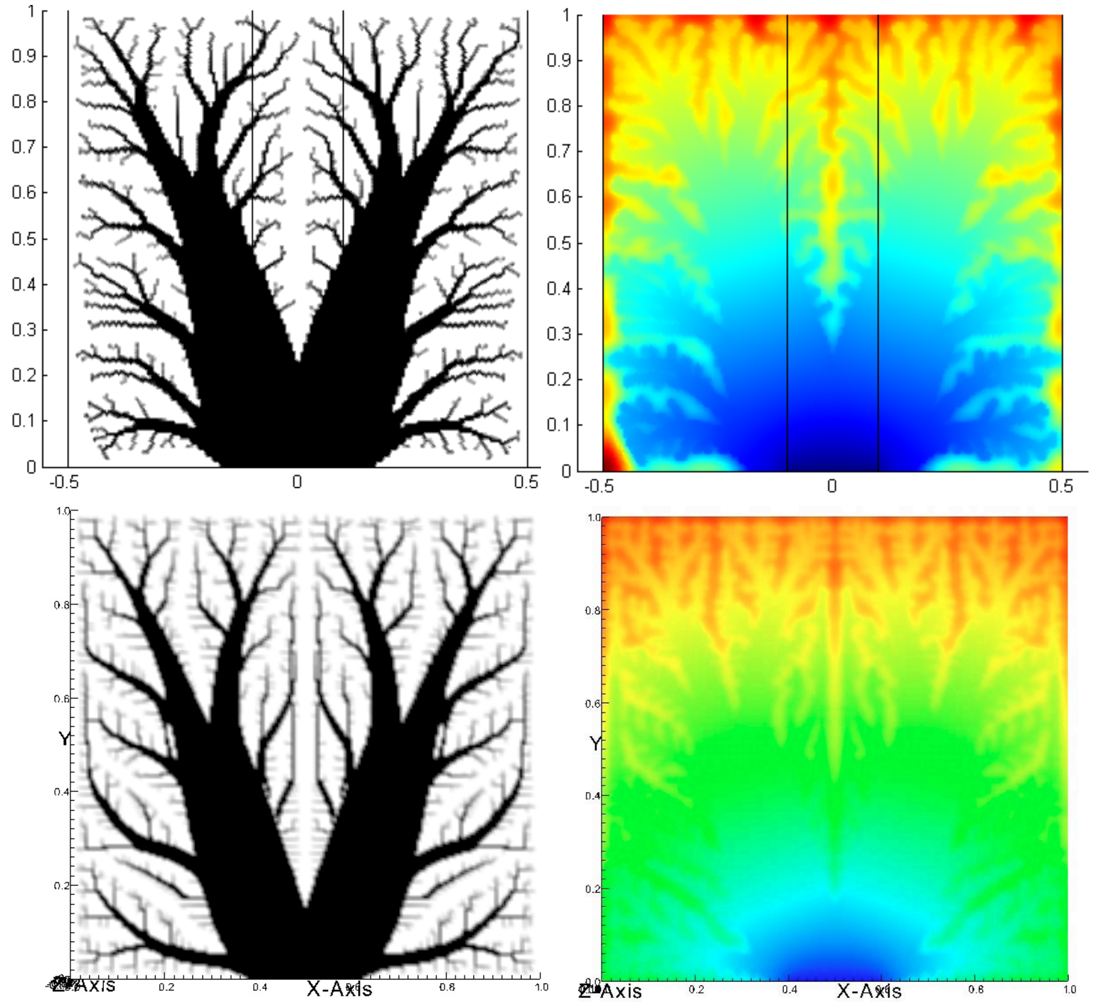
Bendsøe - Sigmund, Springer (2004)



FVM >< FEM optimization results

- **COMSOL (FEM) + MMA**
  - Unstructured mesh
- **OpenFOAM (FVM) + IPOPT**
  - Slightly different parameter values
  - Square mesh elements

Design field: **Temperature field:**



New: FVM & TopOpt & conduction

- **General form of balance law (steady state)**

$$\nabla \cdot \big(F(u, \nabla u)\big) + s(u) = 0$$

- **Previously studied: FVM & TopOpt for flow problems**
  - **Design field enters the source term s**

$$\rho(\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \eta \boldsymbol{\nabla}^2 \mathbf{u} - \underline{\alpha(\gamma)\,\mathbf{u}}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

  - **For instance: Othmer, Int. Jour. Num. Meth. Fluids (2008).**

- **FVM & TopOpt for conduction problems**
  - **Design field enters the flux term F**

$$-\boldsymbol{\nabla} \cdot \big[\underline{D_{\mathrm{th}}(\gamma)}\boldsymbol{\nabla} T\big] = Q$$

- **Discretization using the FVM**
  - **The flux term is integrated by parts:**
    Sensitivites becomes dependent on the design field gradient,

Sensitivity analysis of conduction problem

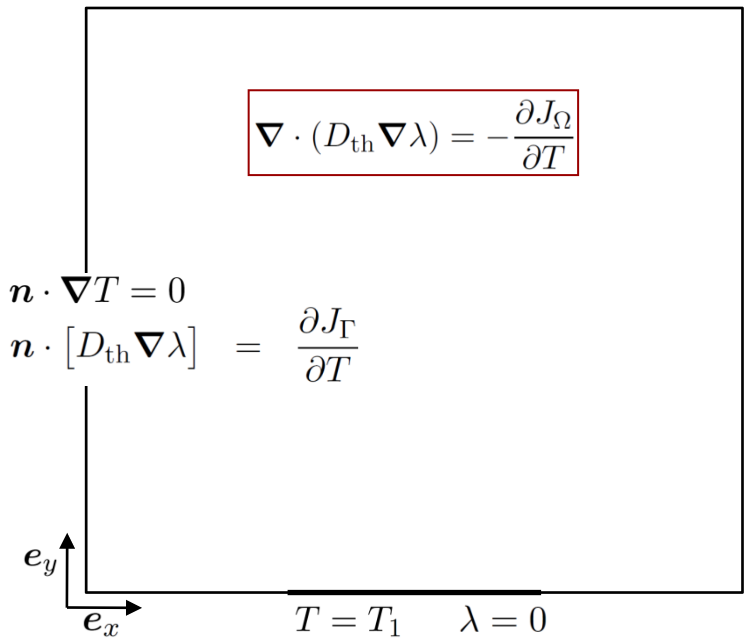
- **Adjoint method**
  - **Define a cost function**

    $$J = \int_{\Gamma} \mathrm{d}\Gamma J_{\Gamma} + \int_{\Omega} \mathrm{d}\Omega J_{\Omega}$$

  - **Define Lagrange function by introducing Lagrange multiplier λ**

    $$\mathcal{L} = J + \int_{\Omega} \mathrm{d}\Omega\, \lambda R$$

  - **Solve the adjoint problem for λ (adjoint temperature)**



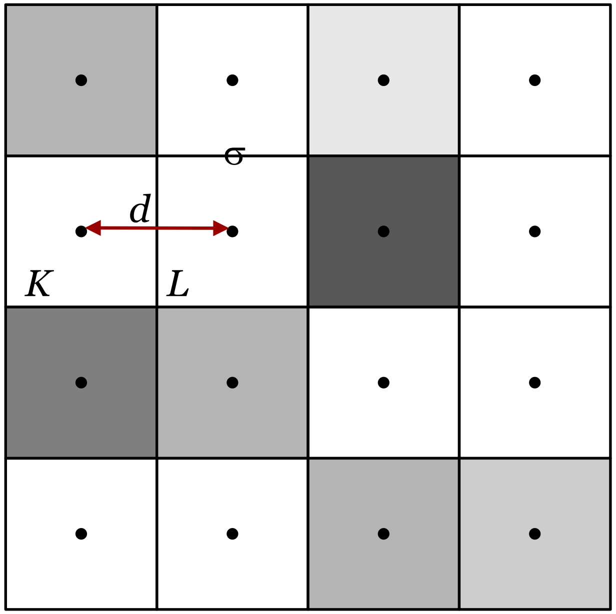
  - **Calculate sensitivity**

    $$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\gamma_i} = \int_{\Omega} \mathrm{d}\Omega\, \lambda \boldsymbol{\nabla} \cdot \left[\frac{\partial D_{\mathrm{th}}(\gamma)}{\partial \gamma_i} \boldsymbol{\nabla} T \right]$$

Continuous vs. discrete adjoint

- **FVM**
  - Gradient approximation on cell edges
  - Imprecise for discontinuous fields
- **Continuous adjoint**
  - Optimize (differentiate) then discretize
  - Sensitivity: $\dfrac{\mathrm{d}\mathcal{L}}{\mathrm{d}\gamma_i} = -\displaystyle\int_\Omega \mathrm{d}\Omega \, \frac{\partial D_{\mathrm{th}}(\gamma)}{\partial \gamma_i} \nabla\lambda \cdot \nabla T$
- **Sensitivity for flow problem**
  - Adjoint and primary flow velocity; u, v
  - Sensitivity: $\dfrac{\mathrm{d}\mathcal{L}}{\mathrm{d}\gamma_i} = \displaystyle\int_\Omega \mathrm{d}\Omega \, \frac{\partial \alpha(\gamma)}{\partial \gamma_i} \boldsymbol{u} \cdot \boldsymbol{v}$
- **Discrete adjoint**
  - Discretize then optimize (differentiate)
  - Sensitivity: $\dfrac{\mathrm{d}\mathcal{L}}{\mathrm{d}\gamma_i} = \displaystyle\sum_{\sigma_i} \frac{|\sigma_i|}{d_{KL}} \frac{\partial D_{\mathrm{th}}}{\partial \gamma_i} (T_K - T_L)(\lambda_K - \lambda_L)$

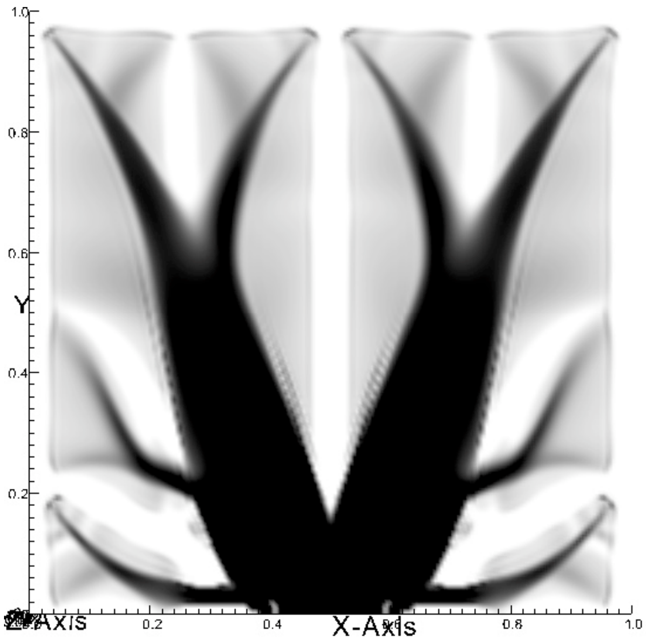
Discretized design field γ



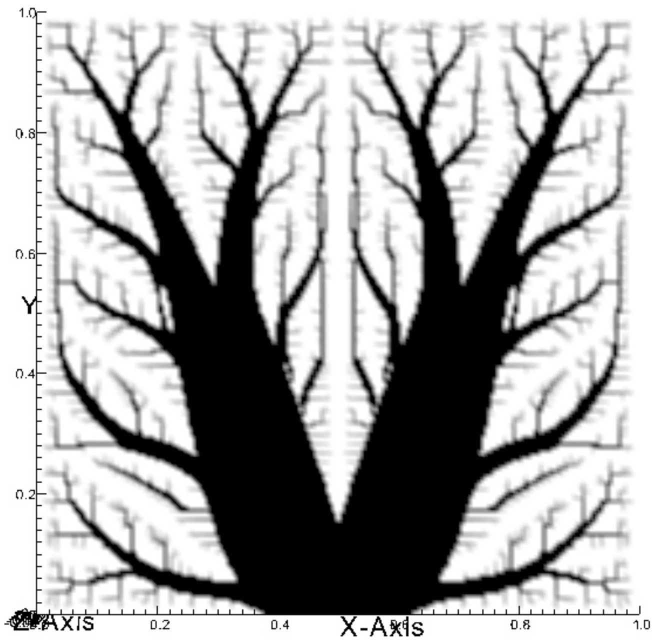
Discrete >< continuous optimizations

- Different optimization output depending on parameters

Continuous adjoint



Discrete adjoint



Further numerical considerations

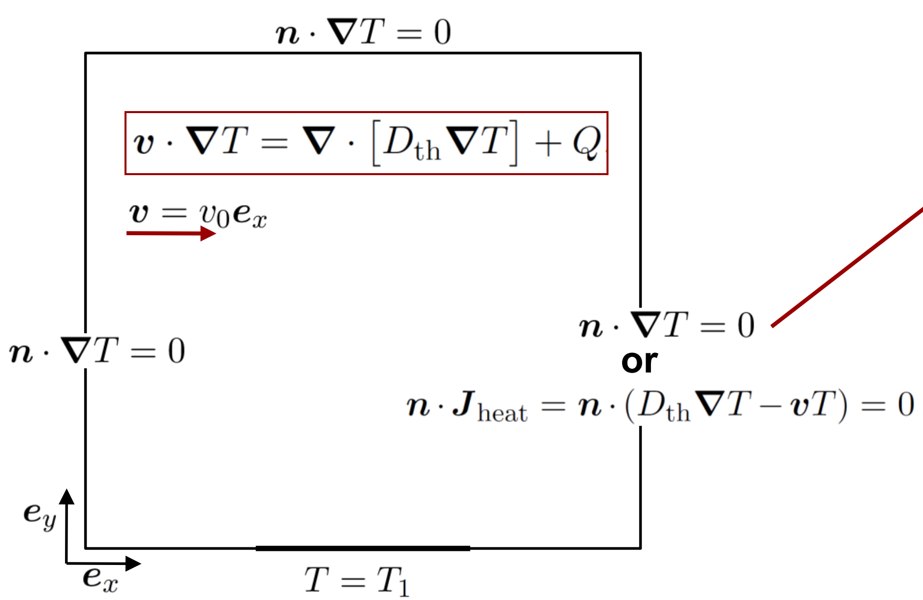
- **Mesh convergence**
  - **Proof lacking: Discretized continuous adjoint sensitivty should converge to continuous expression**
  - **Expect unaltered topology, but feature refinement can cause problems**
    - Continuous adjoint: Volume integration -> small sensitivity change
    - Discrete adjoint: Cell edge integration -> large sensitivity change

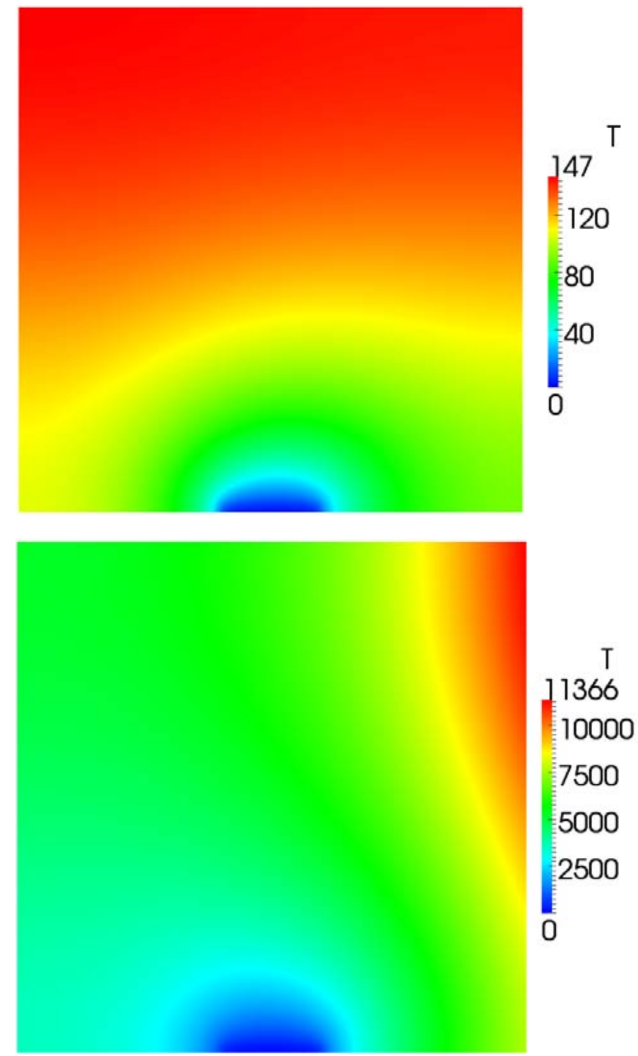
- **IPOPT**
  - **Robust optimization routine**
  - **Poor hessian approximations on top of uncertain gradient approximations**
  - **MMA might be more suitable (no hessian needed)**

- **Higher order approximations of fluxes**
  - **Unsuitable for fields with large discontinuities across cell boundaries**

Conduction – convection model

- **Constant convection term added**
- **Insulation BC: Robin condition**
  - **Non-trivial implementation in OpenFOAM**

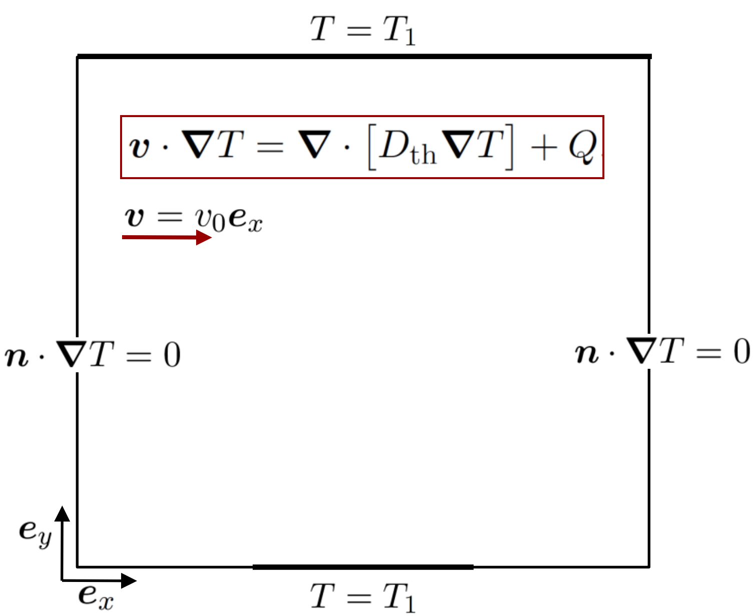




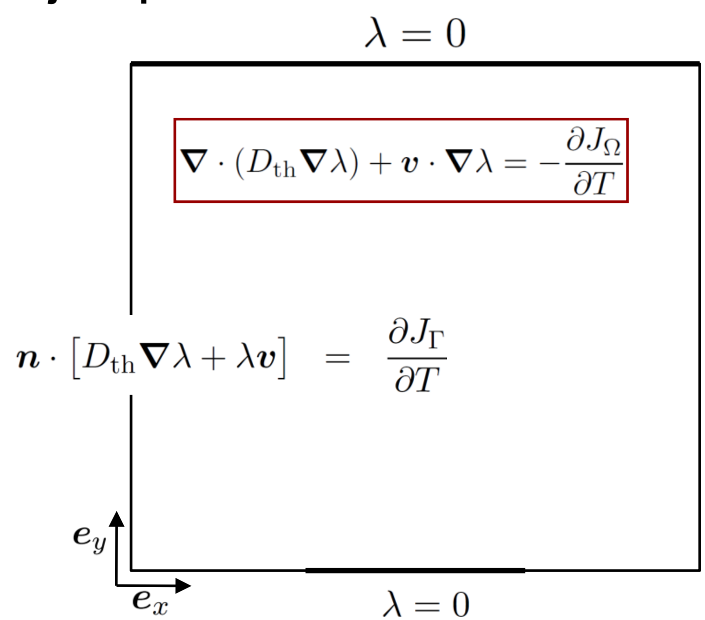
Adjoint equations for convection-conduction problem

- Robin BC in the adjoint problem
- Heat sink added at upper boundary

Primary problem:



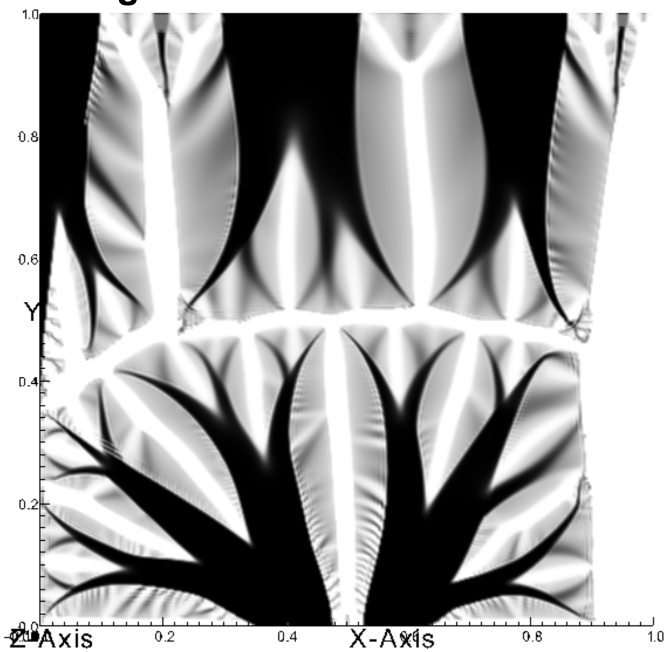
Adjoint problem:



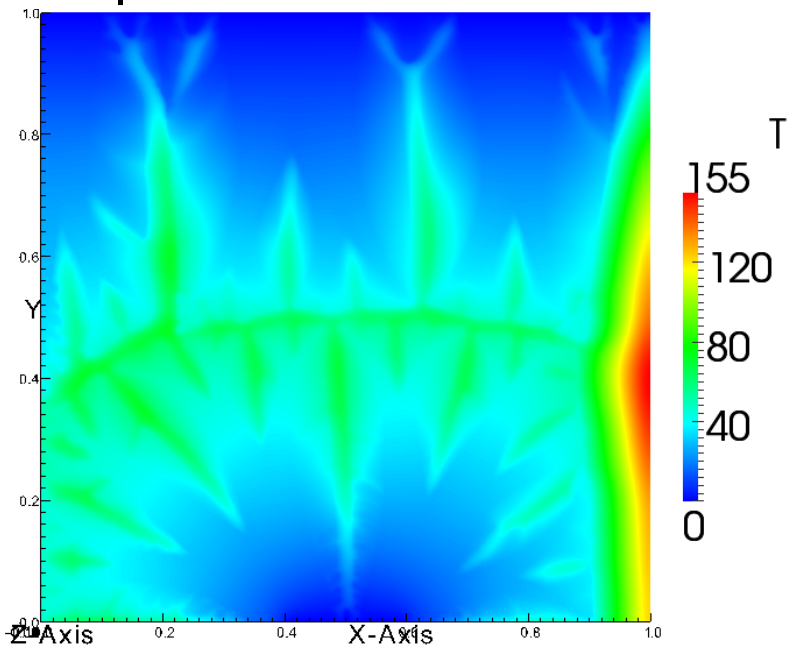
Preliminary conduction-convection optimization

- Minimize the weighted average temperature
- Continuous adjoint

Design field



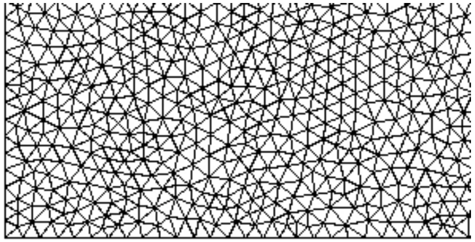
Temperature field



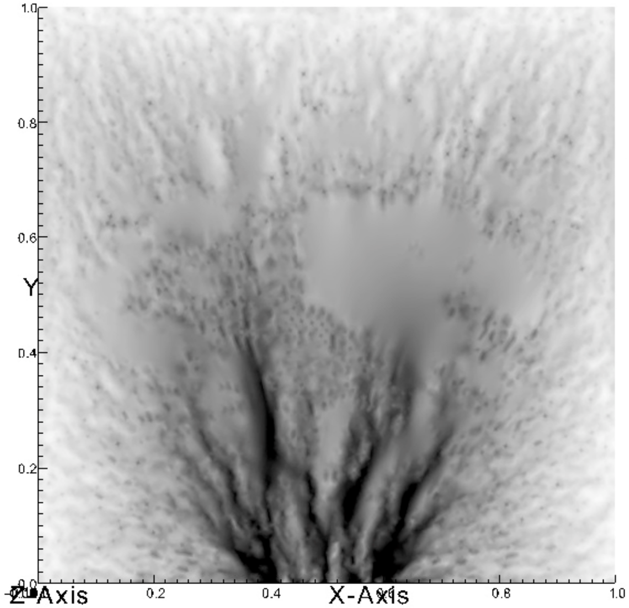
Unstructured mesh – preliminary results

- **Pure conduction problem**
- **Influence of adjoint method**

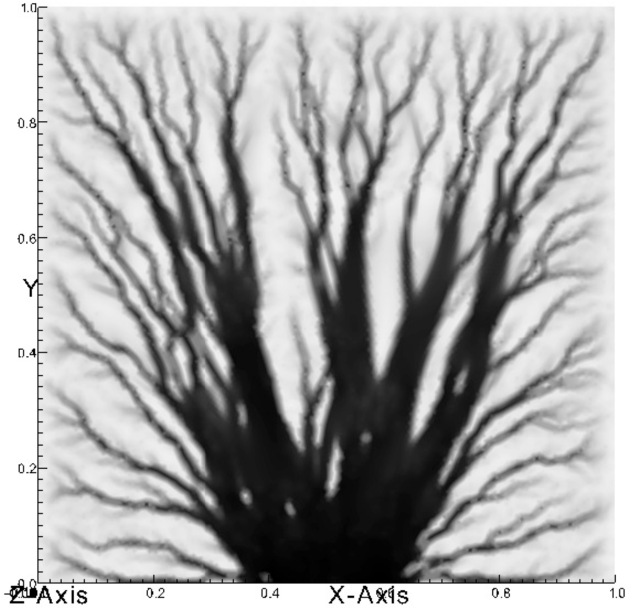
Section of mesh



Continuous adjoint



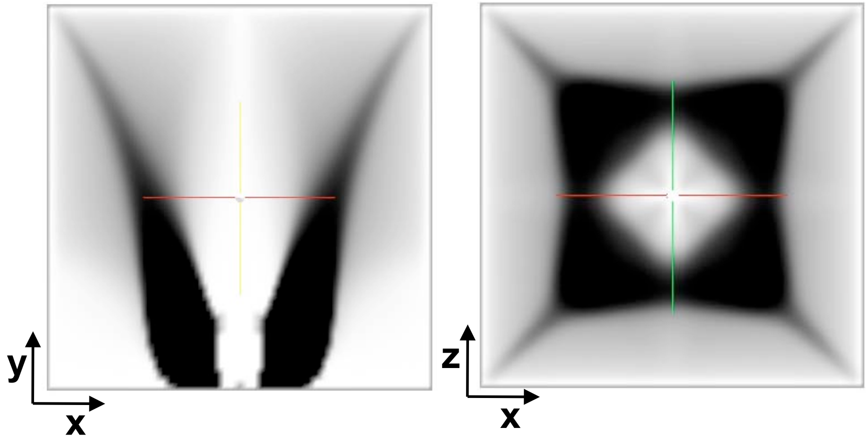
Discrete adjoint



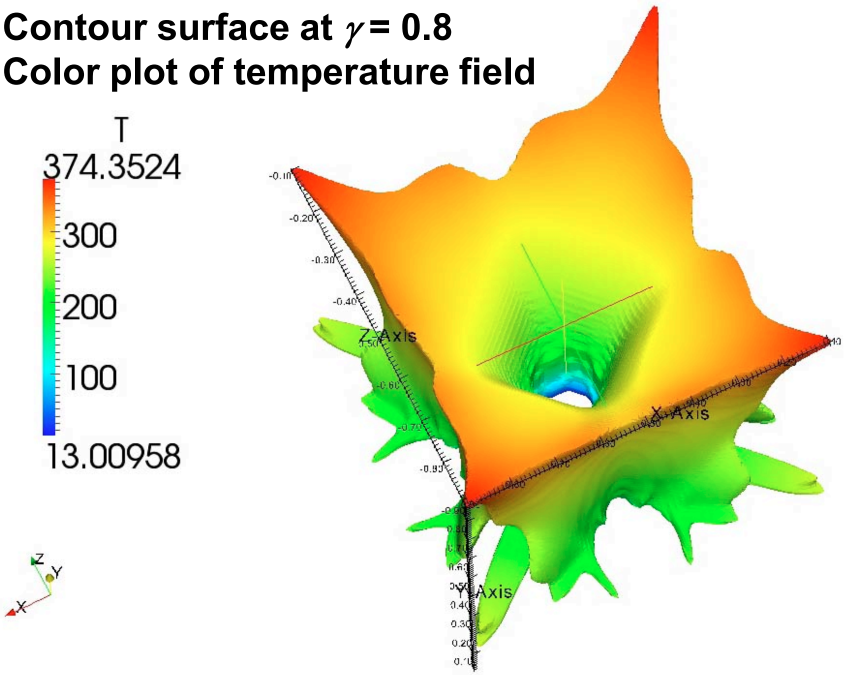
Preliminary 3D result

- **Pure conduction problem**

Cross-sectional plots of design field γ:



Contour surface at $\gamma = 0.8$
Color plot of temperature field



- **Parallelization**
  - Decomposition of computational domain (OpenFOAM)
  - Parallelization of optimization routine (MMA)

Summary and outlook

- **Summary**
  - Topology optimization of heat conduction problem with IPOPT & OpenFOAM
  - Comparison with COMSOL & MMA
  - Identified numerical issues for FVM based topology optimization of diffusion problems
  - Implemented discrete adjoint method for regular (and unstructured) meshes
  - Implementation of optimization routine for constant convection problem
  - Preliminary results of: conduction-convection problem, 3D heat conduction problem, unstructured mesh

- **Outlook**
  - Parallelization of optimization routine (MMA)
  - Coupling to the Navier-Stokes equation
  - Optimization of fully coupled heat transfer problem for a simple model case