Automatic Signature Verification: The State of the Art

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***Abstract*—In recent years, along with the extraordinary diffusion of the Internet and a growing need for personal verification in many daily applications, automatic signature verification is being considered with renewed interest. This paper presents the state of the art in automatic signature verification. It addresses the most valuable results obtained so far and highlights the most profitable directions of research to date. It includes a comprehensive bibliography of more than 300 selected references as an aid for researchers working in the field.**

***Index Terms*—Biometry, personal verification, signature verification, system security.**

I. Introduction

THE SECURITY requirements of the today's society have placed biometrics at the center of a large debate, as it is becoming a key aspect in a multitude of applications [19], [262], [370]. The term biometrics refers to individual recognition based on a person's distinguishing characteristics. While other techniques use the possession of a token (i.e., badge, ID card, etc.) or the knowledge of something (i.e., a password, key phase, etc.) to perform personal recognition, biometric techniques offer the potential to use the inherent characteristics of the person to be recognized to perform this task. Thus, biometric attributes do not suffer from the disadvantages of either the token-based approaches, whose attributes can be lost or stolen, and knowledge-based approaches, whose attributes can be forgotten [137], [325].

A biometric system can either verify or identify. In verification mode, it authenticates the person's identity on the basis of his/her claimed identity. Instead, in identification mode, it establishes the person's identity (among those enrolled in a database) without the subjects having to claim their identity [139], [325]. Depending on the personal traits considered, two types of biometrics can be defined: physiological or behavioral. The former are based on the measurement of biological traits of users, like, for instance, fingerprint, face, hand geometry, retina, and iris. The latter consider behavioral traits of users, such as voice or handwritten signature [19], [139], [322], [325], [370].

The assessment of biometrics is a multifaceted problem [139], [326], [336]. For instance, a biometric trait should be *universal*, i.e., each person should possess the trait; *unique*, i.e., no two persons should share the same trait; *permanent*, i.e., the trait should neither change nor be alterable; *collectable*, i.e., the trait can be obtained easily. In addition, biometric system design should also address other desirable features such as accuracy, cost and speed effectiveness, acceptability by the users, and so on [127], [322].

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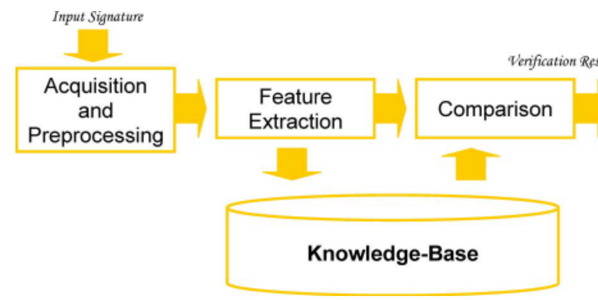


Fig. 1. Process of signature verification.

Although a wide set of biometrics has been considered so far, it is worth noting that no trait is able to completely satisfy all the desirable characteristics required for a biometric system [137]. Thus, the assessment of a biometric trait is strongly dependent on the specific application since it involves not only technical issues but also social and cultural aspects [137], [322], [325].

Handwritten signatures occupy a very special place in this wide set of biometric traits [78], [81], [165], [248], [258]. This is mainly due to the fact that handwritten signatures have long been established as the most widespread means of personal verification. Signatures are generally recognized as a legal means of verifying an individual's identity by administrative and financial institutions [225], [336]. Moreover, verification by signature analysis requires no invasive measurements and people are familiar with the use of signatures in their daily life [259].

Unfortunately, a handwritten signature is the result of a complex process depending on the psychophysical state of the signer and the conditions under which the signature apposition process occurs. Therefore, although complex theories have been proposed to model the psychophysical mechanisms underlying handwriting [253]–[256] and the ink-depository processes [62], [99], [100], [101], signature verification still remains an open challenge since a signature is judged to be genuine or a forgery only on the basis of a few reference specimens [250]. Fig. 1 sketches the three main phases of automatic signature verification: data acquisition and preprocessing, feature extraction, and classification. During enrolment phase, the input signatures are processed and their personal features are extracted and stored into the knowledge base. During the classification phase, personal features extracted from an inputted signature are compared against the information in the knowledge base, in order to judge the authenticity of the inputted signature.

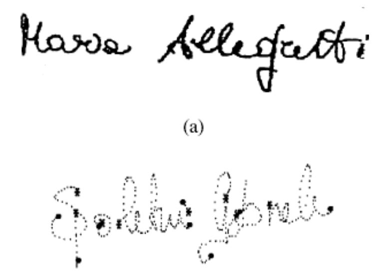
Automatic signature verification involves aspects from disciplines ranging from human anatomy to engineering, from neuroscience to computer science and system science [196]. Because of this fact, in recent years, studies on signature verification have attracted researchers from different fields, working for universities and companies, which are interested in not only the scientific challenges but also the valuable applications this field offers [229]. Comprehensive survey papers reported the progress in the field of automatic signature verification until 1993 [165], [258], [291]. In 1994, a special issue and a book collecting the most relevant research activities were published [251]. Successively, various papers have summarized the increasing research efforts in the field [52], [58], [224], [248], [280] also with respect to the more general area of handwriting analysis and processing [259].

In conjunction with the recent and extraordinary growth of the Internet, automatic signature verification is being considered with new interest. The creation of specific laws and regulations, which have been approved in many countries [173], [336], and the attention that several national associations and international institutes have given to the standardization of signature data interchange formats [10], [135], [136] are evidence of the renewed attention in this field. The aim of these efforts is to facilitate the integration of signature verification technologies into other standard equipment to form complete solutions for a wide range of commercial applications such as banking, insurance, health care, ID security, document management, e-commerce, and retail point-of-sale (POS) [78], [259], [320].

This paper presents the state of the art in automatic signature verification, with specific attention to the most recent advancements. Following an introduction of the phases of the signature verification process, the main contributions of research activities in recent years are described and the most promising trends are discussed. Specifically, Section II presents the main aspects related to data acquisition and preprocessing and Section III discusses the feature extraction phase. Section IV describes research activities concerning the classification phase while Section V summarizes the performance of systems for automatic signature verification reported in the literature. A brief discussion on the applications of automatic signature verification and the most promising research directions are reported in Section VI, along with the conclusions of this paper. A bibliography of more than 300 references is also provided for the more interested reader. It includes the most relevant papers recently published as well as some older papers, which can help give a comprehensive outline of developments in this field of research.

II. Data Acquisition and Preprocessing

On the basis of the data acquisition method, two categories of systems for handwritten signature verification can be identified: static (offline) systems and dynamic (online) systems [132]. Static systems use offline acquisition devices that perform data acquisition after the writing process has been completed. In this case, the signature is represented as a gray level image $\{S(x,y)\}_{0\le x\le X, 0\le y\le Y}$, where $S(x,y)$ denotes the gray level at the position (x,y) of the image. Instead, dynamic systems use online acquisition devices that generate electronic



(a)

(b)

Fig. 2. Static/dynamic signatures. (a) Static signature. (b) Dynamic signature ("$*$" : pen-down; "•" : pen-up).

signals representative of the signature during the writing process. In this case, the signature is represented as a sequence $\{S(n)\}_{n=0,1,\ldots,N}$, where $S(n)$ is the signal value sampled at time $n\Delta t$ of the signing process ($0 \le n \le n$), Δt being the sampling period. Therefore, the offline case involves the treatment of the spatioluminance of a signature image [see Fig. 2(a)], whereas the online case concerns the treatment of a spatiotemporal representation of the signature [see Fig. 2(b)].

The most traditional online acquisition devices are digitizing tablets [115]. Of course, the use of digitizing tablets is far from being natural and many attempts have been made to produce electronic pens that are more acceptable to users while being easy to integrate into current systems [121], [300], [314]. Electronic pens with touch-sensitive screens and digital-ink technologies that avoid signer disorientation by providing immediate feedback to the writer are good examples of such efforts [5], [6]. Electronic pens are also capable of detecting position, velocity, acceleration, pressure, pen inclination, and writing forces, with the use of strain gauges [46], magnetoelastic sensors [374], shift of resonance frequency [237], and laser diodes [300]. Some input devices use ink pen, which is exactly like using a conventional pen on standard paper positioned on the tablet. In this case, the pen produces conventional handwriting using ink, while producing an exact electronic replica of the actual handwriting. The advantage is the possibility to record online and offline data at the same time and to allow very natural writing since an almost standard pen and paper are used [106], [239]. In general, the development of the digitizing devices, ranging from the traditional table-based tablets to the recent handy digitizer tablets [158], personal digital assistant (PDA) [266], and input devices for mobile computing [5], [6], [72], [74], [261], poses new problems concerning device interoperability, that is, the capability of a verification system to adapt to the data obtained from different devices. One example of this is mouse-based signature verification that has been the object of specific research due to its relevance in Internet-based transactions [173], [313]. Other approaches capture handwriting by computer vision techniques. For instance, a special stylus conveying a small charge-coupled device (CCD) camera that captures a series of snapshots of the writing has been recently proposed [219]. The system recovers the whole handwritten trace by analyzing the sequence of successive snapshots. The stylus is also provided with a stress sensor for detecting the pressure applied on the ballpoint and determining the pen-up/pen-down information. There are also alternative approaches that do not require the use of a special stylus, and instead exploit a video camera that is focused

TABLE I
SEGMENTATION TECHNIQUES

Technique	Category	References
Segmentation by Pen-down/Pen-up Signals	Online	G. Dimauro et al. [54, 56], Herbst and Liu [121], R. Plamondon [252], C. Schmidt and K.-F. Kraiss [298], Y. Xuhua et al. [352, 353, 354]
Segmentation by Velocity Signal Analysis	Online	H.Y. Kwon et al. [162], R. Plamondon et al [260]
Segmentation by Perceptually relevant points.	Online	J.J. Brault and R. Plamondon [21], M.M.Shafiei and H.R. Rabiee [299], K.W. Yue and W.S. Wijesoma [369]
Segmentation by Dynamic Time Warping	Online	L. Bovino et al. [18], S. Chen and S. N. Srihari [33], V. Di Lecce et al. [50], G. Dimauro et al. [55], J. Lee et al. [166], W.-S. Lee et al. [172], T.H. Rhee et al. [275]
Segmentation by Connected Components	Offline	G. Congedo et al. [41], G. Dimauro et al. [57, 59]
Segmentation by Tree Structure Analysis	Offline	M. Ammar et al [8]
Segmentation by Statistics of Directional Data	Offline	K. Huang and H.Yan [127], R. Sabourin and R.Plamondon [287, 289]

on the user while writing on a piece of paper with a normal pen [24], [210], [355]. In this way, handwriting is recovered from the spatiotemporal representation given by the sequence of images. This approach can be the simplest way for a user to interact with the computer by using handwriting, and its potential has been specifically demonstrated in the domain of automatic signature verification [207], [209], [211]. In addition, a handglove device for virtual reality applications has been used for online signature verification [317]. This device can provide data on both the dynamics of the pen motion during signing and the individual's hand shape.

In the preprocessing phase, the enhancement of the input data is generally based on techniques originating from standard signal processing algorithms [242].

When static signatures are considered, typical preprocessing algorithms concern signature extraction [59], [61], noise removal by median filters [15], [17], [126] and morphological operators [126], [263], signature size normalization [17], [263], binarization [126], thinning [17], [359], and smearing [126], [283]. In this field, an important issue is the treatment of static signature images on bank checks, since bank check processing still remains an open challenge for the scientific community [59]. In fact, bank check images are very complex because they generally contain a color pictorial background, several logos, and many preprinted guidelines. Thus, the treatment of signature images extracted from the bank check is very difficult and the development of signature verification systems with the accuracy required of banks and other financial institutions is an area of continued research [39], [59], [60], [61], [171], [236], [367]. For this purpose, specific hybrid systems have been developed, which combine online and offline information for handwritten signature verification. The online reference signature, acquired through a digitizing tablet, serves for the preprocessing of the corresponding scanned offline signature image. This kind of hybrid system is well suited for a banking environment where the presence of the customer is needed to open a new account, but is unnecessary during the verification of signatures on checks and other documents [375], [376].

Typical preprocessing algorithms for dynamic signature verification involve filtering, noise reduction, and smoothing. For this purpose, Fourier transform [146], [147], [379], mathematical morphology [115], and Gaussian functions [37], [138], [180] have been used. Signature normalization procedures using global reference systems (center of mass and principal axes of inertia) [131] and Fourier transform [7], [146], [147], [149], [203], [273] have been considered to standardize signatures in the domain of position, size, orientation, and time duration.

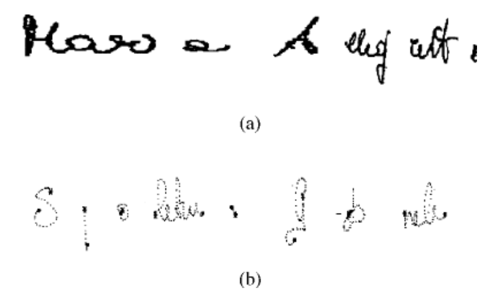


Fig. 3. Examples of signature segmentation. (a) Offline signature segmentation by connected components. (b) Online signature segmentation by components ("$*$" : pen-down; "•" : pen-up).

A crucial preprocessing step, that strongly influences all the successive phases of signature verification, is segmentation. Signature segmentation is a complex task since different signatures produced by the same writer can differ from each other due to local stretching, compression, omission or additional parts. Because of this, specific attention has been devoted to signature segmentation, and several techniques have been proposed. In general, some segmentation techniques derive from specific characteristics of handwritten signatures and reflect specific handwriting models [54], [56], [252], [260]. Other techniques provide segmentation results well suited for particular techniques used for signature verification [55], [172]. Table I reports some of the most relevant techniques for signature segmentation.

The simplest segmentation approaches for static signatures derive from structural descriptions. Some approaches perform structural analysis through the identification of connected components obtained by contour-following algorithms [41], [57], [59]. Fig. 3(a) shows the signature in Fig. 2(a) segmented into connected components. Other approaches describe a signature by a tree structure, obtained through the analysis of horizontal and vertical projection histograms, which identifies fundamental segments in the static image [8]. Offline signature segmentation by statistics of directional data has also been considered [287, 289]. This approach permits the extraction of textured regions that are characterized by local uniformity in the orientation of the gradient, evaluated with the Sobel operator.

Concerning dynamic signatures, some segmentation techniques have been derived directly from the acquired signals representative of the input signature. A widespread segmentation technique that uses pressure information is based on the consideration that the signature can be regarded as a sequence of writing units, delimited by abrupt interruptions [54], [56]; writing

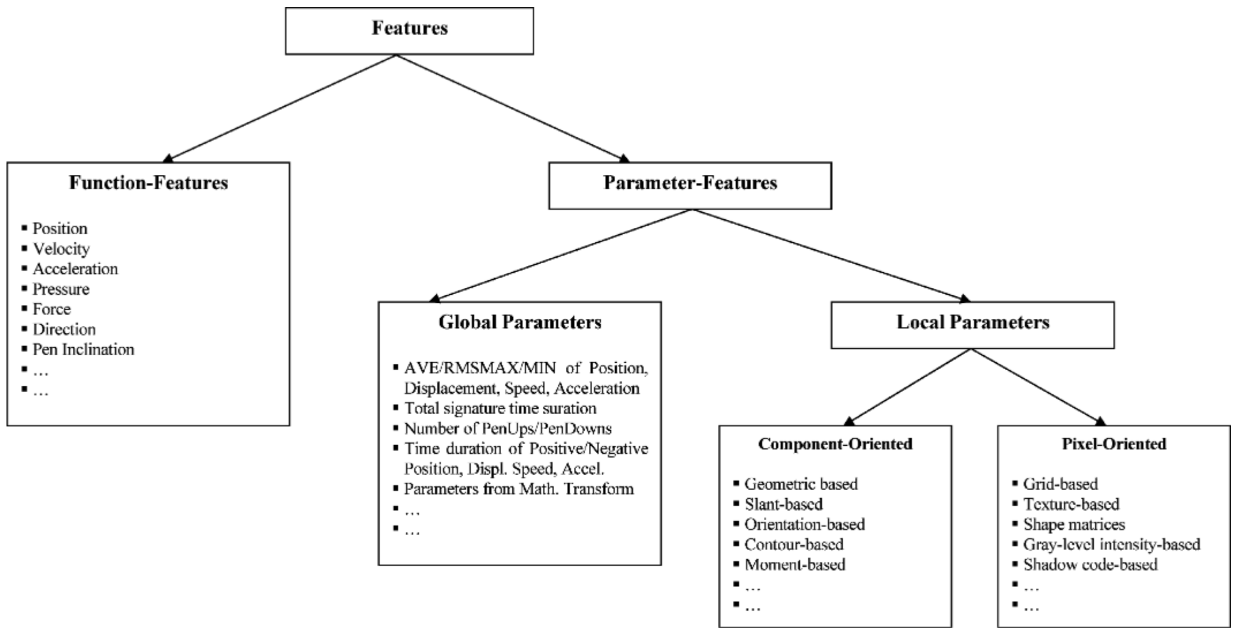


Fig. 4. Features categories.

units are the regular parts of the signature, while interruptions are the singularities of the signature. Thus, pen-up/pen-down signals are used to segment a signature into *components*, where each *component* is a piece of the written trace between a pen-down and a pen-up movement [54], [56], [121], [252], [298]. Furthermore, only a finite set of *components* can be generated by each writer, as demonstrated by the experimental evidence that singularities can occur only in definite positions in the signature of an individual [56]. Fig. 3(b) shows the signature of Fig. 2(b) segmented into *components.* Other approaches exclusively use pen-up strokes for signature verification, since pen-up strokes can be memorized by the computer but are invisible to humans. Hence, possibility of imitating these strokes deliberately is low [352]–[354].

Other segmentation techniques use curvilinear and angular velocity signals [260]. In other cases, signature segmentation is performed by the analysis of the velocity signals, also using static features, when necessary [162].

A different segmentation technique is based on the detection of perceptually important points of a signature [21]. The importance of a point depends on the change of the writing angle between the selected point and the neighbor. A modified version of this technique considers the end points of pen-down strokes as significant splitting points [299]. Other approaches use perceptually important points for segmenting signatures while consider the evolutionary-distance measure, based on arc length distance, for segment association [369].

In order to allow the segmentation of two or more signatures into the same number of perfectly corresponding segments, dynamic time warping (DTW) has been widely used for signature segmentation [55], [166], [172], [275]. After the splitting of a first signature, according to uniform spatial criteria [172] or the position of geometric extremes [55], [166], DTW is applied to determine the corresponding set of points on other specimens. A model-guided segmentation technique has also been proposed [275]. This uses DTW to segment an input signature according to its correspondence with the reference model.

III. Feature Extraction

As shown in Fig. 4, two types of features can be used for signature verification: functions or parameters. When function features are used, the signature is usually characterized in terms of a time function whose values constitute the feature set. When parameter features are used, the signature is characterized as a vector of elements, each one representative of the value of a feature. In general, function features allow better performance than parameters, but they usually require time-consuming procedures for matching [258]. Furthermore, parameters are generally classified into two main categories: global and local. Global parameters concern the whole signature; typical global parameters are total time duration of a signature, number of pen lifts, number of components, global orientation of the signature, coefficients obtained by mathematical transforms, etc. Local parameters concern features extracted from specific parts of the signature. Depending on the level of detail considered, local parameters can be divided into component-oriented parameters, which are extracted at the level of each component (i.e., height to width ratio of the stroke, relative positions of the strokes, stroke orientation, etc.), and pixel-oriented parameters, which are extracted at pixel level (i.e., grid-based information, pixel density, gray-level intensity, texture, etc.). It is worth noting that some parameters, which are generally considered to be global features, can also be applied locally, and *vice versa*. For instance, contour-based features can be extracted at global level (i.e., envelopes extracted at the level of the whole signature) or at local level (i.e., at the level of each connected component).

Table II presents some of the most common function features found in the literature. Position, velocity, and acceleration functions are widely used for online signature verification. Position

TABLE II
FUNCTION FEATURES

Functions	Category	References
Position	Online / Offline	G. Congedo et al. [40], J. Fierrez-Aguilar et al. [94], Y. Hongo et al. [122], Y. Kato et al. [150], Y. Komiya et al. [159], S. Krawczyk and A. K. Jain [160], J. P. Leszcyska [177], Mizukami et al [199, 200], H. Morita et al. [205], D.Muramatsu and T.Matsumoto [212, 213, 214], I. Nakanishi et al. [221, 222, 223], T. Ohishi et al. [233, 234, 235], J. Ortega-Garcia et al. [238], J.D. Penagos at al [245], D. Sakamoto et al. [292, 293], Y. Sato and K. Kogure [296], Q.-Z.Wu et al. [346]
Velocity	Online	A. I. Al-Shoshan [7], G. Congedo et al. [40], V. Di Lecce et al. [50, 51], M. Fuentes et al. [104], K. Huang and H. Yan [129], A.K. Jain et al. [138], G.V. Kiran et al. [158], J. Ortega-Garcia et al. [238], J.D. Penagos at al [245], T. Qu et al. [266, 267], C. Schmidt and K.-F. Kraiss [298], J. Sternby [312], Q.-Z.Wu et al. [346], K. Yu et al. [368], K. Zhang et al. [372]
Acceleration	Online	G. Congedo et al. [40], N.M. Herbst and C.N.Liu [121], G.V. Kiran et al. [158], J.S.Lew [178, 179], J.D. Penagos et al. [245], C. Schmidt and K.-F. Kraiss [298], A.F.Syukri et al. [313]
Pressure	Online	J. Fierrez-Aguilar et al. [94], Y. Hongo et al. [122], K. Huang and H. Yan [129], Y. Kato et al. [150], M. Kawamoto et al. [151], Y. Komiya et al. [159], S. Krawczyk and A. K. Jain [160], H. Morita et al. [205], T. Ohishi et al. [233, 234, 235], J. Ortega-Garcia et al. [238], J.D. Penagos at al [245], T. Qu et al. [266, 267], D. Sakamoto et al. [292, 293], Y. Sato and K. Kogure [296], C. Schmidt and K.-F. Kraiss [298], J. Sternby [312], K. Tanabe et al. [315], K. Yu et al. [368]
Force	Online	H.D Crane and J.S. Ostrem [46], R. Martens and L. Claesen [187, 188, 189]
Direction of pen movement	Online	M. Fuentes et al. [104], J. J. Igarza et al. [130], I. Nakanishi et al. [222, 223], J. Ortega-Garcia et al. [239], I. Yoshimura and M. Yoshimura [363], M. Yoshimura et al. [366]
Pen inclination	Online	J. J. Igarza et al. [130], Y. Kato et al. [150], M. Kawamoto et al. [151], Y. Komiya et al. [159], R. Martens and L. Claesen [187, 188, 189], H. Morita at al. [205], T. Ohishi et al. [233, 234, 235], J. Ortega-Garcia et al. [238], D. Sakamoto et al. [292, 293], J. Sternby [312], K. Yu et al. [368]

function is conveyed directly by the acquisition device whereas velocity and acceleration functions can be provided by both the acquisition device [121], [178] and numerically derived from position [40], [51], [346]. In recent years, pressure and force functions have been used frequently and specific devices have been developed to capture these functions directly during the signing process [46], [219], [235], [237], [300], [374]. In particular, pressure information, which can be registered with respect to various velocity bands, has been exploited for signature verification in order to take advantage of interfeature dependencies [154]. Furthermore, direction of pen movement [363], [366] and pen inclination [130], [151], [238] have also been successfully considered to improve the performance in online signature verification, whereas pen trajectory functions have been extracted from static signatures, in order to exploit the potential of dynamic information for offline signature verification as well [226]. Recent studies also demonstrate that signature verification can be successfully performed by means of "motif" series, which are characteristic subsequences extracted from function features [109].

In general, position, velocity, and pen inclination functions are considered among the most consistent features in online signature verification, when a distance-based consistency model is applied. This model starts from the consideration that the characteristics of a feature must also be estimated by using the distance measure associated to the feature itself [174].

Table III shows some parameter features that have been widely considered for automatic signature verification. Some parameters are specifically devoted to dynamic signature verification. This is the case of some global parameters that describe the signature apposition process, as the total signature time duration [146], [147], [170], [266], the pen-down time ratio [146], [147], [227], [335], and the number of pen lifts (pen-down, pen-up) [82], [166], [169], [170]. Other parameters are numerically derived from time functions representative of a signature, like, for instance, the average (AVE), the root mean square (rms), and the maximum (MAX) and minimum (MIN) values of position, displacement, speed, and acceleration [169], [170], [227]. In other cases, the parameters—that have been used for both dynamic and static signature verification—are determined as coefficients obtained from mathematical tools as Fourier [41], [54], [56], [57], [59], [194], [268], [345], [347], Hadamard [228], cosine [193], wavelet [49], [75], [76], [176], [189], [194], [195], [220], [274], [323], [332], [356], Radom [38], and fractal [127], [206] transforms.

Other parameters in Table III are more widely used for static signature verification, when dynamic information is not available. For example, typical local features extracted from a signature at the component level are geometric-based parameters, such as signature image area, signature height and width, length to width ratio, middle zone width to signature width radio, number of characteristic points (end points, cross-points, cusps, loops, etc.), and so on [8], [17], [290]. Other well-known parameters based on slant [8], [17], [59], [270], [301], orientation [290], contour [15], [26], [230], [231], [274], direction [66]–[68], [149], [282], [301], [350], and curvature [138], [145] have also been considered. Conversely, typical parameters extracted at pixel level are grid-based features. When grid-based parameters are used, the signature image is divided into rectangular regions and well-defined image characteristics, such as ink-distribution [17], [301] or normalized vector angle [185], are evaluated in each region. Grid features and global geometric features are used to build multiscale verification functions [263]–[265]. Texture features have also been extracted, based on the co-occurrence matrices of the signature image [17], shape matrices [283], and gray-level intensity features that provide useful pressure information [44], [126]. The extended shadow code has been considered as a feature vector to incorporate both local and global information into the verification decision [284]. A morphological shape descriptor used in signature verification is the *pecstrum*, which is computed by measuring the result of successive morphological openings of

TABLE III
PARAMETER FEATURES

Parameters	Category	References
Total signature time duration	Online	R.S.Kashi et al. [146, 147], J.Lee et al. [166], L.L.Lee et al. [169, 170], W.Nelson et al. [227], T.Qu et al. [266], W.S.Wijesoma et al. [335]
Pen-down time ratio	Online	R.S.Kashi et al. [146, 147], W. Nelson et al. [227], W. S. Wijesoma et al. [335]
Number of PenUps/Pen Downs	Online	M.C.Fairhust and S. Ng [82], G.V. Kiran et al. [158], J. Lee et al. [166], L.L.Lee et al. [169, 170], T. Qu et al. [266]
AVE/ RMS/ MAX/ MIN of Posit., Displ., Speed, Accel.	Online / Offline	R. S. A. Araujo et al [11], M.C.Fairhust and S. Ng [82], M. Fuentes et al. [104], R.S.Kashi et al. [146, 147], M.A. Khan et al. [154], J. Lee et al. [166], L.L.Lee et al. [169, 170], W. Nelson et al. [227], T. Qu et al. [266], W. S. Wijesoma et al. [335]
Time duration of Positive/Negative Posit., Displ., Speed, Accel.	Online	R. S. A. Araujo et al [11], M.C.Fairhust and S. Ng [82], R.S.Kashi et al. [146, 147], J. Lee et al. [166], L.L.Lee et al. [169, 170], W. Nelson et al. [227], W. S. Wijesoma et al. [335]
X-Y correlation of Posit., Displ., Speed, Accel.	Online	A.N. Abu-Rezq and A.S. Tolba [2], M. Fuentes et al. [104], R.S.Kashi et l. [146, 147], W. Nelson et al. [227]
Fourier Transform	Online / Offline	G. Congedo et al. [41], G. Dimauro et al. [54, 56, 57, 59], D.K.McCormack et al. [194], Z.-H. Quan et al. [268], C.-J. Wen et al. [332], Q.Z. Wu et al. [345, 347], J. Yi et al. [361]
Hadamard Transform	Offline	W.F. Nemcek and W.C. Lin [228]
Cosine Transform	Online	T. Matsuura and T.S. Yu [193]
Wavelet Transform	Online / Offline	P.S.Deng et al. [49], E.A.Fadhel and P.Bhattacharyya [75, 76], D.Letjman and S.George [176], R.Martens and L.Claesen [189], D.K. McCormack et al. [194], D.K.McCormack and J.F.Pedersen [195], I. Nakanishi et al. [220, 221, 222, 223], V.E.Ramesh and M.Narasimha Murty [274], A. Vergara da Silva and D. Santana de Freitas [323], Z. Yang and C.-C. Jay Kuo [356]
Radom transform	Offline	J. Coetzer et al. [38]
Fractal Transform	Offline	K. Huang and H. Yan [127], S. Mozaffari et al. [206]
Direction-based	Online / Offline	S. Armand et al. [13], N.-J. Cheng et al. [36], J.P.Drouhard et al [66, 67, 68], M.C.Fairhust and S. Ng [82], K. Huang and H. Yan [128], M. Kalera et al. [145], R.S.Kashi et al. [146, 147, 149], J. Lee et al. [166], H. Lv et al. [184], T. Matsuura and S. Yamamoto [192], W. Nelson et al. [227], Y. Qi and B.R. Hunt [264], T. Qu et al. [266], S.K. Ramanujan et al. [273], R.Sabourin and J.P.Drouhard [282], M. Shridhar et al. [301], S. N. Srihari et al. [307, 308], H. Srinivasan et al. [309, 310, 311], W.S.Wijesoma et al. [335], X.-H. Xiao and G. Leedham [350], L.Yang et al. [357], E.N. Zois et al. [378], M. Zou et al. [379]
Geometric-based	Offline	M. Ammar [8], S. Ando and M. Nakajima [9], H. Baltzakis and N. Papamarkos [17], L. P. Cordella et al. [44], G. Dimauro et al. [59], J.K. Guo et al. [114], K.Han and I.K.Sethi [116, 117, 118], K. Huang and H.Yan [126], Y. Qi and B.R. Hunt [263, 264, 265], V.E.Ramesh and M.Narasimha Murty [274], Y. Xuhua et al. [354], X. Ye et al. [358]
Curvature-based	Online / Offline	A.K. Jain et al. [138], E.J.R. Justino et al. [143], M. Kalera et al. [145], S. N. Srihari et al. [307, 308], H. Srinivasan et al. [309, 310, 311], Y. Xuhua et al. [354]
Structure-based	Offline	G. Dimauro et al. [59], M. Kalera et al. [145], R. Sabourin at al. [290], S. N. Srihari et al. [307, 308], H. Srinivasan et al. [309, 310, 311]
Graphometric-based	Offline	E.J. R. Justino at al. [141, 142, 143, 144], L.S. Oliveira et al. [236], C. Santos et al. [295]
Peripheral-based	Offline	B. Fang and Y.Y. Tang [85]
Projection-based	Offline	A.N. Abu-Rezq and A.S. Tolba [2], R. Bajaj and S. Chaudhury [15], H. Baltzakis and N. Papamarkos [17], G. Dimauro et al. [59], B. Fang et al. [83, 84], Y. Qi and B.R. Hunt [263, 264, 265], C. Quek and R.W.Zhou [270]
Slant-based	Offline	M.Ammar et al. [8], H.Baltzakis and N.Papamarkos [17],G.Dimauro et al. [59], E.J.R.Justino et al. [143],C.Quek and R.W.Zhou [270], M.Shridhar et al. [301]
Orientation-based	Offline	R. Sabourin et al. [280]
Contour-based	Offline	R. Bajaj and S. Chaudhury [15], H. Cardot et al. [26], S. Chen and S. N. Srihari [33], M.A. Ferrer et al. [91], F. Nouboud [230], F. Nouboud and R. Plamondon [231], I. Pavlidis et al. [243], V.E.Ramesh and M.Narasimha Murty [274]
Grid–based	Offline	H. Baltzakis and N. Papamarkos [17], A.El Yacoubi et al. [71], K. Huang and H.Yan [126], V. K. Madasu et al. [185], F. Nouboud and R. Plamondon [231], Y.Y.Qi and B.R. Hunt [263, 264, 265], M. Shridhar et al. [301], C. Simon et al. [302], L. Wan et al. [329], X.-H. Xiao and G. Leedham [350]
Moment-based	Online / Offline	A.N. Abu-Rezq and A.S. Tolba [2], R.C. Doria et al [64], M.C.Fairhust and S. Ng [82], R.S.Kashi et al. [146, 147], C.-L. Liu et al. [183], H. Lv et al. [184], V.E.Ramesh and M.Narasimha Murty [274]
Texture-based	Offline	H. Baltzakis and N. Papamarkos [17], L. Wan et al. [329]
Shape Matrices	Offline	R. Sabourin et al. [283]
Gray-level intensity-based	Offline	L. P. Cordella et al. [44], K. Huang and H.Yan [126], H. Lv et al. [184]
Shadow code-based	Offline	F. Nouboud and R. Plamondon [231], R. Sabourin et al. [281, 284, 285], C. Simon et al. [302]
Smoothing-based	Offline	B. Fang et al. [87]
Pattern Spectrum	Offline	R.Sabourin et al. [286, 288]

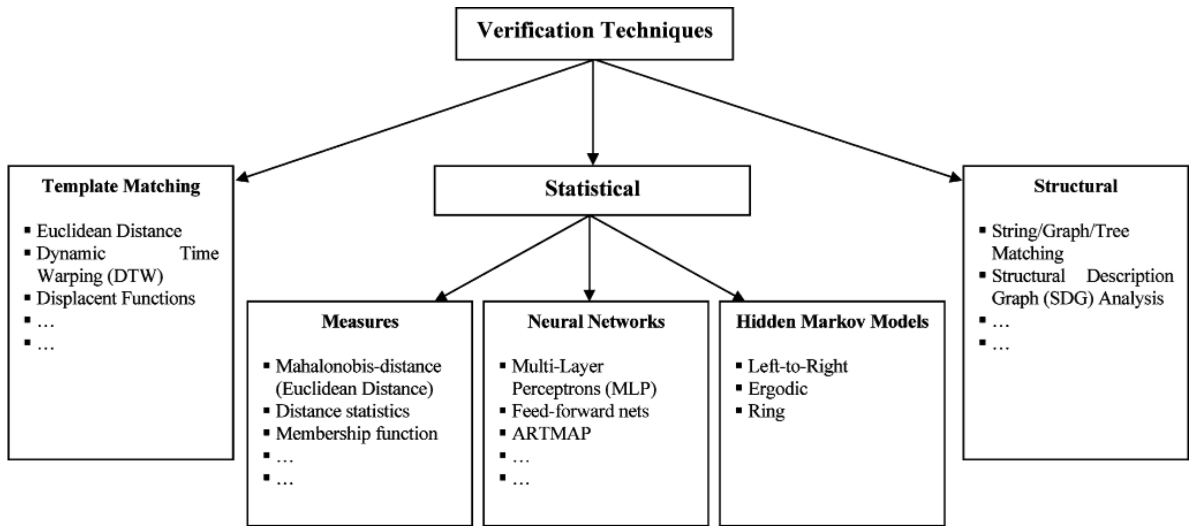


Fig. 5. Signature verification techniques.

the image, as the size of the structuring element increases [286]. The sequences of openings so obtained are called *granulometries* [288]. A smoothness index has been used for detecting skilled forgeries in offline signature verification. This technique was inspired by expert examiners who observed that well-forged signatures are generally less smooth on a detailed scale than the genuine ones [87]. According to an expert forensic approach [295], [304], graphometric-based parameters have also been considered, including static features (caliber, proportionality, etc.) and pseudodynamic features (apparent pressure, stroke curvature, and regularity) [141], [142], [144], [295]. Indeed, it is worth noting that research in automatic signature verification has been strongly influenced by the work of forensic document examiners, as discussed in some excellent papers [23], [99], [246], [305], [306]. For instance, starting from a static signature image, pseudodynamic features can be used to extract information on the dynamics of the underlying signing process. This is considered by forensic experts to be a fundamental aspect concerning the authorship of the sample in question [99], [101], [304]. In general, although not every feature analyzed by a forensic examiner can easily be represented as a parameter feature extracted by a computer program—and *vice versa* [246], [305], it is quite easy to find close relationships between many parameter features and some of the main features used by forensic experts [70], [99], [101], [236], [295], [303]–[305].

Whatever feature set is considered, the evidence that an individual's signature is unique has led many researchers to devote specific attention to the selection of the most suitable features for a signer. Indeed, signatures from different writers generally contain very few common characteristics, and thus, the use of a universally applied feature set is not effective. Feature selection in the domain of signature verification is also required because system efficiency, processing cost, and memory requirement are strictly dependent on the cardinality of the feature set [77], [80], [152], [276]. Therefore, the smaller the feature vector, the greater the number of individuals that can be enrolled in the system and the faster speeds that can be achieved in the verification process [77], [78]. In recent years, several techniques have been proposed for feature selection based on principal component analysis (PCA) and self-organizing feature maps [317], sequential forward search/sequential backward search (SFS/SBS) [80], inter–intra class distance radios (ICDRs) [82], and analysis of feature variability [227], [252]. Forgery-based feature analysis has also been proposed to select feature sets well suited for random and skilled forgery, respectively. This approach has been motivated by evidence that some features are best suited for distinguishing skilled forgeries from genuine signatures whereas other features are better at distinguishing random forgeries [275].

Other approaches use the same features set for each person and face the problem of personalized feature selection by assigning a different weight to each feature [157]. Neural networks (NNs) [168] and genetic algorithms (GAs) have been widely used for determining genetically optimized weighted parameters [274], as well as for selecting optimal functions [191], personalized parameters [334], [352]–[354], or signature strokes to be used for verification [325], [326].

IV. Classification

In the verification process, the authenticity of the test signature is evaluated by matching its features against those stored in the knowledge base developed during the enrolment stage. This process produces a single response (Boolean value) that states the authenticity of the test signature. The verification process involves many critical aspects that ranges from the technique for signature matching to the strategy used for the development of the knowledge base.

Fig. 5 shows some of the most relevant approaches to signature verification, although blended solutions can be adopted in several cases. When template matching techniques are considered, a questioned sample is matched against templates of authentic/forgery signatures. In this case, the most common approaches use DTW for signature matching. When statistical approaches are used, distance-based classifiers can be considered. NNs have also been widely used for signature verification, due to their capabilities in learning and generalizing. More recently, special attention has been devoted to the use of hidden Markov models (HMMs) for both offline and online signature verification. Syntactic approaches are generally related to structural representations of signatures, which are described through their elementary elements (also called "primitives"), and compared through graph or three matching techniques.

The classification techniques most common in the literature are reported in Table IV. When functions are considered, the matching problem can be complicated by random variations, due to the writer's pauses or hesitations. These variations can create portions of signals, such as deletions, additions, and gaps, which complicate the problem of matching. DTW allows the compression or expansion of the time axis of two time sequences representative of the signatures to obtain the minimum of a given distance value [32], [177], [339], [363], [366], [373]. More precisely, let $T : (T_1,\ T_2, \ldots, T_{N_T})$ and $R : (R_1,\ R_2, \ldots, R_{N_R})$ be two online signatures, the DTW is used to determine the optimal warping function $W^*(T,R)$ minimizing a well-defined dissimilarity measure $D_{W(T,R)} = \sum_{k=1}^{K} \mathrm{d}(c_k)$, where $c_k = (i_k,\ j_k)$ (k, i_k, j_k integers, $1 \le k \le K$, $1 \le i_k \le N_T$, $1 \le j_k \le N_R$) and $\mathrm{d}(c_k) = \mathrm{d}(T_{i_k},\ R_{j_k})$ is a distance measure between the samples of T and R. A detailed discussion on DTW, which was initially used in the field of speech processing, is beyond the aim of this paper and further information can be found in the literature [272].

In the field of automatic signature verification, although the superiority of DTW has not been proven with respect to other comparison techniques, such as regional correlation and skeletal tree matching [241], [249], DTW has been extensively used and continuous [207]–[209] and parallel [14] implementations have been investigated. In addition, several techniques for signature data reduction based on GAs [337], [338], PCA [155], [180], minor component analysis (MCA) [180], linear regression (LR) [175], polygonal approximation (PA) [337], [338], extreme points (EPs) [90], and random [337], [338] selection have been considered. Stroke-based DTW has also been investigated [339]. This process starts from the consideration that a comparison between the complete time sequences will not only result in higher computational load but also lead to a loss of the information related to the structural organization of the signatures. In order to avoid deformation of reference signatures when matched against test specimens, a well-suited form of asymmetric DTW was defined [186], [187], [189]. Other template matching approaches can use well-defined distortion measures [344], similarity measures [347], displacement functions [199], [200], relaxation matching [128], accumulated position and velocity distances based on split-and-merge mechanisms [346], fuzzy logic [185], and pattern matching [283], [318].

When parameters are used as features, statistical-based techniques are generally chosen. The most common approaches use Mahalanobis and Euclidean distances: Mahalanobis distance is used when the full covariance matrix is available for each signature class [85], [186], [188], [189], [268], [371]; Euclidean distance is considered when only the mean vector of the class is known [54], [56], [57], [273], [288], [295]. Membership functions [266] and other distance statistics [145], [310] have also been used.

NNs have been widely used for automatic signature verification for a long time, as [165] demonstrates. Table IV shows some of the NN models that have been used recently: Bayesian NNs [30], [351], multilayer perceptrons (MLPs) [7], [15], [17], [126], [167], [345], [350], time-delay NNs [22], [167], ARTMAP NNs [215]–[217], backpropagation neural networks (BPNs) [13], [15], [47], [66]–[68], self-organizing maps [1], [2], and radial basis functions (RBFs) [13], [109], [203], [232], [316]. Fuzzy NN, which combine the advantages of both NNs and fuzzy rule-based systems, has also been considered [102], [270], [353]. In order to improve effectiveness in using NNs, suitable transformed versions of signatures have been proposed and used for input [37]. A transform can reproduce a time-series pattern assuming a constant linear velocity to model the temporal characteristics of the signing process; another transform can map the signal onto a horizontal versus vertical velocity plane, where the variation of the velocities over time is represented as a visible shape. Instead, other approaches first modify the test signature to the template signature by dynamic programming (DP) matching, and then, use an NN to compare dynamic information along the matched points of the signatures [316]. Although NNs have demonstrated good capabilities in generalization [75], they require large amounts of learning data that are not always available [156]. To this purpose, the use of synthetically generated signatures has also been proposed [126].

Recently, intensive research has been devoted to HMMs. These models have found to be well suited for signature modeling since they are highly adaptable to personal variability [104], [190], [321], [357]. Strictly speaking, a HMM is a double stochastic approach in which one underlying yet unobservable process may be estimated through a set of processes that produce a sequence of observations. A comprehensive discussion on HMM is beyond the aim of this paper and can be found in the literature [271]. Concerning the field of signature verification, various HMM topologies have been considered so far, as Fig. 6 shows. Most approaches use the left-to-right HMM topology, since it is considered well suited for signature modeling [71], [91], [130], [146], [321], [333], [379]. Ergodic topology has also been considered for both online and offline signatures verification [269], [333]. Furthermore, in order to guarantee invariance to signature rotation, ring topology has been adopted, which is equivalent to left-to-right topology and a transition from the last state to the first state is allowed [38]. However, independent of the topology used, HMMs seem to be superior to other signature modeling techniques based on structural descriptions [128], [129] and fuzzy approaches [119], [185]. Some results have also demonstrated that HMM-based systems for offline signature verification can outperform human verifiers [39]. Furthermore, recent approaches use HMM in combination with autoregressive models while the signature is decomposed into pseudo-stationary segments and represented by a one-dimension spatial stochastic sequence [202]. The effect of interpersonal and intrapersonal variability on HMM has also been investigated [141], as well as the possibility of automatically and dynamically deriving various author-dependent parameters by *cross-validation* [71].

Support vector machines (SVMs) are another promising statistical approach to signature verification. An SVM is a new classification technique in the field of statistical learning theory and it has been successfully applied in many pattern recognition applications. An SVM can map input vectors to a higher dimensional space in which clusters may be determined by a maximal

TABLE IV
COMPARISON TECHNIQUES

Technique		Category	References
Euclidean Distance		Online / Offline	R. S. A. Araujo et al [11], G. Dimauro et al. [54, 56, 57], M.A. Ferrer et al. [91], M.A. Khan et al. [154], S. Ramanujan et al. [273], R. Sabourin et al [288], C. Santos et al. [295], L. Wan et al. [329]
Mahalanobis Distance		Online / Offline	B. Fang et al. [85], S. Krawczyk and A. K. Jain [160], R. Martens and L. Claesen [186, 188, 189], Z.-H. Quan et al. [268], K. Zhang et al. [371]
Pattern Matching		Offline	C.-C. Lien et al. [181], R. Sabourin et al. [283], K. Ueda [318]
Membership functions		Online	T. Qu et al. [266]
Distance Statistics		Offline	M. Kalera et al. [145], H. Srinivasan et al. [310]
Dynamic Similarity Measure		Online	Q. Z. Wu et al. [347]
Dynamic Time Warping (DTW)	Continuous	Online	L. Bovino et al. [18], Y.Chen and X.Ding [32], G. Congedo et al. [40], V. Di Lecce et al. [50, 51], G. Dimauro et al. [53, 55, 59], K. Huang and H. Yan [129], J.P. Leszcyska [177], M.E. Munich and P. Perona [207, 208, 209], I.Yoshimura and M. Yoshimura [363], M. Yoshimura et al. [366]
	Parallel	Online	Y.J. Bae and M.C. Fairhurst [14]
	GA-based	Online	M. Wirotius et al. [337, 338]
	PCA-based	Online	A. Kholmatov and B. Yanikoglu [155], B.Li et al. [180]
	MCA-based	Online	B.Li et al. [180]
	LR-based	Online	H. Lei et al. [175]
	PA-based	Online	M. Wirotius et al. [337, 338]
	EP-based	Online	H. Feng and C.C. Wah [90]
	Random-based	Online	M. Wirotius et al. [337, 338]
	Stoke-based	Online	B. Wirtz [339]
	Asymmetric	Online	R. Martens and L.Claesen [186, 187, 189]
Dynamic Programming		Online / Offline	B. Fang et al. [83, 84], J. K. Guo et al. [114], J. Lee et al. [166], I. Nakanishi et al. [220], F. Nouboud [230], F. Nouboud and R. Plamondon [231]
Correlation		Online	J.B.Fasquel, M.Bruynooghe [88], K.K. Lau et al. [163], J.S. Lew [179], M.L. Molina et al. [204], V. S. Nalwa [224], M. Perizeau and R. Plamondon [241], C.-J. Wen et al. [332]
Relaxation Matching		Offline	K. Huang and H. Yan [128], C.-F. Lin and C.-W. Chen [182]
Bayesian approach		Offline	D. Muramatsu at al. [212]
Split-and-Merge		Online	Q.Z. Wu et al. [346]
String / Graph / Tree Matching		Online / Offline	Y. Chen and X. Ding [31], S. Chen and S. N. Srihari [33, 34, 35], N.-J. Cheng et al. [36], K. Han and I. K. Sethi [118], A. K. Jain et al [138], I. Pavlidis et al. [243, 244], M. Perizeau and R. Plamondon [241], X.-H. Xiao and R.W. Dai [349]
Structural Description Graph		Online/Offline	L. Bovino et al. [18], G. Dimauro et al. [56], K. Huang and H.Yan [129]
Displacement Function		Offline	Y. Mizukami et al. [199, 200]
Fuzzy Logic		Offline	M. Hanmandlu et al. [120], V. K. Madasu et al. [185], W. S. Wijesoma et al. [335], K. Zhang et al. [372]
Support Vector Machine (SVM)		Online / Offline	M.A. Ferrer et al. [91], M. Fuentes et al. [104], E. J.R. Justino et al. [143], A. Kholmatov and B. Yanikoglu [155], H. Lv et al. [184], S.N. Srihari et al [308]
Neural Network (NN)	Bayesian	Online / Offline	H.D.Chang et al. [30], X.-H.Xiao and G. Leedham [351]
	Multi-Layer Perceptrons (MLP)	Online / Offline	A. I. Al-Shoshan [7], R. Bajaj and S. Chaudhury [15], H. Baltzakis and N. Papamarkos [17], H. Cardot et al. [26, 27], L.P. Cordella et al. [44, 45], E. A. Fadhel and P. Bhattacharyya [75], M. Fuentes et al. [104], K.Huang et al [123, 124, 125, 126], L. L. Lee [168], W.-S. Lee et al. [172], C. Sansone and M. Vento [294], C. Santos et al. [295], Q.-Z. Wu et al. [343, 344], X.-H. Xiao and G. Leedham [350]
	Time-Delay	Online / Offline	J. Bromely et al. [22], L. L. Lee [167]
	ARTMAP	Online / Offline	N.A. Murshed et al. [215, 216, 217, 199]
	Backpropagation Network (BPN)	Online / Offline	S. Armand et al. [13], R. Bajaj and S. Chaudhury [15], A.M. Darwish and G.A. Auda [47], J.P.Drouhard et al. [66, 67, 68], D.Z. Letjman and S.E. George [176], N.A. Murshed et al. [218], R. Sabourin and J.P. Drouhard [282]
	Self-organizing Map	Online / Offline	A. Abu-Rezq and A.S. Tolba [1, 2], H. Cardot et al. [26, 27], A.S. Tolba [317], T. Wessels and C.W. Omlin [333]
	Fuzzy Nets	Online / Offline	K. Franke et al. [102], C. Quek and R.W.Zhou [270], S. Watanabe et al. [330, 331], Y. Xuhua et al. [353, 354]
	Radial Basis Functions (RBF)	Online / Offline	S. Armand et al. [13], H. Baltzakis and N. Papamarkos [17], C. Gruber et al. [109], M.L. Molina et al [203], N. F. O'Brien and S. C. Gustafson [232], M. Tanaka et al. [316]
Hidden Markov Models (HMM)	Left-to-right topology	Online / Offline	J.G.A. Dolfing et al. [63], A. El-Yacoubi et al. [71], M.A. Ferrer et al. [91], J. Fierrez-Aguilar et al. [94, 96, 97], M. Fuentes et al. [104], J. J. Igarza et al [130, 131], E.J.R. Justino et al. [141, 142, 143], R.S. Kashi et al. [146, 147], D.Muramatsu and T.Matsumoto, [213, 214], J. Ortega-Garcia et al. [238], S.K. Ramanujan et al. [273], G.Rigoll and A. Kosmala [277], M.M.Shafiei and H.R. Rabiee [299], B. Van et al. [321], T. Wessels and C.W. Omlin [333], L. Yang et al. [357], H.S. Yoon et al. [362], M. Zou et al. [379]
	Ergodic topology	Online / Offline	Z. –H. Quan and K.-H. Liu [269], T. Wessels and C.W. Omlin [333]
	Ring topology	Offline	J. Coetzer et al. [38]

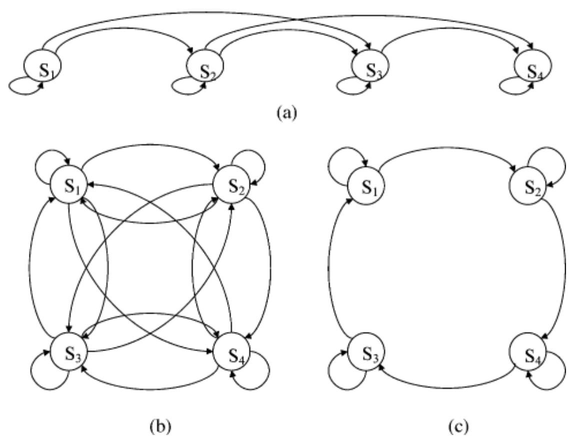


Fig. 6. HMM topologies. (a) Left-to-right. (b) Ergodic. (c) Ring.

separating hyperplane [25]. SVMs have been used successfully in both offline [91], [143], [184] and online [104], [155] signature verification.

Structural approaches mainly concern string, graph, and tree matching techniques and are generally used in combination with other techniques. For instance, string matching [31], [349] is used not only for signature verification but also for signature identification purposes, via advanced local associative indexing [118]. In other cases, the *structural description graph* is used to verify the structural organization of a questioned signature [18], [56], [129], as Fig. 7 illustrates.

In recent years, multiexpert (ME) approaches have been investigated to improve signature verification performance. For this purpose, serial [55], [161], [294], [371], parallel [59], [265], or hybrid strategies [44], [45] have been used and well-defined techniques for reliability estimation have been adopted [43]. Among the others, hybrid combination strategies seem to be particularly suited for signature verification since they attempt to achieve the performance advantages of serial approaches in fast rejecting very poor forgeries while retaining the reliability of parallel combination schemes [44], [45].

Since an ME verification system should combine decisions from complementary signature verifiers, sets of verifiers based on global and local strategies [92], [95] and feature sets [123], [125], parameter features and function features [260], static and dynamic features [50], [51] have been used. Several decision combination schemes have been implemented, ranging from majority voting [4], [50], [51], [59], [274] to Borda count [12], from simple and weighted averaging [18] to Dempster–Shafer evidence theory [12] and NNs [15], [17], [26]. The boosting algorithm has been used to train and integrate different classifiers, for both verification of online [122] and offline [329] signatures.

In addition, ME approaches have been used for stroke-based signature verification in which the verification of a signature is performed by the analysis of its elements. Stroke-based signature verification can lead to lower error rates compared to global approaches, since a large amount of personal information is conveyed in specific parts of the signature and cannot be detected when the signature is viewed as a whole [8], [21], [54]–[57], [59], [126], [164], [278], [298]. Furthermore, the verification at stroke level can be performed by DTW [41], [50], [51], [55], also considering multiple function features for stroke representation (like position, velocity, and acceleration) in order to verify both the shape and dynamics of each part of the signature [18].

Along with the matching techniques, attention has been given to knowledge-base development also in relation to learning strategies [308], [310], [311] and signature modeling techniques [248], [308]. In particular, special attention has been given to writer-dependent learning strategies using only genuine specimens [156], [215], [216], [217], [328]. In this case, a first approach uses a single prototype of genuine signatures for each writer, and several techniques have been proposed for the development of the optimal average prototype for a signer, including shape and dynamic feature combination [298], time- and position-based averaging [340], or selecting the genuine specimen with the smallest average difference, when compared to the other true signatures available [156]. After the prototype has been determined, the decision threshold is generally defined on the basis of the difference values that can be determined from the genuine signatures [156]. A second approach uses a set of genuine signatures for reference. In this case, a crucial problem concerns the selection of the optimal subset of reference signatures, among the specimens available. When static signature verification is considered, the validity of the reference model has been evaluated according to specific quality criteria, as for instance, intraclass variability that should be as low as possible [3], [78], [79]. In dynamic signature verification, the selection of the best subset of reference signatures has been performed on the basis of the analysis of variance within samples [112] or by considering the stability regions in the signatures, determined by a well-defined analysis of local stability [40], [51]. The selection of the best subset of reference signatures can be avoided at the cost of using multiple models for signature verification [148], [197], [198]. Furthermore, knowledge-base development involves the problem of having a lack of sufficient reference data to characterize a given signature class, as is generally the case of many practical applications. Thus, specific research has been devoted to feature modeling [158], [279], also using regularization techniques that estimate the statistical significance of small-size training sets [85], [86], [276]. Other approaches propose the generation of additional training samples from the existing ones by convolutions [48], elastic matching [85], [86], and perturbations [126].

Finally, promising research has recently been devoted to the investigation of different type, complexity, and stability of signatures. These aspects have great theoretical and practical relevance since they highlight the large difference between humans and machines in perceiving, processing, and verifying signatures, while providing fundamental information for developing the next generation systems, with high adaptive capabilities. For instance, short signatures could convey less information than long signatures, resulting in less accurate verification results [20]. Similarly, people with common names could be more likely to share similar signatures with other individuals—at least concerning shape characteristics. In both cases, the system should be able to adapt itself to the characteristics of the enrolled individuals [278].

The complexity of a signature has been quantified by estimating the difficulty for its imitation, obtained as the result of the estimated difficulty in perceiving, preparing, and executing each stroke of the signature itself [20].

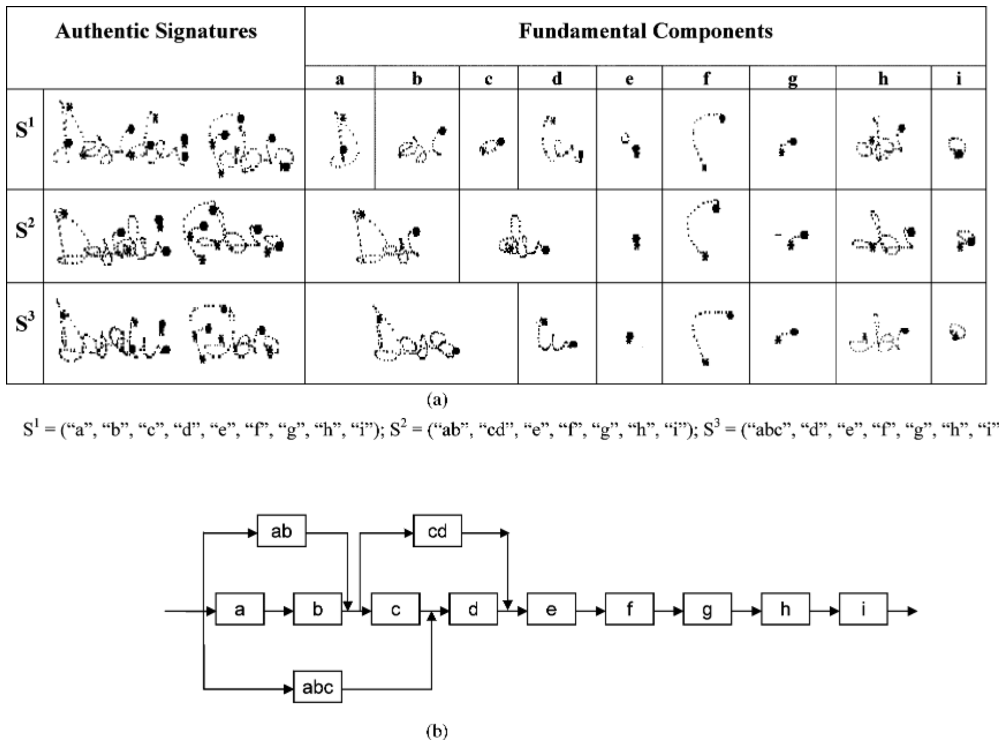


Fig. 7. Structural description of signatures. (a) Description of authentic signatures by components. (b) Structural description graph.

Concerning signature stability, a local stability function can be obtained by using DTW to match a genuine signature against other authentic specimens [42], [53], [129]. Each matching is used to identify the *direct matching points* (DMPs), which are unambiguously matched points of the genuine signature. Thus, a DMP can indicate the presence of a small stable region of the signature, since no significant distortion has been locally detected. More formally, let $T : (T_1,\, T_2, \ldots, T_{N_T})$ be an authentic signature and $R^i : (R^i_1, R^i_2, \ldots, R^i_{N_{R_i}}),\ \ i = 1$, $2, \ldots, n$ be a set of n additional genuine specimens. For each couple $(T,\, R^i)$, $i = 1, 2, \ldots, n$, the optimal warping function $W^*(T,\, R^i)$ can be determined by means of DTW. From $W^*(T,$ $R^i)$, the DMP of T with respect to R^i are identified as the points of T that have a one-to-one coupling with a point of R^i. In other words, let T_p be a point of T coupled with R^i_q of R^i; T_p is DMP of T with respect to R^i if and only if:

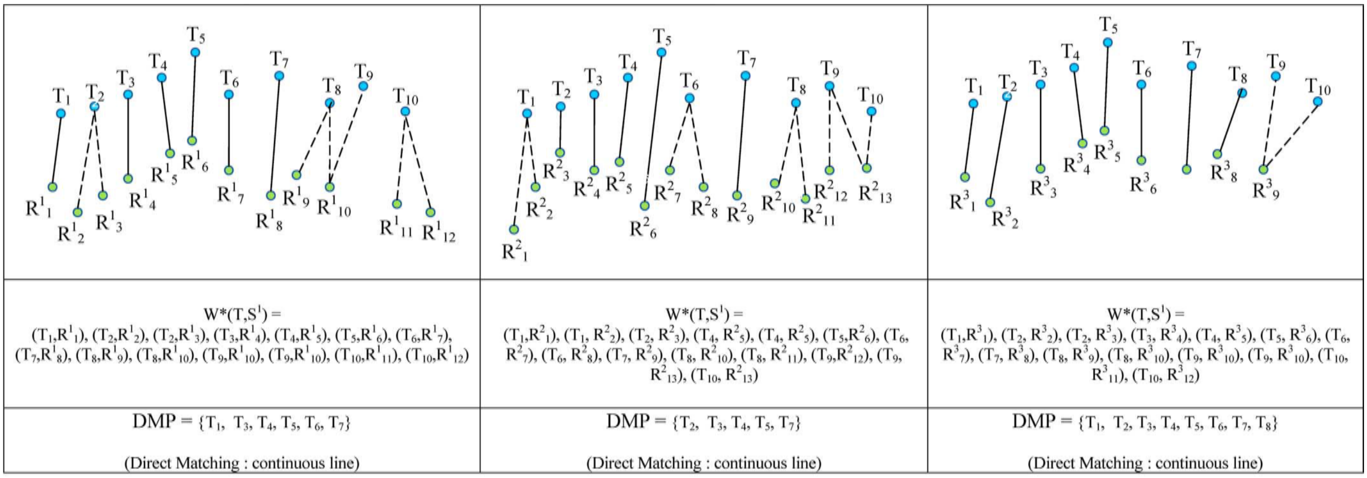
1) $\forall \underline{p} = 1, \ldots, N_T$, $\underline{p} \neq p$, it results that $T_{\underline{p}}$ is not coupled with R^i_q;
2) $\forall \underline{q} = 1, \ldots, N_{Ri}$, $\underline{q} \neq q$, it results that $R^i_{\underline{q}}$ is not coupled with T_p.

A DMP indicates the existence of a small part of the signature T that is roughly similar to the corresponding part of the signature R^i, in the domain specified by the distance used for the DTW. Therefore, for each sample of T, a score is introduced according to its type of coupling with respect to the points of R^i [42], [53]: $\text{Score}^i(T_p) = 1$, if T_p is a DMP; $\text{Score}^i(T_p) = 0$, otherwise. The local stability function of T is defined as $I(T_p) = 1/n \sum_{i=1}^{n} \text{Score}^i(T_p), p = 1, 2, \ldots, N_T$; hence, $I(T_p)$ $\in [0, 1]$, $p = 1, 2, \ldots, n$. Fig. 8 schematically shows a simple example in which the local stability of a short sequence T is evaluated by considering the corresponding sequences S^i, $i =$ $1, 2, 3$.

Following this procedure, Fig. 9 shows the analysis of stability for an entire test signature [see Fig. 9(a)] and the identification of low- and high-stability regions. More precisely, from the consideration that the value of local stability can vary in the range [0,1], low-stability regions are identified as those in which the value of local stability is lower than 0.5, whereas the high-stability regions are identified as those in which the value of local stability is greater than or equal to 0.5 [see Fig. 9(b)].

Furthermore, when the analysis of local stability is used to measure short-term modifications—which depend on the psychological condition of the writer and on the writing conditions—it allows the selection of the best subset of reference signatures [40], [51] and the most effective feature functions for verification aims [51] while providing useful information to weight the verification decision obtained at the stroke level, according to the local stability analysis [53], [129]. Long-term modifications depend on the alteration of the physical writing system of the signer (arm and hand, etc.) as well as on the modification of the motor program in his/her brain. When these modifications are evaluated, useful information can be achieved for updating the reference signature model by including additional information from other new signatures, as they become available [278].

Other types of approaches estimate the stability of a set of common features and the physical characteristics of signatures which they are most related to, in order to obtain global information on signature repeatability that can be used to improve the verification systems [110], [111], [150]. In general, these approaches have shown that there is a set of features that remain stable over long periods, while there are other features that change significantly in time, as a function of signer age. This is the case of features



(a) (b) (c)

	Scores									
	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
Score$^1(\cdot)$	1	0	1	1	1	1	1	0	0	0
Score$^2(\cdot)$	0	1	1	1	1	0	1	0	0	0
Score$^3(\cdot)$	1	1	1	1	1	1	1	1	0	0
Local Stability Value	**0.67**	**0.67**	**1**	**1**	**1**	**0.67**	**1**	**0.33**	**0**	**0**

(d)

Fig. 8. Evaluating the local stability. (a) T versus S^1 matching and DMPs. (b) T versus S^2 matching and DMPs. (c) T versus S^3 matching and DMPs. (d) Computing the local stability.

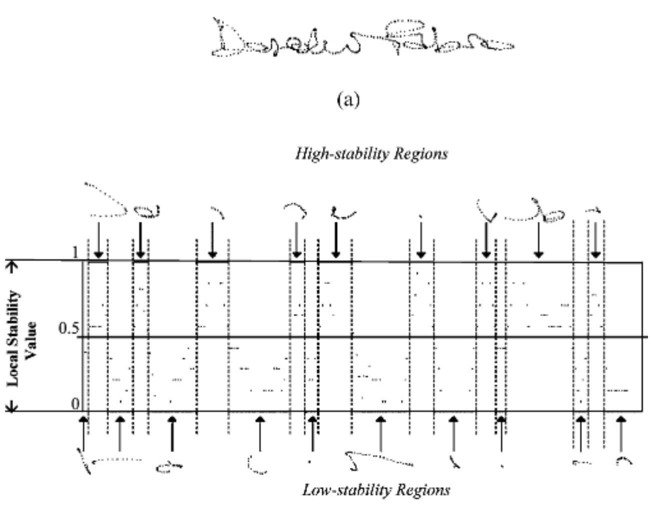


Fig. 9. Analysis of local stability. (a) Test signature. (b) Low- and high-stability regions.

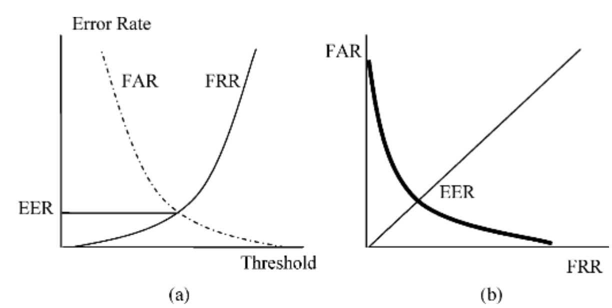


Fig. 10. Performance measures. (a) FAR and FRR. (b) ROC graph.

related to total execution time, velocity, and acceleration [110]. Since intersession variability is one of the most important causes of the deterioration of verification performances, specific parameter-updating approaches have been considered [150].

The enormous differences in the signatures of people from different countries have also required the development of specifically designed solutions. For instance, occidental-style signatures generally consist of signs that could form concatenated text combined with pictorial strokes. In some countries, the habit is to sign with a readable written name whereas in other countries, signatures are not always legible. Many more differences can be expected when considering signatures written by people from non-Western countries. For this purpose, specific approaches have been proposed in the literature for Chinese [30], [36], [163], [182]–[184], [349] and Japanese [318], [364], [365], [367] signatures, which can consist of independent symbols, as well as Arabian/Persian [28], [29], [47], [134] signatures, which are cursive sketches usually independent of the person's name. In general, as the need for cross-cultural applications increases, it is becoming more and more important to evaluate both the extent to which personal background affects signature characteristics and the accuracy of the verification process. For this purpose, a set of metadata, sometimes also called "soft biometrics," is considered. Metadata concern various aspects of a writer background, such as nationality, script language, age, gender, handedness, etc. Some metadata can be estimated by statistically analyzing human handwriting, thus it is possible to adapt signature verification algorithms to the metadata context in order to improve verification performances [140], [297], [326], [342].

V. Performance Evaluation

Automatic signature verification can produce two types of errors: Type I errors concern the false rejections of genuine signatures [false rejection rate (FRR)]; Type II errors concern the false acceptance of forged signatures [false acceptance rate (FAR)]. Therefore, the performance of a signature verification system is generally estimated in terms of FRR and FAR [165], [248], [258]. Depending on the applications, a tradeoff between the two error types must be defined since any reduction of FAR increases FRR, and *vice versa*. In addition, the equal error rate (EER), which is defined as the system error rate when FRR $=$ FAR, is widely considered to be a measure of the overall error of a system [see Fig. 10(a)] [341]. In other cases, the total error rate ε_t, which is defined as $\varepsilon_t = ((\text{FRR} \cdot P(\omega_1)) + (\text{FAR} \cdot P(\omega_2))$—where $P(\omega_1)$ and $P(\omega_2)$ are the *a priori* probabilities of classes of genuine signatures (ω_1) and forgeries (ω_2), is used [281]– [283]. The receiver operating characteristic (ROC) curve analysis is also applied to FRR versus FAR evaluation since it shows the ability of a system to discriminate genuine signatures from forged ones [see Fig. 10(b)] [309], [311].

Unfortunately, the existence of skilled forgeries for a given signature is not certain, nor is the possibility of collecting good quality forgery samples for the test [201], [248]. Since signature forgeries are the results of a behavioral activity, they depend strongly on the type and amount of information provided to forger, as well as his/her training and effort [16]. Thus, the FAR evaluation is difficult and generally imprecise [259], [377]. The traditional method of handling this problem consists of considering different classes of forgeries [248]: *random* forgeries, in which the forger uses his own signature instead of the signature to be tested; *simple* forgeries, in which the forger makes no attempt to simulate or trace a genuine signature; and *free-hand* or *skilled* forgeries, in which the forger tries and practices imitating as closely as possible the static and dynamic information of a genuine signature. Another attempt for grading of forgery quality considers the following four categories: [377]: *accidental* forgeries are those which use arbitrary nonauthentic writing samples against some other reference; *blind* attackers are when the forger only has a textual knowledge about the writing content; *low-force* forgeries occur when the forger is in possession of an offline representation of the signature image; and *brut-force* attackers are when the forger also has the opportunity to observe the dynamics of the writing process.

Tables V and VI summarize the characteristics of some of the most interesting signature verification systems presented in the literature for offline and online signatures, respectively. For each system, some additional information is briefly described in the following. A more detailed description can be found in the literature.

In Table V, Abu-Rezq and Tolba [2] used a neural approach for signature verification based on moment invariant features and projection-based features. Bajaj and Chaudhury [15] used different types of global features: projection based (horizontal and vertical projection) and contour based (upper and lower envelope). Classification was performed by feedforward NN classifiers whereas the classification decisions were combined by a simple-layer feedforward NN (ADALINE). The system of Baltzakis and Papamarkos [17] performed signature verification through global, grid, and texture features. In this case, the classification stage consisted of a two-stage neural scheme, based on RBF. The hybrid ME scheme proposed by Cordella *et al.* [44] was based on two stage cascaded classifiers. It used contour-based features at the first stage and gray-level features at the second, whereas classification was performed by MLP at each stage. In the multiresolution approach of Deng *et al.* [49], curvature data were decomposed into signals using wavelet transforms. A statistical measurement was used to systematically decide which closed contours, and the associated frequency data, of a writer are most stable and discriminating. Based on these data, the optimal threshold value, which controls the accuracy of the feature extraction process, was calculated. Projection-based, slant-based, and geometric-based features and Granlund descriptors (derived by Fourier transform) were used in the ME system of Dimauro *et al.* [59]. This system combined a wholistic approach based on a Euclidean distance classifier, a structural-based approach, and an NN-based approach, using an ARTMAP NN. The results from the three approaches were combined by a voting strategy. Drouhard *et al.* [68] used the directional probability density function (pdf) as a global shape factor and a BPN classifier for signature verification. Some experimental evidence demonstrated that BPN could give almost the same performance as a k-nearest neighbor classifier and was definitely superior to a threshold classifier. In the approach of El-Yacoubi *et al.* [71], pixel density was considered to model offline signatures by HMM-LR. For each writer considered in the enrolment phase, the signer-dependent thresholds were dynamically and automatically derived. Wavelets were used by Fadhel and Bhattacharyya [76] for both data reduction and feature selection. The system proposed used global (wavelet based), statistical, and geometrical features and performed signature verification by a feedforward NN. Fang *et al.* [84] used vertical projection-based features and DTW for signature matching. Fang and Tang [85] considered a set of peripheral features and a Mahalanobis-distance-based threshold classifier. They proposed two methods to face the sparse data problem in offline signature verification. The first one artificially generated additional training samples from the existing training set by an elastic matching technique. The second approach applied regularization technique to the sample covariance matrix. The experimental results showed that both techniques can significantly improve the verification performance. Geometric-based features extracted from contour and stroke analysis were used by Ferrer *et al.* [91]. Euclidean distance classifier, SVM, and HMM-LR were also considered for the verification of both random and simple forgeries. The experimental results indicated HMM superiority with respect to SVM and Euclidean distance classifiers. Huang and Yan [126] presented a system based on geometric features extracted under different scales. The overall match rating was generated by combining the decisions achieved at each scale, by an MLP. The statistical models of Huang and Yan [128] were constructed for pixel distribution and structural description. Both geometric

TABLE V
PERFORMANCES: OFFLINE SYSTEMS

Authors	Main features	Database	Approach	Results
A. N. Abu-Rezq and A. S. Tolba [2]	X-Y correlations, Projection-based, Moment-based	**Training**) 100 (G) (10(G)x10(A)) **Test**) 60 (G) (6(G) x 10(A)	NN	**FRR**: 3% (**FAR** : not estimated)
R. Bajaj and S. Chaudhury [15]	Projection based, Contour based (envelope)	**Test**) 150 (G), 100 (F)	NN	**FRR**: 1%, **FAR**: 3%
H. Baltzakis and N. Papamarkos [17]	Geometric-based, projection-based, slant-based,grid-based, texture-based	**Training**) 1500 (S) (from 115(A)) **Test**) 500 (S)	NN (RBF)	**FRR**: 3%, **FAR**: 9,81%
L. P. Cordella et al. [44]	Contour-based (projections of the outline of the signature) gray-level intensity-based	**FD**) 1960(S) (20 (G),20 (F))x49(A))	NN (MLP) (ME by Cascaded Multiple Experts)	**FRR**: 2,04%, **FAR**: 0,01% (RD), 4,29% (SP), 19,80 (SK)
P. S Deng et al. [49]	Wavelet transform	**Training**) 500 (G) (from 50(A)) **Test**) 500(G), 2500(F)	DTW	**FRR**: 5,60%,**FAR**: 10,98% (English signatures) **FRR**: 6,00%, **FAR**: 7,80% (Chinese signatures)
G. Dimauro et al. [59]	Projection–based, Slant-based, Geometric-based, Fourier Transform (Granlund descriptor)	**Training**) 225 (G) (25(G)x9(A)) **Test**) 450(G) (50(G)x9(A)), 450 (RF) (50(RF)x9(A)), 90(SK) (10(SK)x9(A))	Euclidean Distance, NN (ME by Majority Vote)	**FRR**: 2%, **FAR**: 0,5% (RF), 3.9% (SK) (with 22% Rejection Rate)
J.-P. Drouhard et al. [68]	Direction-based	**Training**) 400 (S) **Test**) 400 (S)	NN (BPN)	ε_t: 3,22% (with $P(\omega_1)=P(\omega_2)=0.5$)
A.El Yacoubi et al. [71]	Grid-based (density of pixels)	**Training**) 1600 (S) (from 40(A)) **Test**) 2400(S) (from 60(A))	HMM (Cross validation)	**FRR**: 0,75%, **FAR**: 0,18% (on training datasets) **FRR**: 1,17%, **FAR**: 0,64% (on test datasets)
E.A. Fadhel and P. Bhattacharyya [76]	Global (Wavelet-based), statistical and geometrical	**FD**) 300 (S) (from 31 (A))	NN	**FRR**: 6,2%,, **FAR**: 5,5%
B. Fang et al. [84]	Projection-based	**FD**) 1320 (G) (from 55(A)), 1320 (F) (from 55(A))	DTW	**FRR**: 22,1%, **FAR**: 23,5%
B. Fang and Y.Y. Tang [85]	Peripheral-based	**Test**) 1320 (G), 1320 (F)	Mahalanobis distance	**EER**: 11,4%
M.A. Ferrer et al. [91]	Geometric-based	**FD**) 3840 (G) (24(G)x160(A)), 4800 (F) (30(F)x160(A))	1) Euclidean Distance, 2) SVM, 3) HMM	**1) FRR**: 5.61%, (16.39%,) **FAR**: 4.96% (15.50%) on RF (SF) **2) FRR**: 3.23%, (15.41%) **FAR**: 2.65% (13.12%) on RF (SF) **3) FRR**: 2.2%, (14.1%) **FAR**: 3.3% (12.6%) on RF (SF)
K. Huang and H. Yan [126]	Geometric based, grid-based	**Test**) 504 (G), 3024 (F)	NN	**FRR**: 11,1%, **FAR**: 11,8%
K. Huang and H. Yan [128]	Geometric-based, Direction based	**Training**) 424 (G) **Test**) 848 (G), 7632 (F)	NN, Structural Matching (ME by Relaxation match.)	**FRR**: 6,3%, **FAR**: 8,2%
E. J. R. Justino et al. [144]	Graphometric-based	**FD1**) 1600(S) (40(S)x40(A)) **FD2**) 2400(S) (40(S)x60(A))	HMM (Cross validation)	**FRR**: 0.75%, **FAR**: 0.22% (FD1) **FRR**: 1%, **FAR**: 0.77% (FD2)
V. K. Madasu et al. [185]	Grid-based (normalized vector angle)	**Training**) 255(G) (17 (G)x5(A)) **Test**) 85(SF), 85(RF), 85(SK)	Fuzzy logic modeling	**FRR**: 0%, **FAR**: 3,5%
Y. Mizukami et al. [199]	Position	**FD**) 400 (S) (200 (G), 200 (F))	Displacement function	**EER**: 24,9%
N. A. Murshed et al. [215]	Grid-based	**FD**) 200 (S) (40(S)x 5(A))	NN (ARTMAP)	**FRR**: 7,27%, **FAR**: 11%
V.E.Ramesh and M.Narasimha Murty [274]	Geometric-based, Moment-based, Contour-based (envelope), wavelet transform	**Training**) 225(G) ((15(G) x15(A), 195(F)(13(F) x15(A)) **Test**) 75(G) (5(G)x15(A)), 150(F)(10(F)x15(A))	Confidence intervals, Minmax, N-dim boundary, NN, Hybrid approach	**FRR**: 10%, **FAR**: 2% (SP)
R. Sabourin et al. [281]	Shadow code-based	**Test**) 800 (S) (40(S)x20(A))	Case a) kNN classifier Case b) min distance classifier	(Case a) ε_t :0,01% (k=1) (with $P(\omega_1)=P(\omega_2)=0.5$) (Case b) ε_t :0,87% (N=4) (with $P(\omega_1)=P(\omega_2)=0.5$)
R. Sabourin and J.-P Drouhard [282]	Direction based (Probability Density Function – PDF)	**Training**) 400 (S) **Test**) 400 (S)	NN	ε_t : 4,07% (with $P(\omega_1)=P(\omega_2)=0.5$)
R. Sabourin et al. [283]	Shape Matrices	**FD**) 800 (S) (from 20(A))	Pattern Matching	ε_t : 0,84%
C. Santos et al. [295]	Graphometric based	**Test**) 300 (G), 600 (F)	Euclidean distance + NN (MLP)	**FRR**: 10,33% **FAR**:4,41% (RD), 1,67% (SP), 15,67% (sim. forgeries)
K. Ueda [318]	Pattern Image	**Test**) 1000 (G), 1000 (F)	Pattern Matching	**EER**: 9,1%
X.-H. Xiao and G. Leedham [350]	Direction based, grid based	**Training**) Genuine samples only (Case 1), Genuine samples + artificial forgeries (Case 2) **Test**) 350 (G), 158 (SK), 230(RF)	NN (MLP)	**FRR**: 10.6%, **FAR**: 38.9% (SK) (Case 1) **FRR**: 9.2%, **FAR**: 17% (SK) (Case 2)

Full Database (FD), Signature (S), Genuine Signatures (G), Forgeries (F), Random Forgeries (RF), Simple Forgeries (SF), Skilled Forgeries (SK), Number of Authors (A)

features and directional frontier features were considered for signature description. The statistical verification algorithm used the geometric features and an MLP for signature verification. For questionable signatures where the pixel feature judgment was inconclusive, a structural matching algorithm was applied, using directional frontier features. Justino *et al.* [144] used HMM-LR with a density-based static feature and a pseudodynamic feature, based on axial slant. In the fuzzy-modeling approach proposed by Madasu *et al.* [185], a well-defined fuzzification function with structural parameters was used for signature verification. In this case, the signature image was partitioned into a fixed number of subimages by a grid-based approach and a normalized