Digital Transformation of Healthcare

Evaluating Predictions

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Objectives

After this lecture students will be able to

- Distinguish between classification and regression metrics
- Compare and contrast the use of different metrics to evaluate predictions
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Metrics for Evaluation of Classification Models

Confusion Matrix

actual outcome

		p	**n**	
predicted outcome	**p′**	TP	FP	$PPV = \frac{TP}{TP + FP}$
	n′	FN	TN	$NPV = \frac{TN}{FN + TN}$
		$Sens = \frac{TP}{TP + FN}$	$Spec = \frac{TN}{FP + TN}$	$Acc = \frac{TP+TN}{TP+TN+FP+FN}$

Confusion Matrix

actual outcome

	p	**n**	
p′	TP	FP	$\text{PPV} = p\left(\mathbf{p} \mid \mathbf{p}'\right)$
n′	FN	TN	$\text{NPV} = p\left(\mathbf{n} \mid \mathbf{n}'\right)$
	$\text{Sens} = p\left(\mathbf{p}' \mid \mathbf{p}\right)$	$\text{Spec} = p\left(\mathbf{n}' \mid \mathbf{n}\right)$	$\text{Acc} = p\left(TP + TN\right)$

predicted outcome

Confusion Matrix

actual outcome

		p	**n**	
predicted outcome	**p′**	TP	FP	$PPV = p(\mathbf{p} \mid \mathbf{p}')$
	n′	FN	TN	$NPV = p(\mathbf{n} \mid \mathbf{n}')$
		$Sens = p(\mathbf{p}' \mid \mathbf{p})$	$Spec = p(\mathbf{n}' \mid \mathbf{n})$	$Acc = p(TP + TN)$

Parameter	Interpretation	Inappropriate for
Accuracy	Overall proximity of test to reality	Imbalanced sample sizes
Sensitivity		
Specificity		
PPV		
NPV		

Confusion Matrix

actual outcome

		p	**n**	
predicted outcome	**p′**	TP	FP	$PPV = p\,(\mathbf{p} \mid \mathbf{p}')$
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Parameter	Interpretation	Inappropriate for
Accuracy	Overall proximity of test to reality	Imbalanced sample sizes
Sensitivity	Chance of a false negative	Expensive testing/Mild disease
Specificity	Chance of a false positive	Cheap testing/Severe disease
PPV	Sensitivity diagnostic utility	Very high prevalence
NPV	Specificity diagnostic utility	Very low prevalence

Combined Statistics

Function of	Metric	Formula
Sensitivity, Specificity	Positive Likelihood Ratio/ROC	$\dfrac{sensitivity}{1 - specificity}$
Sensitivity, Specificity	Negative Likelihood Ratio	$\dfrac{1 - sensitivity}{specificity}$
Sensitivity, PPV	F1 score	$\dfrac{2}{\dfrac{1}{sensitivity} + \dfrac{1}{PPV}}$
TP, TN, FP, FN	Matthews correlation coefficient	$\dfrac{TP \times TN - FP \times FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}$

Likelihood Ratios

Does a test result change the probability that a person has a certain condition?

$$LR+ = \frac{sensitivity}{1 - specificity} = \frac{P(T+ \mid D+)}{P(T+ \mid D-)}$$

$$LR- = \frac{1 - sensitivity}{specificity} = \frac{P(T- \mid D+)}{P(T- \mid D-)}$$

Likelihood Ratios

Likelihood Ratio	Approximate Change in Probability(%)
0.1	-45
0.2	-30
0.5	-15
1	0
2	+15
5	+30
10	+45

Change in post test probability $\approx 0.2 \times \ln LR$ [1]

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[1] McGee, Steven. "Simplifying likelihood ratios." Journal of general internal medicine 17.8 (2002): 647-650. APA
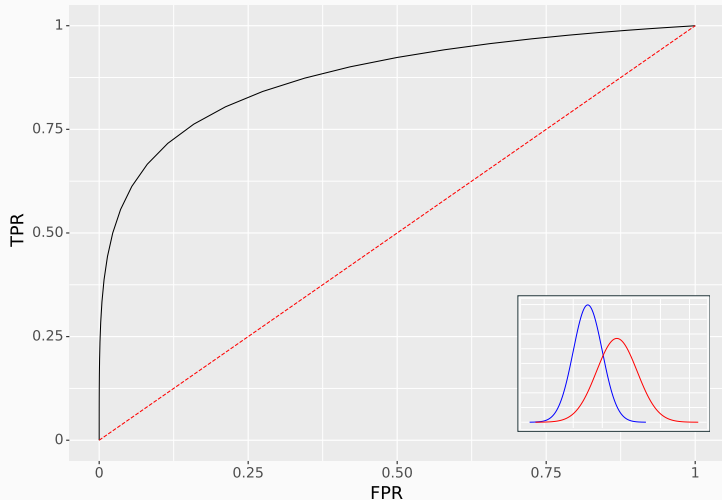
Hypothesis Testing

What does the Area Under the Curve (AUC) correspond to?
What does the diagonal correspond to?
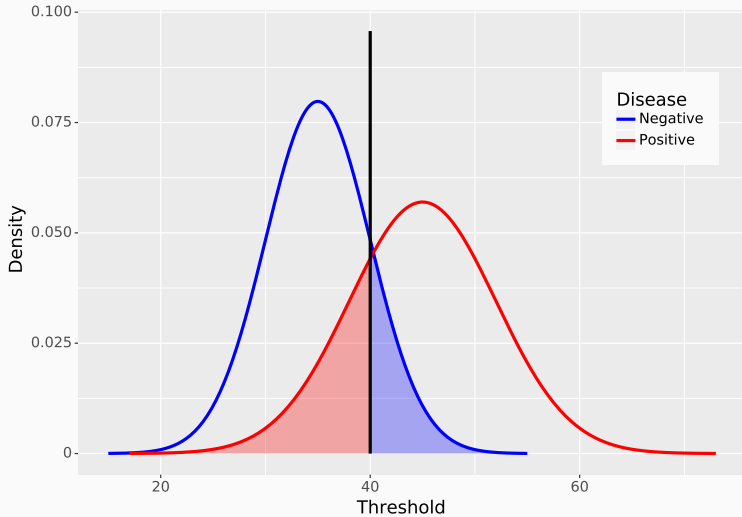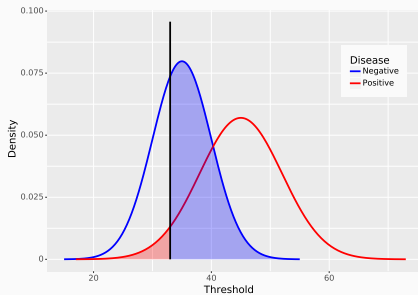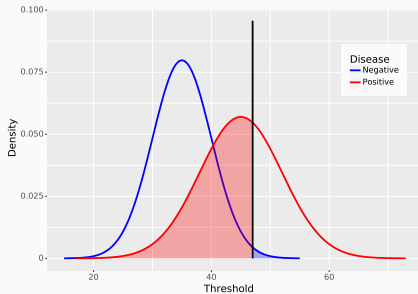
Receiver Operating Characteristic
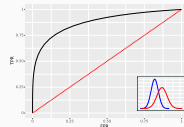
What does the Area Under the Curve (AUC) correspond to?
What does the diagonal correspond to?

1. Given a positive test result what are the chances that the subject is truly positive irrespective of prevalence?
2. PPV is threshold dependent, while AUC is threshold independent but variable dependent

F1 Score

actual outcome

	p	**n**	
p′	TP	FP	$PPV = p(\mathbf{p} \mid \mathbf{p}')$
n′	FN	TN	$NPV = p(\mathbf{n} \mid \mathbf{n}')$
	$Sens = p(\mathbf{p}' \mid \mathbf{p})$	$Spec = p(\mathbf{n}' \mid \mathbf{n})$	$Acc = p(TP + TN)$

(predicted outcome)

$$F1 = \frac{2}{\dfrac{1}{sensitivity} + \dfrac{1}{PPV}} = 2 \times \frac{PPV \cdot sensitivity}{PPV + sensitivity}$$

How does F1 differ from AUC?

1. F1 is sensitivity modified by prevalence. So a low prevalence will hurt your F1 score but might not affect your AUC. F1 is threshold specific and corresponds to a point on the ROC curve

2.

Metrics for Evaluation of Regression Models

Regression Metrics

- What aspects of a model's predictions should I care about?
- What aspects of the model's predictions can I evaluate?

Regression Metrics

- What aspects of a model's predictions should I care about?
- What aspects of the model's predictions can I evaluate?

1. – Accuracy (bias), precision (variance)
   – Average distance of errors
   – Worst case error
   – Do large errors matter more than small errors
   – Maximal distance of errors
   – Difference between my model and some standard model
2. difference, squared difference, min/max, variance of predictions, relative difference (percentage error)

Regression Metrics

$$Mean\ Absolute\ Error = \frac{1}{n}\sum_{i=0}^{n-1}|y_i - \hat{y}_i|$$

$$Mean\ Squared\ Error = \frac{1}{n}\sum_{i=0}^{n-1}(y_i - \hat{y}_i)^2$$

$$R^2 = 1 - \frac{\sum\limits_{i=0}^{n-1}(y_i - \hat{y}_i)^2}{\sum\limits_{i=0}^{n-1}(y_i - \bar{y}_i)^2}$$

$$Explained\ Variance = 1 - \frac{Var\,(y - \hat{y})}{Var\,(y)}$$