Nama: Michael Geraldin Wijaya
NIM: 2602238021

ALIN

1. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix:

A. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ B. $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

2. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix:

A. $\begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ B. $\begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$

---

1. A. Characteristic equation

$$\det(\lambda I - A) = 0$$

$$\lambda I - A = \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{pmatrix}$$

$$\det(\lambda I - A) = 0$$

$$(\lambda-1)^2 = 0$$

$$\lambda^2 - 2\lambda + 1 = 0 \rightarrow \text{Char eq.}$$

$$(\lambda I - A)x = 0$$

$$\left(\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = 1$ → eigenvalue

x_1, x_2 independent → 1

Basis: $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$

B. $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

$$\lambda I - B = \begin{pmatrix} \lambda-1 & 2 \\ 0 & \lambda-1 \end{pmatrix}$$

$\det(\lambda I - B) = 0$

$(\lambda-1)(\lambda-1) = 0$

$(\lambda-1)^2 = 0$

$\lambda^2 - 2\lambda + 1 = 0 \longrightarrow$ char eq.

$\lambda = 1 \longrightarrow$ eigenvalue

$(\lambda I - B)x = 0$

$$\begin{pmatrix} \lambda-1 & 2 \\ 0 & \lambda-1 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2x_2 = 0$ $\quad x_1 = t$

$x_2 = 0$

~~Vectoreigen : $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix} = t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$~~

Basis : $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$

Scanned by TapScanner

2. A. $\begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

$(\lambda I - A) = \begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix}$

$\det(\lambda I - A) = 0$

$\begin{vmatrix} \lambda-4 & 0 & -1 & \lambda-4 & 0 \\ 2 & \lambda-1 & 0 & 2 & \lambda-1 \\ 2 & 0 & \lambda-1 & 2 & 0 \end{vmatrix} = 0$

$(\lambda-4)(\lambda-1)(\lambda-1) + (0)(0)(2) + (-1)(2)(0) - \Big((-1)(\lambda-1)(2) + (\lambda-4)(0)(0) +$

$(0)(2)(\lambda-1)\Big) = 0$

$(\lambda-4)(\lambda^2-2\lambda+1) - \big((-\lambda+1)2\big) = 0$

$\lambda^3 - 2\lambda^2 + \lambda - 4\lambda^2 + 8\lambda - 4 - (-2\lambda+2) = 0$

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 + 2\lambda - 2 = 0$

Char eqvation :

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$(\lambda-1)$	1	-6	11	-6
$\lambda:1$		1	-5	6
	1	-5	6	0

$(\lambda-1)(\lambda^2-5\lambda+6) = 0$

$(\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\lambda = 1 \vee \lambda = 2 \vee \lambda = 3$

eigenvalue

. $\lambda = 1$

$(\lambda I - A)X = 0$

$\begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

x_2 independent , $x_2 = 1$, $x_3 = 0$

$-3x_1 - x_3 = 0$

$B_1 = \left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$ $\quad -3x_1 = 0$

$x_1 = 0$

Scanned by TapScanner

• $\lambda = 2$

$$\begin{pmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2x_1 \quad - x_3 = 0$

$2x_1 + x_2 = 0$

$2x_1 \quad + x_3 = 0$

x_1 independent

$x_1 = -1$

$-2x_1 - x_3 = 0$

$x_3 = -2x_1$

$= 2$

$x_2 = -2x_1$

$= 2$

$B = \left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \right\}$

• $\lambda = 3$

$$\begin{pmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

x_1 independent

$x_1 = -1$

$-x_1 - x_3 = 0$

$x_3 = -x_1$

$= 1$

$2x_1 + 2x_2 = 0$

$2x_2 = -2x_1$

$2x_2 = 2$

$x_2 = 1$

$B = \left\{ \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$

Scanned by TapScanner

B. $\begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$

$$(\lambda I - B) : \begin{pmatrix} \lambda-6 & -3 & 8 \\ 0 & \lambda+2 & 0 \\ -1 & 0 & \lambda+3 \end{pmatrix}$$

$\det(\lambda I - B) : 0$

$$\left| \begin{matrix} \lambda-6 & -3 & 8 & \lambda-6 & -3 \\ 0 & \lambda+2 & 0 & 0 & \lambda+2 \\ -1 & 0 & \lambda+3 & -1 & 0 \end{matrix} \right| : 0$$

$$(\lambda-6)(\lambda+2)(\lambda+3) + (-3)(0)(-1) + (8)(0)(0) - \big((8)(\lambda+2)(-1) + (\lambda-6)(0)(0) + (-3)(0)(\lambda+3) \big) = 0$$

$$(\lambda-6)(\lambda^2+3\lambda+2\lambda+6) - \big(8(-\lambda-2)\big) : 0$$

$$\lambda^3 + 3\lambda^2 + 2\lambda^2 + 6\lambda - 6\lambda^2 - 18\lambda - 12\lambda - 36 - (-8\lambda - 16) : 0$$

~~$\lambda^3 + -16\lambda - 20 : 0$~~

$\lambda^3 - \lambda^2 - 16\lambda - 20 : 0$ $\Longrightarrow$ Characteristic equation

$\lambda - 5$	1	-1	-16	-20
$\lambda : 5$		5	20	20
	1	4	4	0

$$(\lambda-5)(\lambda^2+4\lambda+4) = 0 \iff \lambda : 5 \lor \lambda : -2$$

eigen value =

$$(\lambda-5)(\lambda+2)^2 : 0$$

• $\lambda = 5$

$$\begin{pmatrix} -1 & -3 & 8 \\ 0 & 7 & 0 \\ -1 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -3 & 8 \\ 0 & 7 & 0 \\ 0 & -3 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & -3 & 8 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -3 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$x_2 = 0$

$-x_1 - 3x_2 + 8x_3 = 0$

$-x_1 = -8x_3$

$x_3 = 1$

$-x_1 = -8$

$x_1 = 8$

$-x_1 - 3x_2 + 8x_3 = 0 \Rightarrow -x_1 + 8x_3 = 0$

$7x_2 = 0$

$x_2 = 0$

$-x_1 + 8x_3 = 0$

$\to x_3 = 1$

$-x_1 = -8$

$x_1 = 8$

$B: \left\{ \begin{pmatrix} 8 \\ 0 \\ 1 \end{pmatrix} \right\}$

• $\lambda = -2$

$$\begin{pmatrix} -8 & -3 & 8 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$x_2 = 0$

$-x_1 + x_3 = 0$ $\quad x_1 = 1$

$x_3 = 1$

$B: \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$