No.

Date

ALIN

Nama : Michael Geraldin Wijaya
NIM : 2602232021

Suppose that A, B, C, D, and E are matrices with the following sizes:

I.

A	B	C	D	E
(4×5)	(4×5)	(5×2)	(4×2)	(5×4)

In each part, determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

(a) BA

$\underset{4\times5}{B} \times \underset{4\times5}{A}$ → undefined / tidak bisa didefinisikan

(b) AC + D

$\underset{4\times\cancel{5}}{A} \times \underset{\cancel{5}\times2}{C} + \underset{4\times2}{D}$

$\underset{4\times2}{AC} + \underset{4\times2}{D} = \underset{(4\times2)}{AC+D}$

(c) AE + B

$\underset{4\times\cancel{5}}{A} \times \underset{\cancel{5}\times4}{E} + \underset{4\times5}{B}$

$\underset{4\times4}{AE} + \underset{4\times5}{B}$ → tidak terdefinisi karena baris dan kolom tidak sama

(d) AB + B

$\underset{4\times5}{A} \times \underset{4\times5}{B} + \underset{4\times5}{B}$ → tidak bisa didefinisikan

(e) E(A+B)

$\underset{5\times4}{E} \; (\underset{4\times5}{A} + \underset{4\times5}{B})$

$\underset{5\times\cancel{4}}{E} \; \underset{\cancel{4}\times5}{(A+B)} = \underset{(5\times5)}{E(A+B)}$

(f) E(AC)

$\underset{5\times4}{E} \; (\underset{4\times\cancel{5}}{A} \times \underset{\cancel{5}\times2}{C})$

$\underset{5\times\cancel{4}}{E} \; \underset{\cancel{4}\times2}{(AC)} = \underset{5\times2}{E(AC)}$

(g) E^TA

$\underset{4\times5}{E^T} \times \underset{4\times5}{A}$ => tidak bisa didefinisikan

(h) $(A^T + E)D$

$(\underset{5\times4}{A^T} + \underset{5\times4}{E}) \; \underset{4\times2}{D}$

$\underset{5\times\cancel{4}}{(A^T+E)} \; \underset{\cancel{4}\times2}{D} = \underset{(5\times2)}{(A^T+E)D}$

No.

Date

II. In each part, find matrices A, x, and b that express the given system of linear equations as a single matrix equation $Ax = b$, and write out this matrix equation.

(a) $2x_1 - 3x_2 + 5x_3 = 7$
$9x_1 - x_2 + x_3 = -1$
$x_1 + 5x_2 + 4x_3 = 0$

$$\begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \; b = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

(b) $4x_1 \qquad - 3x_3 + x_4 = 1$
$5x_1 + x_2 \qquad - 8x_4 = 3$
$2x_1 - 5x_2 + 9x_3 - x_4 = 0$
$3x_2 - x_3 + 7x_4 = 2$

$$\begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

III. $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}$, $a = 4$ $b = -7$

1.) Show that

(a) $A + (B + C) = (A + B) + C$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} + \left(\begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}\right) = \left(\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}\right) + \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}$$

No.

Date

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 8 & -5 & -2 \\ 1 & 8 & 6 \\ 7 & -2 & 15 \end{bmatrix} = \begin{bmatrix} 10 & -4 & -2 \\ 0 & 5 & 7 \\ 2 & -6 & 10 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -6 & 1 \\ 1 & 12 & 11 \\ 5 & -1 & 19 \end{bmatrix} = \begin{bmatrix} 10 & -6 & 1 \\ 1 & 12 & 11 \\ 5 & -1 & 19 \end{bmatrix}$$

(b) $(AB)C = A(BC)$

$$\begin{bmatrix} 28 & -28 & 6 \\ 20 & -31 & 38 \\ 0 & -21 & 36 \end{bmatrix} \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} -18 & -62 & -33 \\ 7 & 17 & 22 \\ 11 & -27 & 38 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -222 & 26 \\ 83 & -67 & 278 \\ 87 & 33 & 240 \end{bmatrix} = \begin{bmatrix} -10 & -222 & 26 \\ 83 & -67 & 278 \\ 87 & 33 & 240 \end{bmatrix}$$

(c) $(a+b)C = aC + bC$

$$-3 \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -8 & 12 \\ 4 & 28 & 16 \\ 12 & 20 & 36 \end{bmatrix} + \begin{bmatrix} 0 & 14 & -21 \\ -7 & -49 & -28 \\ -21 & -35 & -63 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 & -9 \\ -3 & -21 & -12 \\ -9 & -15 & -27 \end{bmatrix} = \begin{bmatrix} 0 & 6 & -9 \\ -3 & -21 & -12 \\ -9 & -15 & -27 \end{bmatrix}$$

(d) $a(B-C) = aB - aC$

$$4 \begin{bmatrix} 8 & -1 & -8 \\ -1 & -6 & -2 \\ 1 & -12 & -3 \end{bmatrix} = \begin{bmatrix} 32 & -12 & -20 \\ 0 & 4 & 8 \\ 16 & -28 & 24 \end{bmatrix} - \begin{bmatrix} 0 & -8 & 12 \\ 4 & 28 & 16 \\ 12 & 20 & 36 \end{bmatrix}$$

$$\begin{bmatrix} 32 & -4 & -32 \\ -4 & -24 & -8 \\ 4 & -48 & -12 \end{bmatrix} = \begin{bmatrix} 36 & -4 & -32 \\ -4 & -24 & -8 \\ 4 & -48 & -12 \end{bmatrix}$$

2) Using the matrices and scalars in Exercise 1, verify that

(a) $a(BC) = (aB)C = B(aC)$

$$4\begin{bmatrix} -18 & -62 & -33 \\ 7 & 17 & 22 \\ 11 & -27 & 38 \end{bmatrix} = \begin{bmatrix} 32 & -12 & -20 \\ 0 & 4 & 8 \\ 16 & -28 & 24 \end{bmatrix}\begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}\begin{bmatrix} 0 & -8 & 12 \\ 4 & 28 & 16 \\ 12 & 20 & 36 \end{bmatrix}$$

$$\begin{bmatrix} -72 & -248 & -132 \\ 28 & 68 & 88 \\ 44 & -108 & 152 \end{bmatrix} = \begin{bmatrix} -72 & -248 & -132 \\ 28 & 68 & 88 \\ 44 & -108 & 152 \end{bmatrix} = \begin{bmatrix} -72 & -248 & -132 \\ 28 & 68 & 88 \\ 44 & -108 & 152 \end{bmatrix}$$

(b) $A(B-C) = AB - AC$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}\begin{bmatrix} 8 & -1 & -8 \\ -1 & -6 & -2 \\ 1 & -12 & -3 \end{bmatrix} = \begin{bmatrix} 28 & -28 & 6 \\ 20 & -31 & 38 \\ 0 & -21 & 36 \end{bmatrix} - \begin{bmatrix} 8 & 4 & 29 \\ 19 & 63 & 61 \\ 13 & 31 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -32 & -23 \\ 1 & -84 & -23 \\ -13 & -53 & 2 \end{bmatrix} = \begin{bmatrix} 20 & -32 & -23 \\ 1 & -84 & -23 \\ -13 & -52 & 2 \end{bmatrix}$$

(c) $(B+C)A = BA + CA$

$$\begin{bmatrix} 8 & -5 & -2 \\ 1 & 8 & 6 \\ 7 & -2 & 15 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 26 & -25 & -11 \\ -4 & 6 & 13 \\ -4 & -26 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -5 & 2 \\ -6 & 31 & 54 \\ -12 & 26 & 70 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -30 & -9 \\ -10 & 37 & 67 \\ -16 & 0 & 71 \end{bmatrix} = \begin{bmatrix} 20 & -30 & -9 \\ -10 & 37 & 67 \\ -16 & 0 & 71 \end{bmatrix}$$

(d) $a(bC) = (ab)C$

$$4\begin{bmatrix} 0 & 14 & -21 \\ -7 & -49 & -28 \\ -21 & -35 & -63 \end{bmatrix} \qquad -28\begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 56 & -84 \\ -28 & -196 & -112 \\ -84 & -140 & \end{bmatrix} = \begin{bmatrix} 0 & 56 & -84 \\ -28 & -196 & -112 \\ -84 & -140 & -252 \end{bmatrix}$$

No.

Date

3.) Using the matrices and scalars in Exercise 1, Verify that

(a) $(A^T)^T = A$

$$\begin{bmatrix} 2 & 0 & -2 \\ -1 & 4 & 1 \\ 3 & 5 & 4 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}$$

(b) $(A+B)^T = A^T + B^T$

$$\begin{bmatrix} 10 & -4 & -2 \\ 0 & 5 & 7 \\ 2 & -6 & 10 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -2 \\ -1 & 4 & 1 \\ 3 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 4 \\ -3 & 1 & -7 \\ -5 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 0 & 2 \\ -4 & 5 & -6 \\ -2 & 7 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 2 \\ -4 & 5 & -6 \\ -2 & 7 & 10 \end{bmatrix}$$

(c) $(aC)^T = aC^T$

$$\begin{bmatrix} 0 & -8 & 12 \\ 4 & 28 & 16 \\ 12 & 20 & 36 \end{bmatrix}^T = 4\begin{bmatrix} 0 & 1 & 3 \\ -2 & 7 & 5 \\ 3 & 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 12 \\ -8 & 28 & 20 \\ 12 & 16 & 36 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 12 \\ -8 & 28 & 20 \\ 12 & 16 & 36 \end{bmatrix}$$

(d) $(AB)^T = B^T A^T$

$$\begin{bmatrix} 28 & -28 & 6 \\ 20 & -31 & 38 \\ 0 & -21 & 36 \end{bmatrix}^T = \begin{bmatrix} 8 & 0 & 4 \\ -3 & 1 & -7 \\ -5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 & -2 \\ -1 & 4 & 1 \\ 3 & 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 28 & 20 & 0 \\ -28 & -31 & -21 \\ 6 & 38 & 36 \end{bmatrix} = \begin{bmatrix} 28 & 20 & 0 \\ -28 & -31 & -21 \\ 6 & 38 & 36 \end{bmatrix}$$

IV. Calculate the matrices $A \times B$!

$$A = \begin{bmatrix} 1 & 3 & -2 & 3 & -3 \\ 2 & 4 & -2 & 2 & 1 \\ 0 & -2 & 1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 3 \\ 2 & 2 \\ 1 & 2 \end{bmatrix}$$

$$(3 \times \cancel{5}) \rightarrow (3 \times 2) \leftarrow (\cancel{5} \times 2)$$

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \qquad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_1B_1 + A_2B_2 \\ A_3B_1 + A_4B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -3 \\ 9 & 4 \\ 2 & 5 \end{bmatrix}$$

$$(3 \times 2)$$