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Sciencetific Comp

$f(x) = 1,75x^4 - 5\cos(3x)$

First

a. 2 Points $\quad f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{h}$ Forward

$x_i = 1,5$ $\quad x_{i+1} = x_i + h$ $\quad f(1,6) = 1,75(1,6)^4 - 5\cos(3(1,6))$

$h = 0,1$ $\quad = 1,5 + 0,1$ $\quad = 11,03131$

$= 1,6$ $\quad f(1,5) = 9,91335$

$f'(1,5) \approx \frac{f(1,6) - f(1,5)}{0,1} \approx 11,1796$

3 Points $\quad f'(x_i) \approx \frac{-3f(x_i) + 4(f(x_{i+1}) - f(x_{i+2})}{2h} + O(h^2)$

$x_{i+2} = x_i + 2h$

$= 1,7$ $\quad f(1,7) = 1,75(1,7)^4 - 5\cos(3(1,7))$

$= 12,72629$

$f'(1,5) \approx \frac{-3f(1,5) + 4f(1,6) - f(1,7)}{2(0,1)} \approx 8,2945$

Backward
2 Points $\quad f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{h}$

$x_{i-1} = x_i - h$ $\quad f(1,4) = 1,75(1,4)^4 - 5\cos(3(1,4))$

$= 1,5 - 0,1$ $\quad = 9,1741$

$= 1,4$

$f'(1,5) \approx \frac{f(1,5) - f(1,4)}{h} \approx \frac{9,91335 - 9,1741}{0,1} = 7,3925$

3 Points $\quad f'(x_i) \approx \frac{3f(x_i) - 4(f(x_{i-1})) + f(x_{i-2})}{2h}$

$x_{i-2} = 1,5 - 0,2$

$= 1,3$ $\quad f(1,3) = 1,75(1,3)^4 - 5\cos(3(1,3))$

$= 8,62784$

$f'(1,5) \approx \frac{3(9,91335) - 4(9,1741) + 8,62784}{2(0,1)} \approx 8,35745$

b. 2 Points

Central

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

$$f'(1,5) \approx \frac{f(1,6) - f(1,4)}{0,2} \approx \frac{11,03131 - 9,1741}{0,2} \approx 9,2805$$

4 Points

$$f'(x_i) \approx \frac{f(x_{i-2}) - 8f(x_{i-1}) + 8f(x_{i+1}) - f(x_{i+2})}{12h}$$

$$f'(1,5) \approx \frac{f(1,3) - 8f(1,4) + 8f(1,6) - f(1,7)}{12(0,1)}$$

$$\approx \frac{8,62784 - 8(9,1741) + 8(11,03131) - 12,72629}{1,2}$$

$$\approx 8,966025$$

Second

c. Forward

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

$$f''(1,5) = \frac{f(1,7) - 2f(1,6) + f(1,5)}{(0,1)^2} =$$

$$= \frac{12,72629 - 2(11,03131) + 9,91335}{0,01}$$

$$= 57,702$$

Backward

$$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}$$

$$f''(1,5) = \frac{f(1,3) - 2f(1,4) + f(1,5)}{(0,1)^2}$$

$$= \frac{8,62784 - 2(9,1741) + 9,91335}{0,01}$$

$$= 19,299$$

Central

$$f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2}$$

$$= \frac{f(1.6) + f(1.4) - 2f(1.5)}{(0.1)^2}$$

$$= \frac{11.03131 + 9.1741 - 2(9.91335)}{0.01}$$

$$= 37.871$$

d. Exact value or true value of $f(1.5)$

$$f(x) = 1.75x^4 - 5\cos(3x)$$
$$f'(x) = 7x^3 + 5\sin(3x)(3)$$
$$f'(x) = 7x^3 + 15\sin(3x)$$

$$f'(1.5) = 7(1.5)^3 + 15\sin(3(1.5))$$
$$f'(1.5) = 8.96205$$

Absolute relative error

Forward :

First derivative

$$|\varepsilon_t| = \left| \frac{\text{true value} - \text{approximate value}}{\text{true value}} \right| \times 100\%$$

2 points

$$= \left| \frac{8.96205 - 11.1796}{8.96205} \right| \times 100\% = 24.74\%$$

3 points

$$= \left| \frac{8.96205 - 8.2945}{8.96205} \right| \times 100\% = 7.44863\%$$

Date

Backward °

2 points

$$= \left| \frac{8.96205 - 7,3925}{8.96205} \right| \cdot 100\% = 17,51\ \%$$

3 points

$$= \left| \frac{8,96205 - 8.35745}{8,96205} \right| \cdot 100\% = 6,74622\ \%$$

Central °

2 Points

$$= \left| \frac{8.96205 - 9,28605}{8.96205} \right| \cdot 100\% = 3,615\%$$

4 Points

$$= \left| \frac{8.96205 - 8,966025}{8.96205} \right| \cdot 100\% = 0,044\ \%$$

Second derivative

Exact value of $f''(1,5)$

$f''(x) = 21x^2 + 45\cos(3x)$

$f''(1,5) = 21(1,5)^2 + 45\cos(3(1,5))$

$= 37,76419$

forward °

$$= \left| \frac{37,76419 - 57,702}{37,76419} \right| \cdot 100\% = 52,79\ \%$$

Backward %

$$= \left| \frac{37,7619 - 19,299}{37,7619} \right| . 100\% = 48,89\ \%$$

Central %

$$= \left| \frac{37,7619 - 37,871}{37,7619} \right| . 100\% = 0,28\ \%$$