ALIN

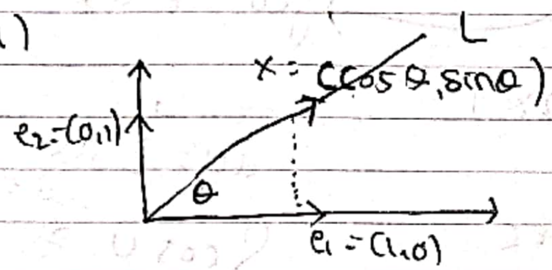
Nama : Michael Geraldin Wijaya
NIM : 2602238021

Lat Soal

1. a. Find the orthogonal projections of the standard unit vectors $e_1 = (1,0)$ and $e_2 = (0,1)$ onto the line L that makes an angle θ with the positive x-axis

b. Use the result in part (a) to find the standard matrix for the operator $T: R^2 \to R^2$ that maps each point orthogonally onto L.

=> (a)



$x = (\cos\theta, \sin\theta)$ adalah ~~unit~~ vektor sepanjang L

maka

orthogonal projection of e_1 along x

$$\text{proj}_x e_1 = \frac{e_1 \cdot x}{\|x\|^2} x = \frac{(1.\cos\theta) + (0.\sin\theta)}{\sqrt{\cos^2\theta + \sin^2\theta}} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \cos\theta \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{pmatrix}$$

$$= (\cos^2\theta, \sin\theta\cos\theta)$$

e_2 along x

$$\text{proj}_x e_2 = \frac{e_2 \cdot x}{\|x\|^2} x = \frac{(0.\cos\theta) + (1.\sin\theta)}{\sqrt{\cos^2\theta + \sin^2\theta}} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \sin\theta \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin\theta\cos\theta \\ \sin^2\theta \end{pmatrix} = (\sin\theta\cos\theta, \sin^2\theta)$$

(b)

$$\left.\begin{array}{l} \text{Proj}_x e_1 \\ \text{Proj}_x e_2 \end{array}\right\} \Rightarrow Y = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

2. Find the vector component of U along a and the vector component of U orthogonal to a.

$U = (3, 1, -7)$ $\quad a = (1, 0, 5)$

$\Rightarrow$ Vector component of U along a

$$\text{Proj}_a U = \frac{U \cdot a}{|a|^2} a$$

$$= \frac{(3.1) + (1.0) + ((-7).(5))}{(\sqrt{1^2+0^2+5^2})^2} (1, 0, 5)$$

$$= \frac{-32}{26} (1, 0, 5)$$

$$= -\frac{16}{13} (1, 0, 5)$$

$$= \left(-\frac{16}{13}, 0, -\frac{80}{13}\right)$$

Vector component of U orthogonal to a

$$U - \text{Proj}_a U = (3, 1, -7) - \left(-\frac{16}{13}, 0, -\frac{80}{13}\right)$$

$$= \left(\frac{55}{13}, 1, -\frac{11}{13}\right)$$

3. Determine whether the vectors

$V_1 = (1, -2, 3)$, $V_2 = (5, 6, -1)$ $\quad V_3 = (3, 2, 1)$

are linearly independent or linearly dependent in R^3

No.

Date

$x_1V_1 + \dots + x_pV_p = 0$

$$\begin{array}{c} \\ \\ -3R_1+R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 5 & 3 & 0 \\ -2 & 6 & 2 & 0 \\ 3 & -1 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ 2R_1+R_2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 5 & 3 & 0 \\ -2 & 6 & 2 & 0 \\ 0 & -16 & -8 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_2+R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 5 & 3 & 0 \\ 0 & 16 & 8 & 0 \\ 0 & -16 & -8 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{8}R_2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 5 & 3 & 0 \\ 0 & 16 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

karena independent maka setidaknya salah satu vektor - vektor tersebut adalah linear combination (kombinasi linear) dari yang lain.

$\Rightarrow x_1V_1 + x_2V_2 + x_3V_3 = 0$

$-\frac{1}{2}pV_1 + (-\frac{1}{2}pV_2) + pV_3 = 0 \quad : p$

$\rightarrow -\frac{1}{2}V_1 - \frac{1}{2}V_2 + V_3 = 0$

$\Leftrightarrow V_3 = \frac{1}{2}V_1 + \frac{1}{2}V_2$

$x_1 + 5x_2 + 3x_3 = 0$

$2x_2 + x_3 = 0$

$x_3 = p$

$2x_2 = -p$

$x_2 = -\frac{1}{2}p$

$x_1 - \frac{5}{2}p + 3p = 0$

$x_1 = -\frac{1}{2}p$

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

Trivial solution

Karena sistem tersebut solusinya trivial solution maka V_1, V_2, dan V_3 itu independent (bebas).