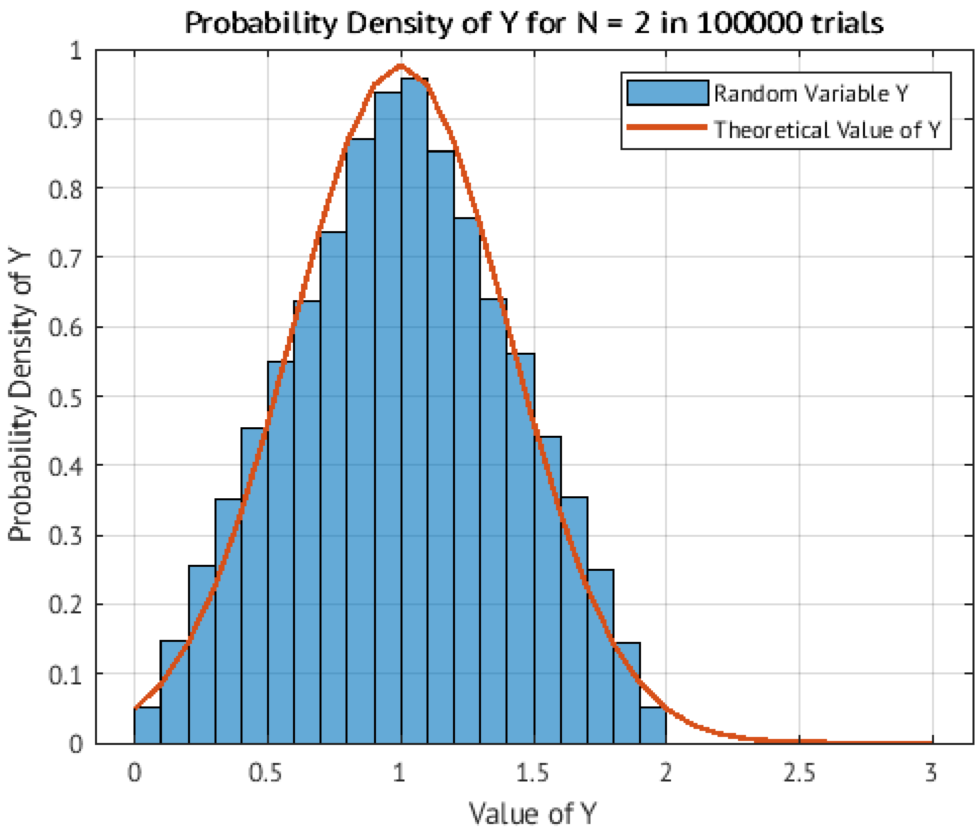
```
%S21 CMPE320 Project3 Solns
% Modified for S21, EFCL 4/15/2021

close all;
clear;

Ntrials = 100000;
kplot=0;

% This next line is what MATLAB calls an anonymous function, which is a
% function that we use only in one script. the function call is
% (x,mu,sig2), and the returned value is a number.  sig2 = sigma^2

fgauss = @(x,mu,sig2) exp(-0.5*((x-mu).^2/sig2))/sqrt(2*pi*sig2);
```

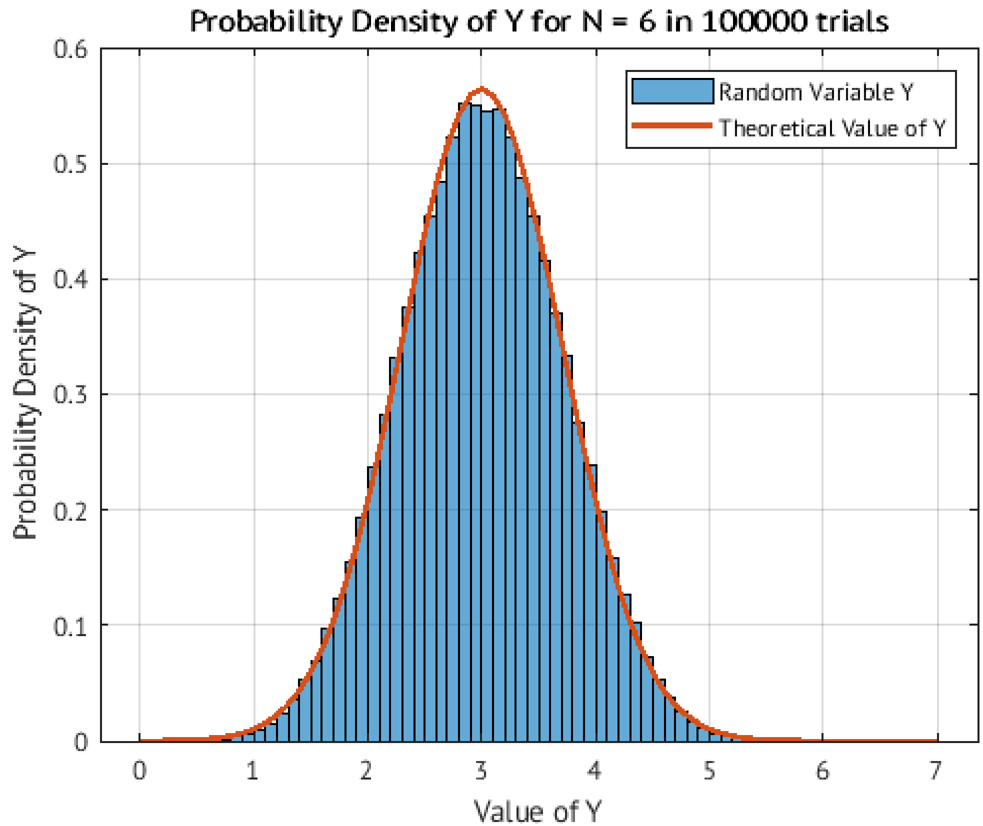
Problem 2.1 Uniformly distributed

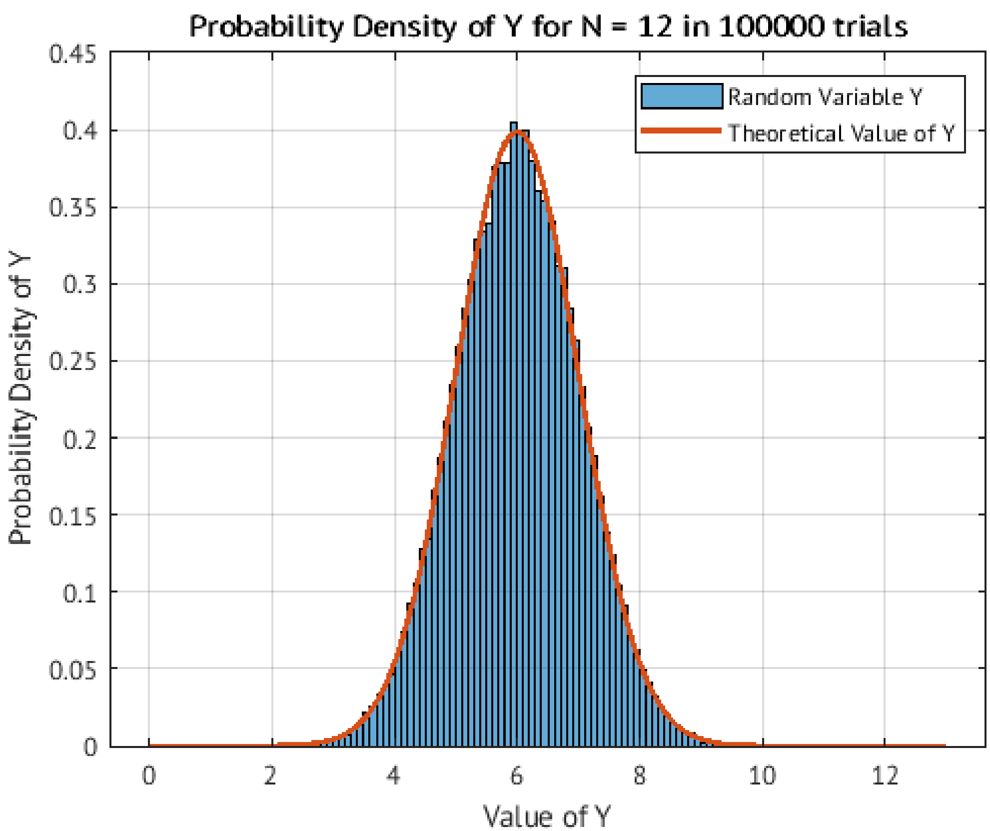
```
disp(['---------------']);
disp(['Section 2.1']);
Nsum = [2,6,12]; % checked  with S21 assignment
for k=1:length(Nsum)

    xd = rand(Nsum(k),Ntrials);  % generate [Nsum(k) by Ntrials] array of
 random values
    xs = sum(xd); % "sum" adds down the MATLAB columns, thereby giving us the
 sum of Nsum(k) values
    xmin = 0;
    xmax = Nsum(k); % largest value of the sum
    mu = Nsum(k)*0.5; % 0.5 is the E[X]
    sig2 = Nsum(k)*(1/12); % 1/12 is var[X]
    m = mean(xs); % sample mean
    S = var(xs); % sample variance
    disp(['For ',int2str(Ntrials),' independent trials of the sum of
 ',int2str(Nsum(k)),' iid rv from U(0,1)']);
    disp(['   the theoretical mean is ',num2str(mu),' and the sample mean is
 ',num2str(m)]);
    disp(['   the theoretical variance is ',num2str(sig2),' and the sample
 variance is ',num2str(S)]);

    dx=0.1; % fine grain dx for plotting
    x = [0:dx:Nsum(k)+1]; % fine grain for fY(y)

    % create new figure...
    figure();
    %...and then a new scaled histogram using the values of xs
    scaledHist = histogram(xs, 'BinEdges', x, 'Normalization', 'pdf');

    % And unpack the data using unpackHistogram from Project 2
    [vals, numBins, binCenters] = unpackHistogram(scaledHist);

    % And plot the Gaussian pdf on top of the histogram with labels and
```

```
    hold on;
    plot(x, fgauss(x,mu,sig2), 'LineWidth', 2);
    hold off;
    % grids and the other elements of a professional plot
    xlabel('Value of Y');
    ylabel('Probability Density of Y');
    grid on;
    legend('Random Variable Y', 'Theoretical Value of Y');
    title(['Probability Density of Y for N = ', num2str(Nsum(k)), ' in ',
 num2str(Ntrials), ' trials']);


end;
% there are length(Nsum) plots to this point.
```

```
---------------
Section 2.1
For 100000 independent trials of the sum of 2 iid rv from U(0,1)
   the theoretical mean is 1 and the sample mean is 1
   the theoretical variance is 0.16667 and the sample variance is 0.16649
For 100000 independent trials of the sum of 6 iid rv from U(0,1)
   the theoretical mean is 3 and the sample mean is 3.003
   the theoretical variance is 0.5 and the sample variance is 0.49669
For 100000 independent trials of the sum of 12 iid rv from U(0,1)
   the theoretical mean is 6 and the sample mean is 6.0008
   the theoretical variance is 1 and the sample variance is 1.0008
```




Probability Density of Y for N = 6 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7


Probability Density of Y for N = 12 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12

Problem 2.2 Uniformly distributed discrete

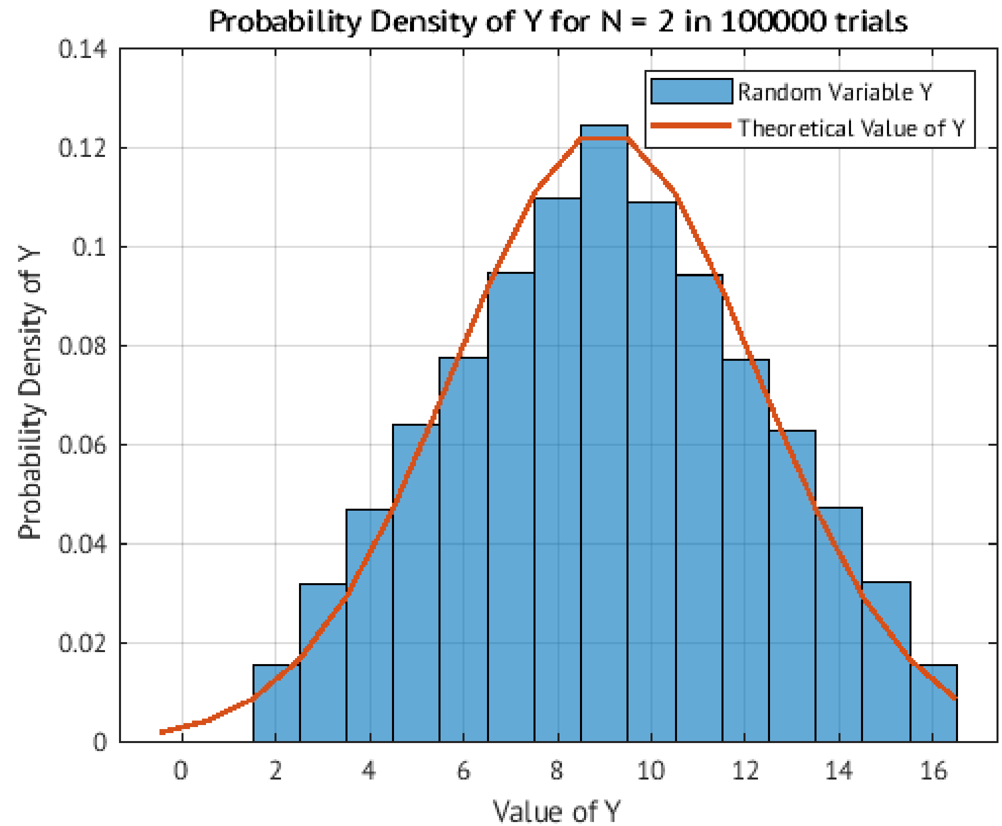
```
disp('---------------');
disp('Section 2.2');
Nsum = [2,20,40]; % checked with S22 assignment
Nsides = 8; %   checked with S22 assignment
for k = 1:length(Nsum)
    xd = randi(Nsides,Nsum(k),Ntrials); % generates Ntrials values between 1
 and Nsides
    xs = sum(xd); % sum of xd values
    % 4.5 is the E[8 sided die], then times amount of dice (Nsum(k))
    mu = Nsum(k) * 4.5; % Mean of uniform distribution
    % E[X^2] = sum( (1..8)^2 / 8 ) = 25.5
    % sample var = 25.5 - 4.5^2 = 5.25, E[X^2] - (E[X])^2 = 5.25
    sig2 = Nsum(k) * 5.25; % Variance of uniform distribution
    m = mean(xs);
    S = var(xs);
    disp(['For ',int2str(Ntrials),' independent trials of the sum of
 ',int2str(Nsum(k)),' iid rv from U(0,1)']);
    disp(['   the theoretical mean is ',num2str(mu),' and the sample mean is
 ',num2str(m)]);
    disp(['   the theoretical variance is ',num2str(sig2),' and the sample
 variance is ',num2str(S)]);

    x = [-0.5:Nsum(k) * Nsides + 0.5];
    % new figure made
    figure();
    % scaled histogram made
    scaledHist = histogram(xs, 'BinEdges', x, 'Normalization', 'pdf');

    % unpack values from histogram
    [vals, numBins, binCenters] = unpackHistogram(scaledHist);

    % And plot the Gaussian pdf on top of the histogram with labels and
    hold on;
    plot(x, fgauss(x,mu,sig2), 'LineWidth', 2);
    hold off;
    % grids and the other elements of a professional plot
    xlabel('Value of Y');
    ylabel('Probability Density of Y');
    grid on;
    legend('Random Variable Y', 'Theoretical Value of Y');
    title(['Probability Density of Y for N = ', num2str(Nsum(k)), ' in ',
 num2str(Ntrials), ' trials']);

end

% Do the experiment again for a large number of trials (Ntrials) and the
% specified number of terms in the sum (Nsum)

%Problem 2.3  Exponentially distributed
disp('---------------');
disp('Section 2.3');
```

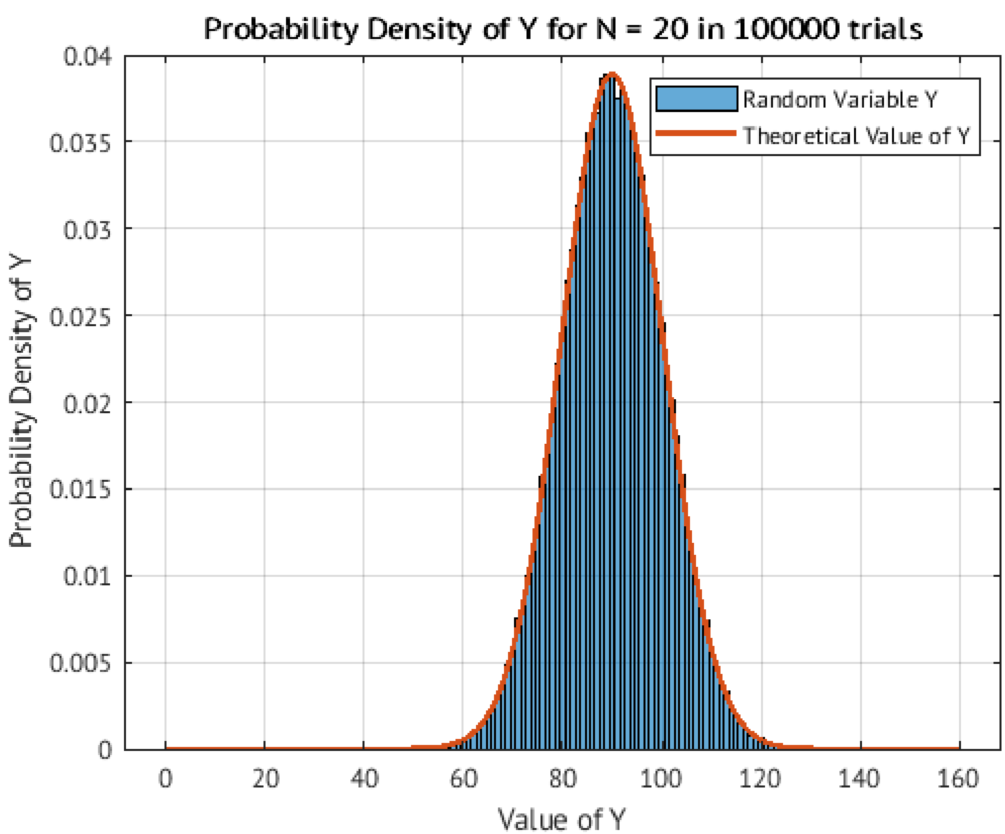
```
Nsum = [5,50,150]; % checked with S22 assignment
lambda=0.5;
for k = 1:length(Nsum)
    xd = randx(Nsum(k), Ntrials, lambda);
    xs = sum(xd);
    mu = Nsum(k) / lambda;
    sig2 = Nsum(k) / (lambda^2);
    m = mean(xs);
    S = var(xs);
    disp(['For ',int2str(Ntrials),' independent trials of the sum of
 ',int2str(Nsum(k)),' iid rv from U(0,1)']);
    disp(['   the theoretical mean is ',num2str(mu),' and the sample mean is
 ',num2str(m)]);
    disp(['   the theoretical variance is ',num2str(sig2),' and the sample
 variance is ',num2str(S)]);
    if (k == 1)
        x = [-0.5:1:Nsum(k) * 8];
    else
        x = [-0.5:1:Nsum(k) * 4];
    end
    figure();

    scaledHist = histogram(xs, 'BinEdges', x, 'Normalization', 'pdf');
    % unpack values from histogram
    [vals, numBins, binCenters] = unpackHistogram(scaledHist);

    hold on;
    plot(x, fgauss(x,mu,sig2), 'LineWidth', 2);
    hold off;
    % grids and the other elements of a professional plot
    xlabel('Value of Y');
    ylabel('Probability Density of Y');
    grid on;
    legend('Random Variable Y', 'Theoretical Value of Y');
    title(['Probability Density of Y for N = ', num2str(Nsum(k)), ' in ',
 num2str(Ntrials), ' trials']);
end
%And again, with samples drawn from randx provided with Project 1.


%Problem 2.4  Sum of iid Bernoulli trials
disp('---------------');
disp('Section 2.4');
Nsum = [4,8,40];  % Checked with Project 2  40 is, in this case, a big number
p = 0.5;
for k = 1:length(Nsum)
    xd = rand(Nsum(k), Ntrials)<= p; % generates numbers between 0 and 1
    xs = sum(xd);
    mu = Nsum(k) * p; % value of p, probability of 1 appearing
    sig2 = Nsum(k) * p * (1 - p); % variance of bernoulli
    m = mean(xs);
    S = var(xs);
    disp(['For ',int2str(Ntrials),' independent trials of the sum of
 ',int2str(Nsum(k)),' iid rv from U(0,1)']);
```

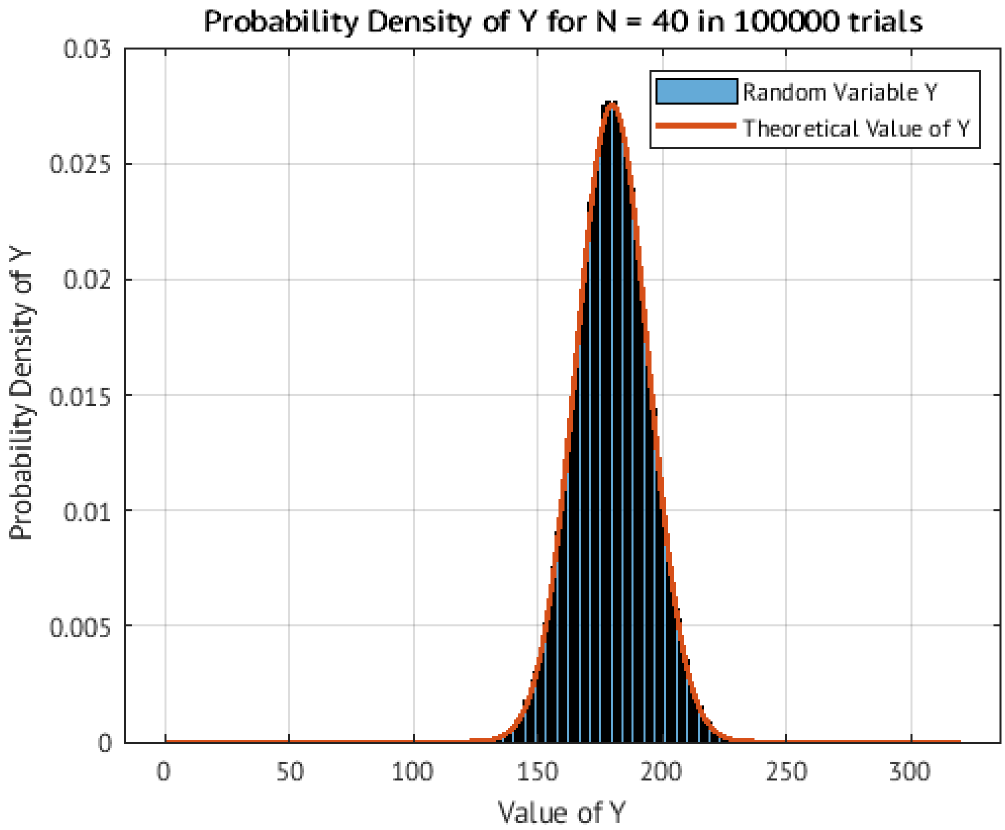
```
    disp(['    the theoretical mean is ',num2str(mu),' and the sample mean is
 ',num2str(m)]);
    disp(['    the theoretical variance is ',num2str(sig2),' and the sample
 variance is ',num2str(S)]);

    figure();
    subplot(2,1,1);
    % subplot 1
    x = [-0.5:Nsum(k) + 0.5];
    scaledHist = histogram(xs, 'BinEdges', x, 'Normalization'
 , 'probability');
    hold on;
    % the 0:Nsum(k) is equivalent to x (used as k for nchoosek) without having
 a negative value
    nchoosek = factorial(Nsum(k)) ./ (factorial(Nsum(k) - (0:Nsum(k))) .*
 factorial(0:Nsum(k)));
    pmfBern =  nchoosek .* p .^ (0:Nsum(k)) .* (1-p) .^ (Nsum(k) -
 (0:Nsum(k)));
    stem((0:Nsum(k)), pmfBern, 'LineWidth', 2);
    hold off;

    % grids and the other elements of a professional plot
    xlabel('Value of Y');
    ylabel('Probability of Y');
    grid on;
    legend('Random Variable Y', 'Theoretical Value of Y');
    title(['Probability of Y for N = ', num2str(Nsum(k)), ' in ',
 num2str(Ntrials), ' trials']);

    % subplot 2
    subplot(2,1,2);
    dx = 1;
    x = [-0.5:dx:Nsum(k) + 0.5];
    scaledHist = histogram(xs, 'BinEdges', x, 'Normalization', 'pdf');
    % unpack values from histogram
    [vals, numBins, binCenters] = unpackHistogram(scaledHist);

    hold on;
    plot(x, fgauss(x,mu,sig2), 'LineWidth', 2);
    hold off;
    % grids and the other elements of a professional plot
    xlabel('Value of Y');
    ylabel('Probability Density of Y');
    grid on;
    legend('Random Variable Y', 'Theoretical Value of Y');
    title(['Probability Density of Y for N = ', num2str(Nsum(k)), ' in ',
 num2str(Ntrials), ' trials']);
end
%Don't forget to plot both the exact (PMF) answer and the approximate
%Central Limit Theorem answer.  Use good practice (i.e., stem(x,y) for the
%PMF, but use plot(x,y) for the pdf.

---------------
```

Section 2.2

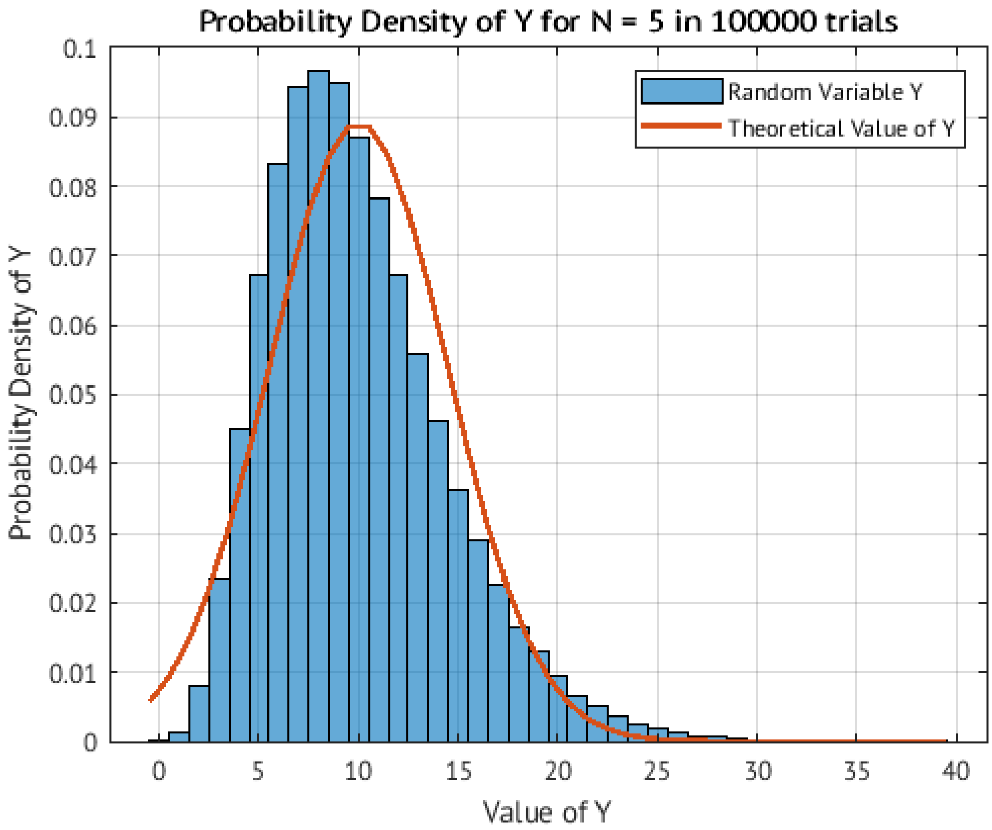
```
For 100000 independent trials of the sum of 2 iid rv from U(0,1)
   the theoretical mean is 9 and the sample mean is 8.9955
   the theoretical variance is 10.5 and the sample variance is 10.5242
For 100000 independent trials of the sum of 20 iid rv from U(0,1)
   the theoretical mean is 90 and the sample mean is 89.9882
   the theoretical variance is 105 and the sample variance is 104.471
For 100000 independent trials of the sum of 40 iid rv from U(0,1)
   the theoretical mean is 180 and the sample mean is 179.9561
   the theoretical variance is 210 and the sample variance is 210.4184
---------------
Section 2.3
For 100000 independent trials of the sum of 5 iid rv from U(0,1)
   the theoretical mean is 10 and the sample mean is 9.9991
   the theoretical variance is 20 and the sample variance is 19.9221
For 100000 independent trials of the sum of 50 iid rv from U(0,1)
   the theoretical mean is 100 and the sample mean is 99.9724
   the theoretical variance is 200 and the sample variance is 199.7858
For 100000 independent trials of the sum of 150 iid rv from U(0,1)
   the theoretical mean is 300 and the sample mean is 300.0927
   the theoretical variance is 600 and the sample variance is 599.0228
---------------
Section 2.4
For 100000 independent trials of the sum of 4 iid rv from U(0,1)
   the theoretical mean is 2 and the sample mean is 1.9977
   the theoretical variance is 1 and the sample variance is 1.0009
For 100000 independent trials of the sum of 8 iid rv from U(0,1)
   the theoretical mean is 4 and the sample mean is 3.9862
   the theoretical variance is 2 and the sample variance is 2.0087
For 100000 independent trials of the sum of 40 iid rv from U(0,1)
   the theoretical mean is 20 and the sample mean is 19.9894
   the theoretical variance is 10 and the sample variance is 9.9683
```


Probability Density of Y for N = 2 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
12
14
16

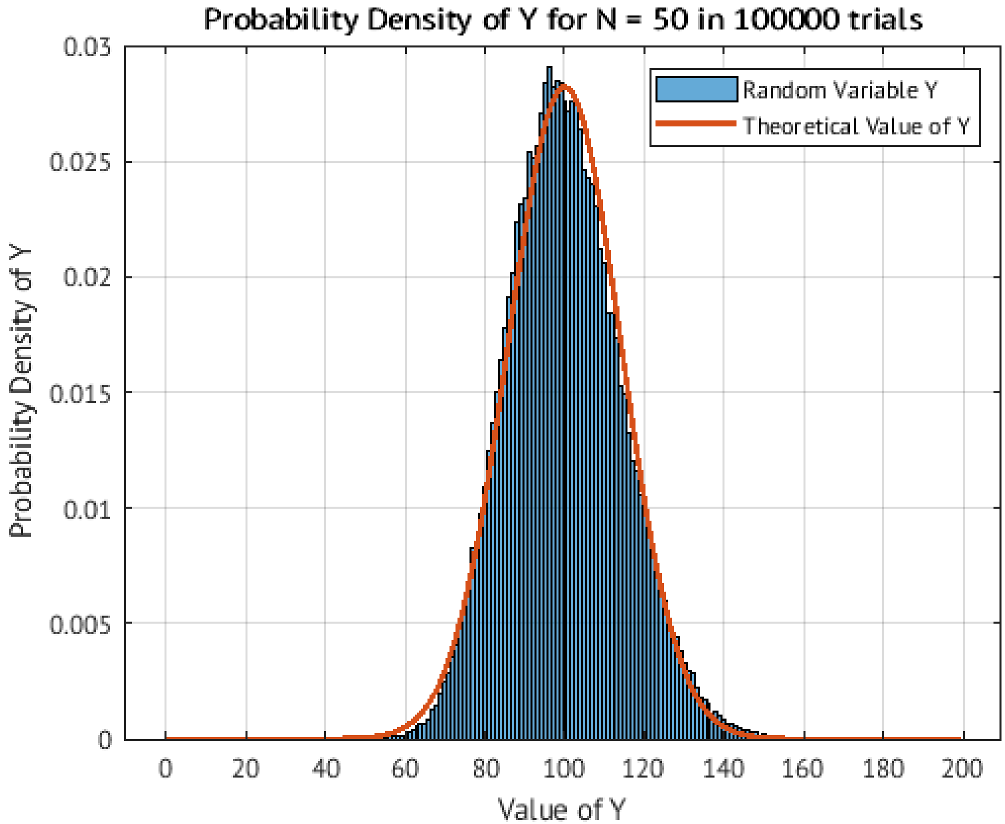

Probability Density of Y for N = 20 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
140
160

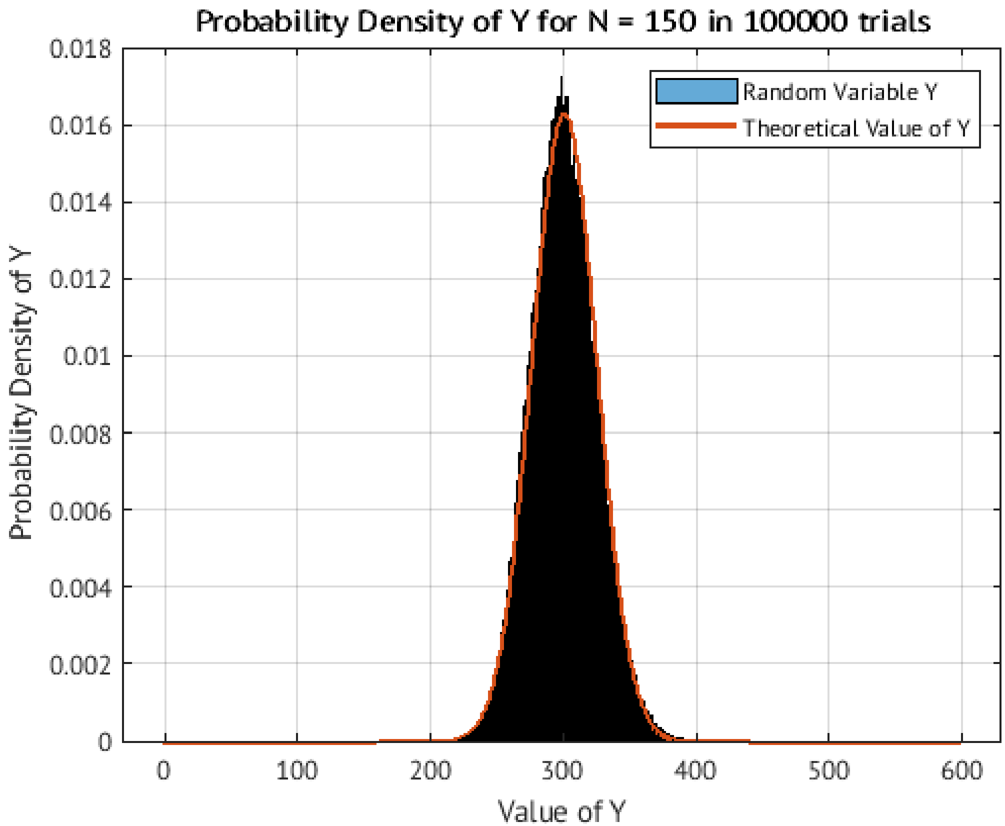


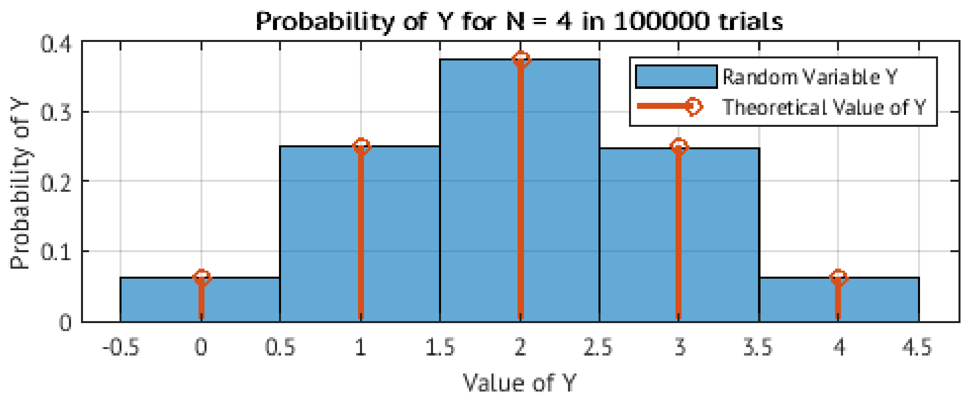
Probability Density of Y for N = 40 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y

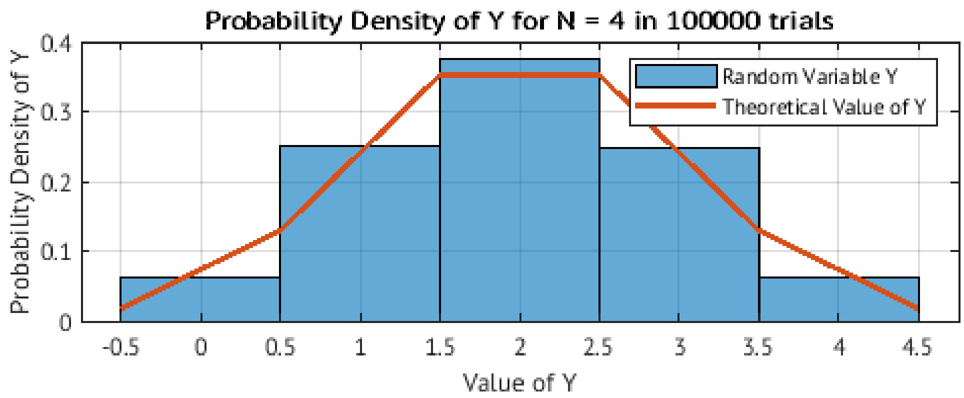


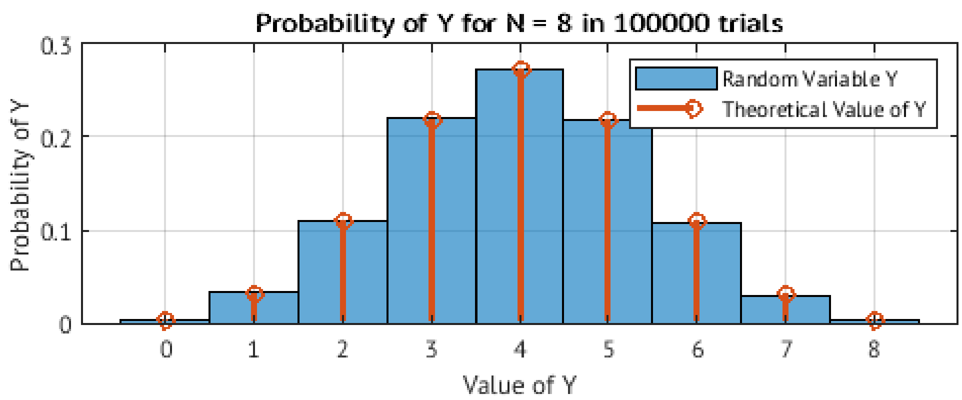
Probability Density of Y for N = 5 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y

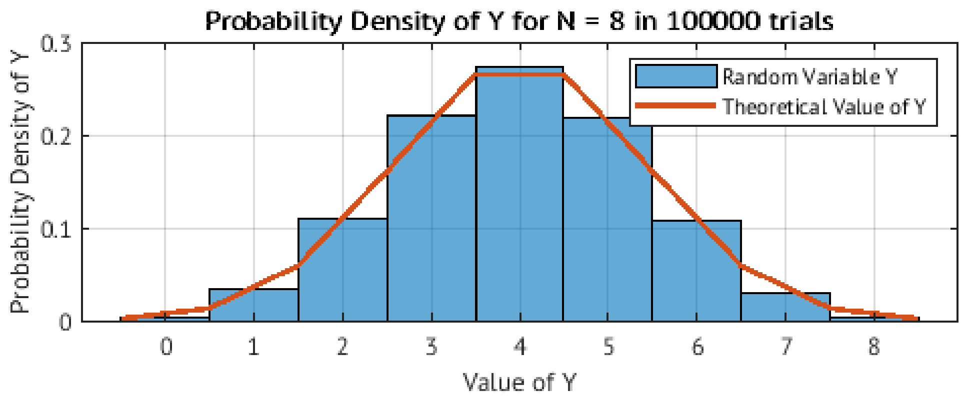

Probability Density of Y for N = 50 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
140
160
180
200

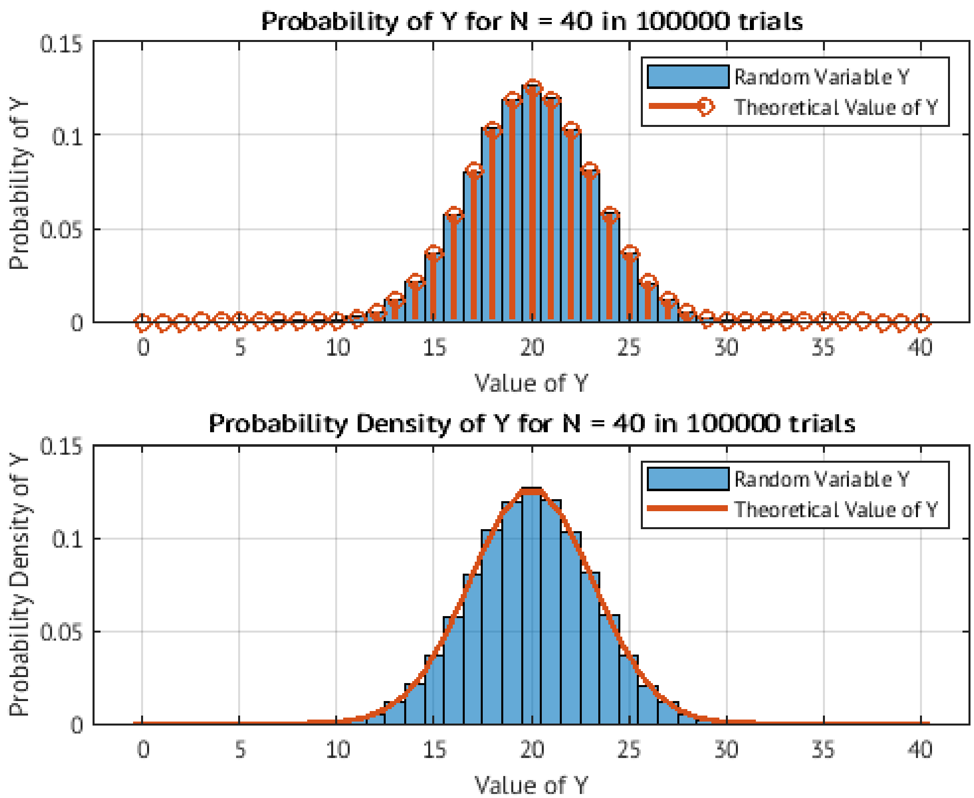

Probability Density of Y for N = 150 in 100000 trials
Random Variable Y
Theoretical Value of Y
Probability Density of Y
Value of Y
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
100
200
300
400
500
600


Probability of Y for N = 4 in 100000 trials
Probability of Y
Value of Y
Random Variable Y
Theoretical Value of Y


Probability Density of Y for N = 4 in 100000 trials
Probability Density of Y
Value of Y
Random Variable Y
Theoretical Value of Y


Probability of Y for N = 8 in 100000 trials
Probability of Y
Value of Y
Random Variable Y
Theoretical Value of Y


Probability Density of Y for N = 8 in 100000 trials
Probability Density of Y
Value of Y
Random Variable Y
Theoretical Value of Y



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