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3-D ATTITUDE REPRESENTATION OF HUMAN JOINTS: A STANDARDIZATION PROPOSAL*

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Abstract—In view of the singularities, asymmetries and other adverse properties of existing, three-dimensional definitions for joint and segment angles, the present paper proposes a new convention for unambiguous and easily interpretable, 3-D joint angles, based on the concept of the attitude 'vector' as derived from Euler's theorem. The suggested standard can be easily explained to non-mathematically trained clinicians, is readily implemented in software, and can be simply related to classical Cardanic/Eulerian angles. For 'planar' rotations about a coordinate system's axes, the proposed convention coincides with the Cardanic convention.

The attitude vector dispenses with the 'gimbal-lock' and non-orthogonality disadvantages of Cardanic/Eulerian conventions; therefore, its components have better metrical properties, and they are less sensitive to measurement errors and to coordinate system uncertainties than Cardanic/Eulerian angles.

A sensitivity analysis and a physical interpretation of the proposed standard are given, and some experimental results that demonstrate its advantages.

INTRODUCTION

In clinical movement analysis, there is an interest to depict 3-D joint and segment angulation in terms of three meaningful and independent quantifiers that denote, respectively, flexion/extension, ab/adduction and endo/exorotation. Regretfully, there does not appear to exist a consensus as to how this should be done. While purely 'planar' rotations with only one non-zero component do not seem to cause much debate, the representation of complex, 3-D angulation when two or three components occur simultaneously can give rise to fierce discussions within the biomechanics community (e.g. Andrews, 1984; Biomch-L, 1990, 1992; Grood *et al.*, 1981).

This situation is aggravated by a tendency in clinical circles to rely on kinematically awkward descriptions of 3-D attitudes, caused by an apparent generalization of the vectorial nature of positions and translations to attitudes and rotations, and by a desire to use a quantitative measure for what the physician is used to observe with the naked eye. For example, Sutherland *et al.* (1988, p. 65) have written:

> "We have chosen the familiar, laboratory-oriented planes of movement used by physicians and physical therapists rather than more complex concepts such as Eulerian movement[‡] which would be familiar only to engineers, mathematicians or physicists."

In this approach, joint angles are obtained by subtracting corresponding segment angles defined in the sagittal, coronal and transverse 'planes of movement'. While this is correct in purely planar movement, it generally yields inappropriate values in the 3-D, spatial case: even if a joint is not moved, change of the laboratory frame of reference with respect to which the segment angles are expressed will result in changes of the joint angles thus defined. Practically, this means that a subject who walks at an angle with respect to a defined X-direction will exhibit joint angles different from those that are found when he walks into the designated X-direction.

The explanation for the above fallacy lies in the non-vectorial behaviour of 3-D rotations: while translations and positions can be expressed in terms of vectors with the usual properties of additivity and commutativity, rotations and attitudes do not exhibit these mathematically and physically attractive properties unless they are very small. In the standard rigid-body transformation describing 'Eulerian movement', where $\mathbf{y}$ is the position coordinate vector of an arbitrary point in the global or proximal coordinate system and $\mathbf{x}$ the corresponding vector in the local or distal coordinate system,

$$\mathbf{y}=R\mathbf{x}+\mathbf{p},\quad R'R=RR'=I,\ |R|=+1. \qquad (1)$$

In equation (1), $\mathbf{p}$ is the local or distal origin's position vector in the global or proximal system, R is an

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‡*Eulerian movement* relates the motion of each segment (rigid body) to another or multiple other rigid bodies in space (Sutherland *et al.*, 1988, p. 65).

orthonormal attitude (orientation) matrix consisting of nine dependent parameters which can be described in terms of at most three independent quantities, and the prime denotes transposition. The term attitude is preferred over orientation since the latter term includes position in photogrammetric parlance, corresponding to the word pose in kinematics.

Given attitude and position parameters $\{R_p, \mathbf{p}_p\}$ for the proximal segment and $\{R_d, \mathbf{p}_d\}$ for the distal segment, where $\mathbf{x}_p$ and $\mathbf{x}_d$ refer to segment-imbedded, Cartesian coordinate systems, and $\mathbf{y}$ to an external, Cartesian coordinate system, the joint parameters $\{R_j, \mathbf{p}_j\}$, i.e. those describing the distal segment's orientation with respect to the proximal one, follow by repeated application of equation (1), with the appropriate substitutions and after elimination of the global coordinate vector $\mathbf{y}$,

$$R_j = R_p' R_d \qquad \mathbf{p}_j = R_p'(\mathbf{p}_d - \mathbf{p}_p). \tag{2}$$

Equation (2) demonstrates the generally multiplicative, non-linear character of the transformation from segment into joint parameters. Only for special cases and angular representations can the non-linear matrix multiplication be expressed in terms of addition or subtraction of rotation angles. In particular, the purely planar case, with rotations about a single coordinate axis, is an example.

Within the scope of this paper, the segment-imbedded and global, Cartesian coordinate systems are mostly assumed to be defined; this includes the choice of the coordinate systems' origins. While their definition is a difficult problem in its own, this paper's focus is on a parsimoneous and well-behaved representation of the segment's or joint's attitude (orientation) in terms of three quantities with sufficient metrical and physical interpretability, similar to the representation of 3-D position. However, the effect of coordinate system ambiguities on the positional and angular quantities is considered because this effect is closely related to the optimality of the angular quantities as such.

CARDANIC/EULERIAN ANGLES

A popular approach for representing the attitude matrix R in a set of three independent and physically realizable angles is based on Cardanic/Eulerian rotation sequences in which, starting from a reference attitude, a current segment or joint attitude is thought to be attained by an ordered sequence of rotations about the axes of a selected, Cartesian coordinate system (Blankevoort *et al.*, 1988; Chao, 1980; Grood and Suntay, 1983; Kadaba *et al.*, 1990; Pennock and Clark, 1990); this may be the global, external coordinate system (in $\mathbf{y}$) or the segment-imbedded, local coordinate system (in $\mathbf{x}$). The attitude matrix R in equation (1) can then be expressed in terms of an ordered matrix product of three elementary attitude matrices.

$$R_{ijk} = R_i(\phi_i) R_j(\phi_j) R_k(\phi_k), \quad ijk \text{ (anti)cyclic}, \qquad \boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)', \tag{3}$$

where i, j and k denote 'planar' rotations about the coordinate axes (1: x; 2: y; 3: z) of either coordinate system, with the component matrices defined as

$$R_1(\phi_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & -\sin\phi_1 \\ 0 & \sin\phi_1 & \cos\phi_1 \end{pmatrix},$$

$$R_2(\phi_2) = \begin{pmatrix} \cos\phi_2 & 0 & \sin\phi_2 \\ 0 & 1 & 0 \\ -\sin\phi_2 & 0 & \cos\phi_2 \end{pmatrix},$$

$$R_3(\phi_3) = \begin{pmatrix} \cos\phi_3 & -\sin\phi_3 & 0 \\ \sin\phi_3 & \cos\phi_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

The angle ϕ_i denotes a rotation of the x-system in equation (1) about the ith coordinate axis, in the right-handed screw sense.

For ijk (anti)cyclic, i.e. all three numbers are different, the name Cardanic (or Bryant) angles is used; for the convention with identical matrices, R_i at the begin and end of the product $R_i(\phi_i)R_j(\phi_j)R_k(\phi_k)$, the term Euler(ian) angles is customary, especially in the technical literature (Selvik, 1989; Wittenburg, 1977). Since Euler angles exhibit a singularity at the reference attitude, this convention—originally proposed by Euler (1748)—is often avoided. Furthermore, the convention $R_i(\phi_i)R_j(\phi_j)R_k(\phi_k)$ is not used as this reduces to $R_i(\phi_i+\phi_j)R_k(\phi_k)$, with merely ϕ_k and the sum $\phi_i+\phi_j$ identifiable from some, but not all, attitude matrices R.

Given a particular sequence ijk in equation (1), the attitude to be represented is attained by rotating from the reference attitude either in the temporal order ϕ_i about the ith axis, then ϕ_j about the jth axis, and finally ϕ_k about the kth axis of the coordinate system in which $\mathbf{x}$ is expressed, or in the reverse order, with opposite signs, i.e. first $-\phi_k$ about the kth axis, then $-\phi_j$ about the jth axis, and finally $-\phi_i$ about the ith axis of the coordinate system in which $\mathbf{y}$ is expressed. This 'reversal' effect follows from the property

$$R_{ijk}(\phi_i, \phi_j, \phi_k) = R_{kji}(-\phi_k, -\phi_j, -\phi_i)' \tag{5}$$

in combination with the fact that single rotations of the $\mathbf{x}$-system with respect to the $\mathbf{y}$-system are equivalent with opposite but equal rotations of the $\mathbf{y}$-system with respect to the $\mathbf{x}$-system.

It might be useful to point out that Cardanic/Eulerian rotation sequences are sometimes described in terms of a primary rotation about some Cartesian coordinate axis, followed by a second rotation about another, displaced coordinate axis, and by a final, third rotation about a doubly displaced coordinate axis. This is equivalent to a sequence of rotations about the axes of one coordinate system as seen by an observer in the other coordinate system. If the observer would be in the system about whose axes a rotation sequence is performed, the term 'displaced' would not apply.

While for a given attitude matrix R, the component values in the 'vector' $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)'$ vary only slightly with changes in the sequence ijk if $|\phi|\ll 1$ rad, significant differences are found if this condition is not met as discussed in detail below. This 'sequence effect' is a key problem in Cardanic/Eulerian attitude representation.

For example, if $\{\phi_1,\phi_2,\phi_3\}\hat{=}\{$flexion/extension, ab/adduction, endo/exorotation$\}$, the values $\{60°, 5°, -10°\}$ under the 123 sequence change into $\{59.63°, 11.15°, -0.72°\}$ under the 231 sequence. Thus, ab/adduction and endo/exorotation may be differently defined under strong knee flexion. However, the values $\{5°, 5°, -10°\}$ under a 123 sequence change into $\{5.05°, 5.86°, -9.53°\}$ under a 231 sequence. Thus, only small differences occur if the knee is almost fully extended (cf. Blankevoort *et al.*, 1988). The consequence of this sequence effect is that different values of 3-D joint angulation are calculated from identical attitudes, and this is likely to affect user training and comparison between reported, clinical data.

From a geometrical point of view, the term vector for $\boldsymbol{\phi}$ above is wrong, as the three angles do not behave in the linear, vectorial sense. However, the term is often used algebraically to denote a one-column matrix. Furthermore, (infinitesimally) small changes in these angles generally behave as vectors; such changes are considered in the following paragraphs.

Cardanic linkage systems

Grood and Suntay (1983) have proposed what they choose to call a 'sequence independent, oblique coordinate system' in which the current pose of a joint (orientation, i.e. position and attitude, cf. Sommer, 1991) is thought to be reached from a predefined reference pose via three elementary, helical displacements about and along an ordered sequence ijk of the axes i, j and k of an (electro)goniometric linkage system. The terminal axes i and k are each imbedded in one of the body segments comprising the joint, and they coincide with prior selected, Cartesian coordinate axes defined in these segments; the intermediate or 'floating' axis j is normal to the two imbedded axes, and coincides with the line of nodes in classical handbook descriptions of Eulerian/Cardanic angles. Except for special values of the intermediate rotation ϕ_j, the two terminal axes are not orthogonal. As regards the rotational aspect, such a linkage system is equivalent to a particular Cardanic/Eulerian sequence in the sense of equation (3).

That the 'terminal axes' are imbedded in the **x**- and **y**-systems of equation (1) follows from the elementary matrices in equation (3): the leftmost, elementary matrix describes a rotation about the ith axis of the **y**-system, while the rightmost, elementary matrix describes a rotation about the kth axis of the **x**-system. The 'floating axis', however, is tied to neither imbedded coordinate system because of the intervening, terminal rotation matrices.

Although temporal sequence dependency of Cardanic and Eulerian rotations is avoided in such goniometrically defined coordinate systems, a sequence effect is now imposed by the choice of imbedded and floating axes. The only difference is that, under the temporal view, rotations occur in a strict temporal order that correspond with a particular, physiologically not necessarily realizable path, while different paths are possible under the geometrical view. Thus, different numerical results may be obtained for specific joint attitudes (given identical segment coordinate systems but different choices for the floating and imbedded axes), and various adverse effects continue to occur, such as gimbal-lock and Codman's paradox discussed below.

The physical interpretation of Cardanic/Eulerian angles under the temporal and geometrical sequence views is straightforward, and many deem this an important asset of this convention, even though temporal, Cardanic sequences may be biomechanically nonrealizable. Indeed, the Committee on Standardization of the International Society of Biomechanics has published a draft proposal on kinematics standardization in which the Chao/Grood and Suntay convention is recommended (Cavanagh and Wu, 1992; ISB, 1992). In many cases, this convention provides reliable results, especially if the 'floating angle' does not become too large; this is the case in typical gait analyses, a major application of 3-D biokinematics, where the floating axis usually corresponds with joint ab/adduction. What happens in the opposite case (e.g. shoulder and complex segment movement) is discussed below.

Non-orthogonality and 'gimbal-lock'

For a given, cyclic convention $ijk = 123$, 231 or 312, the elements of the matrix product in equation (3) can be calculated as sums and differences of goniometric functions of the three chosen Cardanic angles. From a didactical point of view, it is useful to express them in a slightly different way:

$$\begin{aligned} r_{kj}+r_{ji}&=(1+\sin\phi_j)\sin(\phi_i+\phi_k),\\ r_{kj}-r_{ji}&=(1-\sin\phi_j)\sin(\phi_i-\phi_k), \end{aligned} \tag{6}$$

$$\begin{aligned} r_{jj}-r_{ki}&=(1+\sin\phi_j)\cos(\phi_i+\phi_k),\\ r_{jj}+r_{ki}&=(1-\sin\phi_j)\cos(\phi_i-\phi_k), \end{aligned} \tag{7}$$

$$r_{ik}=\sin\phi_j, \tag{8}$$

$$\begin{aligned} -r_{jk}r_{ii}-r_{ik}r_{ij}&=\cos^2\phi_j\sin(\phi_i+\phi_k),\\ r_{ik}r_{ij}-r_{jk}r_{ii}&=\cos^2\phi_j\sin(\phi_i-\phi_k), \end{aligned} \tag{9}$$

$$\begin{aligned} r_{ik}r_{ii}-r_{jk}r_{ij}&=\cos^2\phi_j\cos(\phi_i+\phi_k),\\ r_{jk}r_{ij}+r_{ik}r_{ii}&=\cos^2\phi_j\cos(\phi_i-\phi_k). \end{aligned} \tag{10}$$

If the sequence ijk is anticyclic, i.e. 321, 213 or 132, the cyclic case can be obtained via the intermediate transformation given by equation (5).

As is apparent from these equations, the sum or difference $\phi_i \pm \phi_k$ becomes undefined whenever $\sin\phi_j = \mp 1$. This is the mathematical explanation of the so-called *gimbal-lock* effect (Chao, 1980): in this situation, the terminal, mechanical axes of the equivalent linkage system are parallel, and only the sum or difference of the corresponding angles are defined. Furthermore, equations (6)–(10) have two solutions in the general, non-singular case: $\{\phi_i, \phi_j, \phi_k\}$ and $\{\pi+\phi_i, \pi-\phi_j, \pi+\phi_k\}$ (modulo 2π), a phenomenon known in anatomy as *Codman's paradox* (cf. Codman, 1934; Kapandji, 1982). Under the temporal sequence interpretation, Codman's paradox is the phenomenon that one may reach the same attitude of, say, the arm, from the neutral, anatomical pose either by flexing through 180° or by first abducting through 180° followed by 180° endorotation about the arm's longitudinal axis.

Sensitivity analysis

The imbedded axes under the geometrical interpretation of Cardanic sequences are non-orthogonal unless the intermediate or *floating* angle $\phi_j = n\pi$ ($n = \ldots, -2, -1, 0, 1, 2, \ldots$). If the angles are inexact because of noisy measurements, strong non-orthogonality will result in highly correlated, large errors in the angles ϕ_i and ϕ_k, and thus make meaningful interpretation of graphical joint angle representations difficult; in the extreme situation of gimbal-lock, correlation is perfect, and two of the three angles cannot be distinguished at all any more. Using the error model in Woltring *et al.* (1985) for isotropically distributed, multiplicative angular errors $\Delta\psi$,

$$\hat{R} = \{I + \mathscr{A}(\Delta\psi)\}R, \tag{11}$$

where the skew-symmetric matrix $\mathscr{A}(\mathbf{a})$ with corresponding axial vector $\mathbf{a}$ is defined as

$$\mathscr{A}(\mathbf{a}) = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \quad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \tag{12}$$

and where the error vector $\Delta\psi$ has covariance matrix

$$\Psi = \mathscr{E}(\Delta\psi\Delta\psi') = \sigma_\psi^2 I, \tag{13}$$

the covariance matrix Φ of the three Cardanic angles $(\phi_i, \phi_j, \phi_k)'$ can be derived as

$$\Phi = \frac{\partial\phi}{\partial\Delta\psi}\Psi\left(\frac{\partial\phi}{\partial\Delta\psi}\right)' = \sigma_\psi^2 \begin{pmatrix} \dfrac{1}{\cos^2\phi_j} & 0 & \dfrac{-\sin\phi_j}{\cos^2\phi_j} \\ 0 & 1 & 0 \\ \dfrac{-\sin\phi_j}{\cos^2\phi_j} & 0 & \dfrac{1}{\cos^2\phi_j} \end{pmatrix}. \tag{14}$$

Thus, for any $\phi_j \neq n\pi$ ($n = \ldots, -2, -1, 0, 1, 2, \ldots$), the variance of the two 'terminal' angles ϕ_i and ϕ_k will be larger than the variance of the 'floating' angle ϕ_j. If the floating angle is small, e.g. ab/adduction in knee movement, the effect is rather small; however, it may not be negligible in other joint movements such as in the hip and shoulder, and when representing general segment movement with respect to a laboratory reference system.

One may note that the error model above is realistic for more-or-less isotropically distributed landmarks and landmark errors in photogrammetric movement reconstruction, and for electromagnetic measurement systems. In other cases, the error distribution can be strongly non-isotropic, thus making the equations more complex; yet, similar results will be obtained.

About vectors and non-metrical spaces

The notion of an *axial* vector in equation (12) is one of the perplexities of vector representations in geometry and kinematics, and may serve as an introduction to the attitude vector proposed in the next section. More generally, axial vectors are called pseudovectors. They are characterized by the property that their signs change under an improper rotation, i.e. a coordinate transformation that includes a change from a left-handed to a right-handed coordinate system (or *vice versa*) as when reflecting in a mirror; real vectors do not change their sign.

For example, vector cross-products are pseudovectors if the individual vectors in the cross-product are true vectors. More specifically, coordinate transformation on a vector product is a distributive operation, i.e. $A(\mathbf{a}*\mathbf{b}) = (A\mathbf{a})*(A\mathbf{b})$ where A is a $3*3$ matrix (or tensor), if and only if A is a proper attitude matrix, i.e. $A'A = I$ and $\det(A) = +1$. If $A'A = I$ but $\det(A) = -1$, a sign change occurs as is simply demonstrated by choosing $A = -I$. In the general, affine case of non-metrical transformations where $A'A \neq I$, not only the sign but also the magnitude of the vector product is affected in a different way than the magnitudes of the constituent vectors; see Hoffmann (1966, Ch.5.12) for further comments. Torques and moments, rotation and acceleration velocity vectors, and the attitude vector proposed below are examples of pseudovectors.

When a vector is truly a vector, or how a vector should be properly defined has confused mathematicians and (bio) mechanicians from the beginning. Volkmann (1913, p. 99) gives the following account when he discusses the pseudovector character of moments:

> "This similarity* gives us the right to talk about the components and resultant of a turning moment in the same manner as we talk about the components and resultant of a force. We should, however, be aware of the fact that forces are vectors, while turning moments are not. (...) There are other instances in Physics where entities that are related to rotations are treated as vectors in the above

*In terms of the validity of a general coordinate transformation through multiplication with an attitude matrix.

sense. We wish to denote such entities as *rotators* in order to clarify their relation to the notion of rotation—in the manner of E. Wiechert."

Regretfully, no further reference was provided. The final words in Hoffman (1966) are illuminating, when he states that mathematicians often work with non-metrical spaces, in which vectors do not need to have magnitudes. Perhaps, the Cardanic system can be viewed as a very non-metrical space, because rotations as physically measurable about the axes of this system can be much larger than the total rotation of the moving object. In the next section, an alternative is presented which has better metrical properties and a reasonably clear, physical interpretation.

AN ATTITUDE 'VECTOR'

The Cardanic/Eulerian convention, while popular in standard handbooks and in various commercial systems for 3-D movement analysis, is certainly not the only possibility for parsimoniously describing 3-D attitudes and rotations. Spinors, Rodrigues parameters or Gibbs vectors, quaternions, and the attitude vector proposed in this section have been used in mathematical physics, aerospace navigation, and robotics because of various properties that Cardanic/Eulerian angles lack; see Altmann (1986, 1989) for a survey, including a historical review.

Instead of defining joint pose or movement in terms of a temporally or geometrically ordered sequence of three helical displacements, there are good reasons to view a current pose in terms of a single helical displacement (cf. Woltring *et al.*, 1985), to be decomposed into orthogonal components in either body segment's coordinate system which, apart from a sign inversion, appear to be identical (Woltring and Fioretti, 1989). For attitude representation (position representation is more complicated and, as said before, not discussed in this paper), one can define an attitude 'vector' $\boldsymbol{\theta}=\theta\mathbf{n}$, where $\mathbf{n}$ is the unit direction vector about which the scalar rotation θ occurs, in the right-handed rotation sense; see Fig. 1.

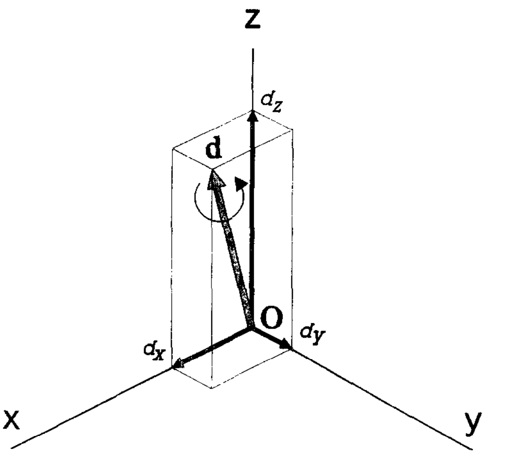

Fig. 1. Vectorial representation and decomposition of translations and rotations. A translation (of a reference point from a reference position) can be represented in terms of a displacement along a directed line in space; a rotation (from a reference attitude) can be similarly represented as a displacement about a directed line in space. If the direction is represented by a unit direction vector $\mathbf{n}$, and the displacement (whether translation or rotation) has amplitude d, the vectorial representation of the displacement is $\mathbf{d}=d\mathbf{n}$, and can be decomposed along the axes of any Cartesian coordinate system. For rotations, the right-handed 'corkscrew' convention applies.

Regularity and globality

Craig (1986, p. 46) calls $\boldsymbol{\theta}$ a compact 3×1 vector description of orientation. Given an attitude matrix R, $\boldsymbol{\theta}$ can be uniquely related to R as

$$R=\cos\theta I+\frac{\sin\theta}{\theta}\mathscr{A}(\boldsymbol{\theta})+\frac{1-\cos\theta}{\theta^2}\boldsymbol{\theta}\boldsymbol{\theta}', \quad \theta=\sqrt{\boldsymbol{\theta}'\boldsymbol{\theta}}, \tag{15}$$

where $0\leqslant\theta\leqslant\pi$ (modulo 2π), and where $\mathscr{A}(\boldsymbol{\theta})$ is the skew-symmetric matrix defined in equation (11). Equation (15) follows from the relations in Spoor and Veldpaus (1980) and Woltring *et al.* (1985), by substituting $\boldsymbol{\theta}=\theta\mathbf{n}$. For small angles, i.e. $\theta\ll 1$ rad, equation (15) reduces to

$$R\approx I+\mathscr{A}(\boldsymbol{\theta}). \tag{16}$$

One can show by construction that $\boldsymbol{\theta}$ is a pseudovector. Furthermore, $\boldsymbol{\theta}$ is an eigenvector of R, with corresponding, real eigenvalue $+1$; $\boldsymbol{\theta}$ does not exhibit the asymmetry between angles of the Cardanic representation where the flocking angle behaves quite differently from the two terminal angles, and there is no gimbal-lock or Codman's paradox. The existence of $\boldsymbol{\theta}$ is a direct consequence of its eigenvector nature by virtue of Euler's theorem (e.g. Wittenburg, 1977, p. 13):

> "Two arbitrarily oriented Cartesian coordinate systems with common origin can be made to coincide with each other by rotating one of them through a certain angle about an axis which is passing through the origin and which has the direction of the unit eigenvector $\mathbf{n}$."

Despite this classical theorem, it is unclear when the attitude vector concept was described for the first time. In a personal communication, Hoschek (1992) mentioned Lagally (1928) as an early source. More recently, workers in aerospace navigation (Bortz, 1971; Laning, 1949) and in robotics (Craig, 1986; Latombe and Mazer, 1980; Liegéois, 1984) have rediscovered the attitude vector, and an early use in biomechanics of Bortz's work was reported by Mital and King (1979).

Closely related to $\boldsymbol{\theta}$ are Rodrigues' parameters, the components of $\tan(\theta/2)\mathbf{n}$ for which Rodrigues (1840) reported a simple relation to 'add' attitude vectors; see Altmann (1986, 1989) for some interesting, historical details. For successive rotations characterized by Rodrigues parameters that are vectorially denoted as **a** and **b**, the Rodrigues parameters for the total attitude follow as

$$\mathbf{c} = \tan \tfrac{1}{2}\theta_c\, \mathbf{n}_c = (\mathbf{a} + \mathbf{b} - \mathbf{a} * \mathbf{b})/(1 - \mathbf{a}'\mathbf{b}), \quad |\mathbf{n}_c| = 1. \tag{17}$$

Note that the Rodrigues parameters utilize only half the total rotation angle, and that they become infinitely large if $\theta \to \pi$.

Unlike **n**, the vector $\boldsymbol{\theta}$ is well-determined from noisy measurements even for small θ; thus, it does not suffer from the four-component redundancy and noisiness drawbacks of using both θ and **n** as proposed by Shiavi *et al.* (1987) and Ramakrishnan and Kadaba (1991). A further advantage is that the attitude vector representation corresponds approximately with the mean value of all valid (i.e. no gimbal-lock) Cardanic representations once Codman's paradox is accounted for, by choosing that set of angles between $-\pi$ and $+\pi$ for which $|\boldsymbol{\theta}|$ is smallest.

The values of the attitude vector components and of the Cardanic angles can be mapped onto each other via the identity $R_{ijk} = R$ in equations (3) and (15). Furthermore, simply related to $\boldsymbol{\theta}$ are Hamiltonian quaternions **q** which are rather common in navigational attitude representations,

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} \theta_{n_1} \\ \theta_{n_2} \\ \theta_{n_3} \end{pmatrix},$$

$$\mathbf{q} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta_{n_1} \\ \sin\theta_{n_2} \\ \sin\theta_{n_3} \end{pmatrix}, \tag{18}$$

while lacking their redundancy disadvantage (four elements instead of three); see Altmann (1986, 1989) for further details. Sometimes, $\theta/2$ rather than θ is used when defining quaternions.

While the components of the attitude vector representation generally do not have a physical interpretation similar to those of the Cardanic convention, they are always defined, their axes are always orthogonal with respect to each other, and they are easily interpreted as the components of the rotation vector $\boldsymbol{\theta}$ in a designated, Cartesian coordinate system such as the global, external **y**-coordinate system or the local, segment-imbedded **x**-coordinate system in equation (1). Furthermore, changes in the attitude vector have a clear, physical interpretation as derived in the following section.

Equation (16) may help to understand the error model of equation (11): the true attitude is distributed by a differential rotation of the **x**-coordinate system in the **y**-coordinate system, where the attitude vector components of the disturbance are isotropically distributed; if their standard deviations are sufficiently small, the first-order approximation of this error model holds.

Sensitivity analysis

In a similar fashion as for Cardanic angles, the covariance matrix of the attitude vector can be derived as

$$\Theta = \frac{\partial \boldsymbol{\theta}}{\partial \Delta \boldsymbol{\psi}} \Psi \left(\frac{\partial \boldsymbol{\theta}}{\partial \Delta \boldsymbol{\psi}}\right)' = \sigma_\psi^2 \left(\mathbf{n}\mathbf{n}' + \left(\frac{\theta/2}{\sin\theta/2}\right)^2 (I - \mathbf{n}\mathbf{n}')\right). \tag{19}$$

It appears that Θ is always well-behaved: the term $((\theta/2)/(\sin\theta/2))^2$ limits to 1 for $\theta = 0$ and to $\frac{1}{4}\pi^2$ for $\theta = \pi$. This is a major advantage with respect to the asymmetric influence of the floating angle ϕ_j in the Cardanic case. In the small-angle case $\theta \ll 1$ rad, $\Theta \approx \Phi \approx \sigma_\psi^2 I$: in this situation, the Cardanic angles and the attitude vector components are identical and uncorrelated in a first-order approximation sense. However, if at least one of the angles differs significantly from zero, the covariance matrices are significantly different; this is even true if the attitude vector components and the Cardanic angles are identical ('planar' attitude matrices, with one angle non-zero and the other two equal to zero) unless the non-zero angle is small.

For a worst-case comparison, consider the situation where only the floating angle varies between 0 and 180°, while the other two angles remain zero. In this case, $\phi_j = \theta_j = \theta$, $\phi_i = \phi_k = \theta_i = \theta_k = 0$, and one can compare the diagonal elements of equations (14) and (33) on a term-by-term basis. It is sufficient to compare σ_{ϕ_i} and σ_{θ_i}, because the same results apply for ϕ_k and θ_k and because $\sigma_{\phi_j} = \sigma_{\theta_j} = \sigma_\psi$ in this case. Up to 90°, σ_{θ_i} grows from σ_ψ to $1.111\sigma_\psi$, while σ_{ϕ_i} goes from σ_ψ to infinity. Up to 140°, $\sigma_{\theta_i} \le \sigma_{\phi_i}$, and between 140 and 180°, $\sigma_{\theta_i}/\sigma_{\phi_i}$ increases with θ from 1 to $\pi/2$; see Fig. 2 for a graphical comparison.

Conversely, if only one of the terminal, Cardanic angles (ϕ_i, say) varies from 0 to 180°, the standard deviations of all Cardanic angles and σ_{θ_i} remain equal to σ_ψ, while σ_{θ_j} and σ_{θ_k} vary from σ_ψ to $(\pi/2)\sigma_\psi$. Thus, the other attitude vector components have a slightly larger standard deviation here, but as above not more than 11% for angles up to 90°. Under no circumstances can the standard deviation of any attitude vector component be larger than $\pi/2$ (1.571) times the standard deviation of some Cardanic angle; usually, its standard deviation will be smaller than the standard deviation of the noisiest Cardanic angle.

Derivatives and physical-metrical interpretation

Within the scope of this paper, emphasizing the 3-D representational aspects of joint and segment attitudes, derivatives are not directly relevant. However, they are needed in sensitivity analyses, and they can help to understand the non-vectorial nature of the attitude vector by describing its relation with the angular velocity vector (Woltring, 1991a). Furthermore, derivatives are useful to understand the phys-

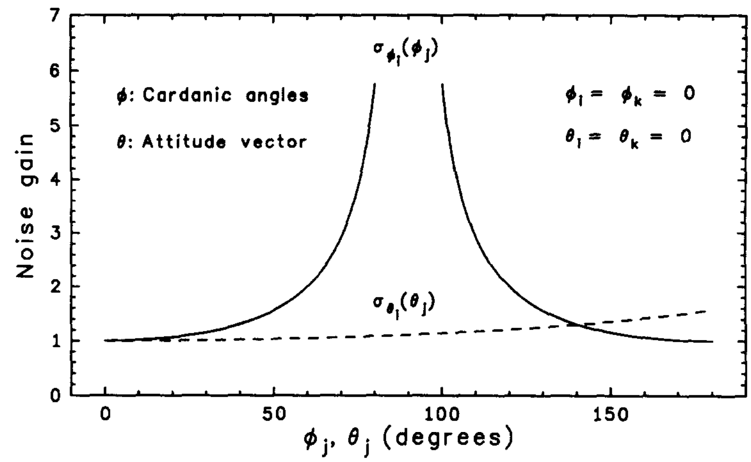

Fig. 2. Noise standard deviation amplification in the terminal angles of the Cardanic convention as a function of the floating angle and in the off-axis of the attitude vector as a function of the total attitude angle, if two of the three components are zero.

ical-metrical effects of small changes in the attitude vector.

Linear derivatives follow by straightforward differentiation of positional data, and angular velocity and acceleration in the planar case follow by direct differentiation of the scalar rotation angle θ. In the 3-D case, the non-vectorial nature of 3-D segment and joint attitudes renders assessment of rotational derivatives more complex, and this is sometimes overlooked by investigators who wish to generalize from planar to spatial analysis. In fact, Bottema and Roth (1979, Ch. VI. 3) have shown that it is fundamentally impossible to find attitude angles such that their derivatives are equal to the rotation velocity vector, by showing that the mixed second derivatives of such angles do not commute.

The rotation velocity vector $\boldsymbol{\omega}$ in the y-coordinate system of the rigid-body model given by equation (1) follows from the classical relation (Poisson's equation, cf. Wittenburg, 1977)

$$\mathscr{A}(\boldsymbol{\omega}) = \dot{R}R'. \tag{20}$$

By differentiating and substituting R as a function of $\boldsymbol{\theta}$, the linear relation

$$\boldsymbol{\omega} = Q(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{21}$$

can be derived between the attitude vector's time derivative $\dot{\boldsymbol{\theta}}$ and the rotation velocity vector $\boldsymbol{\omega}$, where

$$Q(\boldsymbol{\theta}) = \mathbf{nn}' + \frac{\sin\theta/2}{\theta/2}(R(\tfrac{1}{2}\boldsymbol{\theta}) - \mathbf{nn}'). \tag{22}$$

These relations re-emphasize that $\boldsymbol{\theta}$ is not a true vector in the geometrical sense, unlike the position vector $\mathbf{p}$ whose derivative is the translation velocity vector. Similar to $R(\boldsymbol{\theta})$ [equation (15)], $Q(\boldsymbol{\theta})$ is well-behaved for all θ; for small $\theta \ll 1$ rad, equation (21) becomes

$$Q(\boldsymbol{\theta}) \approx I + \tfrac{1}{2}\mathscr{A}(\boldsymbol{\theta}). \tag{23}$$

Thus, if $\theta \ll 1$ rad, $\boldsymbol{\theta}$ behaves as a pseudovector in a first-order approximation sense.

From equation (22), one can interpret the physical effects of small changes in the attitude vector components. Pure magnitude changes in $\boldsymbol{\theta}$ behave as incremental rotations about the attitude vector, with identical amplitude, while changes normal to $\boldsymbol{\theta}$ behave as small rotations, scaled by a factor $(\sin\theta/2)/(\theta/2)$, about the bissectrix between the attitude vector disturbances in both coordinate systems. This follows from the fact that only a rotation $\theta/2$ occurs in $Q(\boldsymbol{\theta})$, while the full rotation θ occurs in $R(\boldsymbol{\theta})$. The scaling of the normal components is limited, though: from 1.0 at $\theta=0$ to a factor $(\sin\pi/2)/(\pi/2) = 2/\pi = 0.65$ at $\theta=\pi$. For $\theta \leqslant \pi/2$, the scaling factor will decrease from 1.0 at $\theta=0$ to $\sqrt{8}/\pi \approx 0.9$ at $\theta=\pi/2$.

Small changes in only one attitude vector component occur about spatial axes that are defined by the columns of $Q(\boldsymbol{\theta})$, with the amount of scaling equal to the lengths of these columns; one can show numerically that these axes are reasonably orthonormal. An extreme case occurs if one attitude component, θ_j say, is zero, while the other two have equal magnitudes. One can define a crosstalk angle χ_{ik} as the deviation from 90° of the angle between the ith and kth columns of equation (22), which can be calculated from equation (22) as $\chi_{ik} = \pi/2 - 2\arctan(\sin\theta/2)/(\theta/2)$. Ideally, this crosstalk angle should be zero. For $\theta \leqslant \pi/2$, χ_{ik} varies between 0 and 6°, while the largest value is 25°, at $\theta=\pi$; see Fig. 3. For large changes in only one component of the attitude vector, the physical path is generally curvilinear about an attitude vector whose direction changes unless this change is along $\boldsymbol{\theta}$; this can be studied via an appropriate transformation of the Rodrigues formula in equation (17).

By contrast, changes in individual Cardanic angle have a constant scaling factor 1 in terms of their effect on changing the segment's or joints's attitude; however, the crosstalk angle between the terminal axes is a piecewise linear function of the floating angle (see Fig. 3), so crosstalk (dependence, correlation) between the terminal angles increases the closer the floating

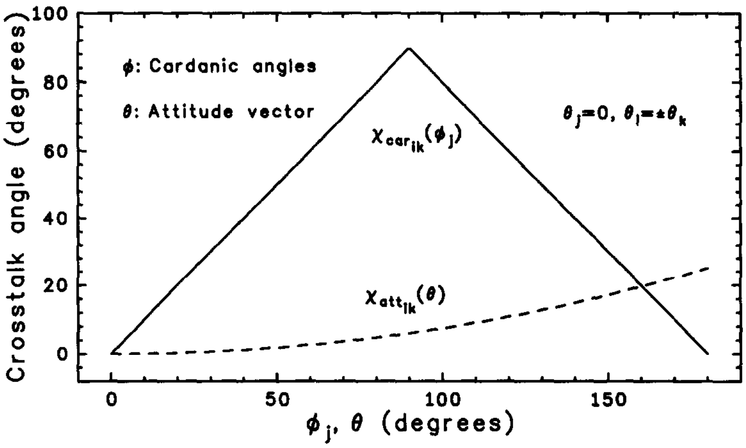

Fig. 3. Crosstalk angle in the Cardanic and attitude vector conventions.

angle is to gimbal-lock. Only for angles beyond 160°, the Cardanic angles show less crosstalk than the worst-case example for the attitude vector.

Summarizing: while changes in Cardanic (and Eulerian) angles can be physically understood as highly correlated about the imbedded and floating axes of the geometrical sequence interpretation unless the floating angle is small, and without scaling changes when individual angles are varied, changes in attitude vector components can be understood as reasonably independent and scale-invariant (metrical) about spatial axes which are, in a manner of speaking, halfway between the two Cartesian coordinate systems whose attitude changes with respect to each other are to be quantified; this halfway nature gives them an attractive joint angle character, as they are not tied to either segment in some biased way.

EXPERIMENTAL COMPARISON

The considerations in the preceding sections were largely theoretical. Fioretti *et al.* (1990)* have compared the calculated joint angle patterns of knee joint motion during healthy gait, for the attitude vector convention (less appropriately called 'helical angles' at the time) and for all six Cardanic conventions. Furthermore, they performed some coordinate system transformations on the femur- and tibia-imbedded coordinate systems in order to study the possibilities of dynamic coordinate system alignment and its influence on the calculated, angular graphs.

Data were collected with a CoSTEL opto-electronic system from body-mounted infrared LEDs, and the attitude matrices $\{R\}$ and position vectors $\{\mathbf{p}\}$ were assessed with the algorithms in Spoor and Veldpaus (1980). Cardanic angles and the attitude vector were calculated using the relations (6)–(10) and (15). Table 1 associates an index with the sequence code *ijk* of equation (3); index 0 corresponds with the attitude vector convention, while -3 corresponds with the 'joint coordinate system' convention of Grood and Suntay (1983). Furthermore, ab/adduction corresponds with the X- or 1-axis, flexion/extension with the Y- or 2-axis, and endo/exorotation with the Z- or 3-axis.

Results are relative to a rather slow walk, characterized by a cadence of 63 steps per minute at an average velocity of 0.48 m s^{-1}. The measurement errors in the 3-D reconstructed coordinates result in errors in the angles of the clusters quantifiable to 0.3°. This is the maximum standard deviation relative to the angles at the vertices of the LED cluster while the subject is walking along the experimental path.

Figures 4–6 show the knee angle trajectories calculated with the various parametrizations. Misalignments between the true anatomical system and the calculated one, giving rise mainly to exaggerated ab/adduction angle, were corrected by coordinate system transformations that either zeroed ab/adduction only (Fig. 5) of both ab/adduction and endo/exorotation (Fig. 6) at maximum flexion. For this purpose, the smallest helical rotation of both segments' imbedded coordinate systems with respect to the global system was chosen that achieved the required result. This approach, while functioning well from a numerical point of view, may be anatomically questionable; however, the palpation approach for identifying the joint's flexion/extension axis is open to similar criticism (cf. Kadaba *et al.*, 1990). Another explanation for the apparent crosstalk from flexion/extension into ab/adduction might be the ill-defined position of the hip joint's mean pivot along the (mean) flexion/extension axis. At least, the 18° ab/adduction found in the uncorrected data of Fig. 4 when maximum flexion occurs is likely to be wrong.

It is evident from the graphs that the flexion/extension angle has the smallest sensitivity to the chosen conventions in the sense that the various curves are

*With the authors' kind consent, this section was largely taken from their work.

Table 1. Index to angular parametrization mapping for Figs 4–6

Sequence *ijk*	213	132	321	θ	123	231	312
Index	−3	−2	−1	0	+1	+2	+3

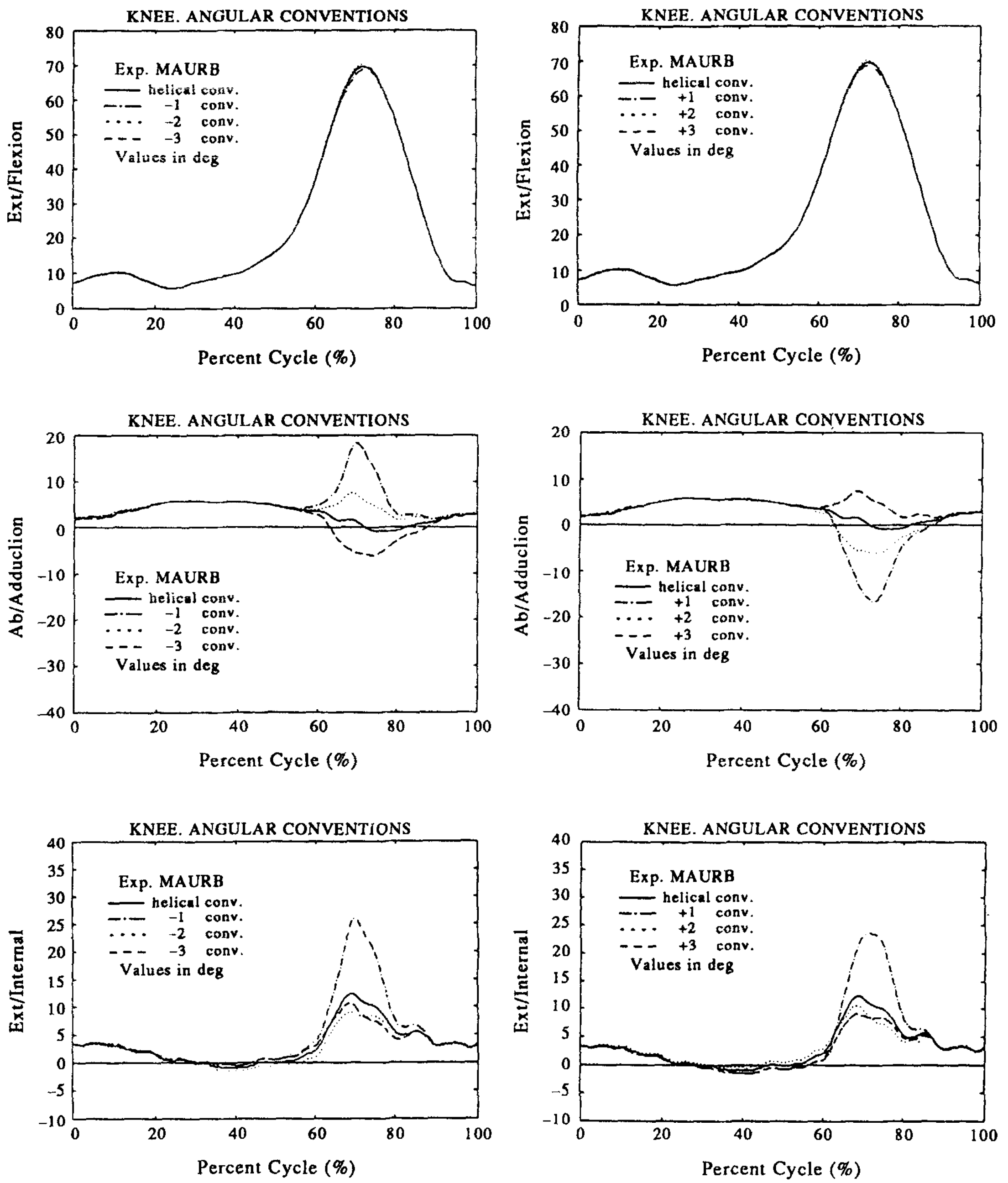

Fig. 4.

almost coincident. This is not true for the ab/adduction and endo/exorotation angles when the designated flexion angle is large. The former angles have limited values with respect to the flexion angle and are more or less sensitive to it (crosstalk), depending on the chosen convention. As expected from the preceding, theoretical paragraphs, this is particularly true for the −1 and +1 conventions characterized by a floating axis (Y) coincident with the designated flexion axis, about which the movement largely occurs. Here,

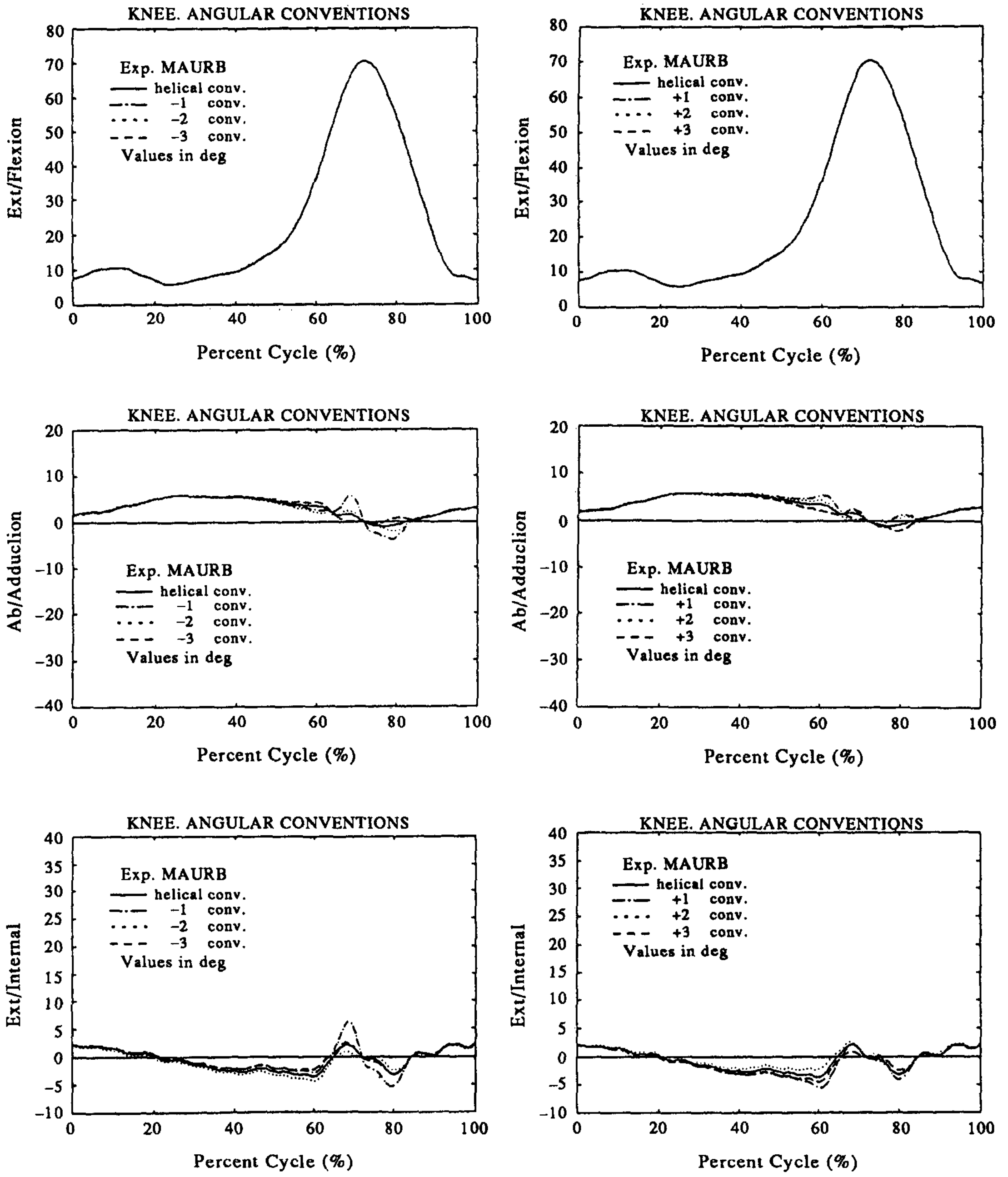

Fig. 5.

the strongly non-orthogonal relation between the equivalent, imbedded axes of a 3-D electrogoniometer explains in a physical manner the strong correlations between the angles that are used to quantify ab/adduction and endo/exorotation when the designated flexion angle is large.

Due to the limited ranges of ab/adduction and endo/exorotation, the -3 and $+2$ conventions and the -2 and $+3$ conventions produce almost identical results. Furthermore, the attitude vector produces results that are approximately average values of the results from the other conventions.

Finally, the corrected curves in Figs 5 and 6 indicate little or virtually no difference for flexion/extension, and rather similar trajectories for the other angles: in this correction, they have been intrinsically decorrelated, by forcing both to zero at maximum flexion.

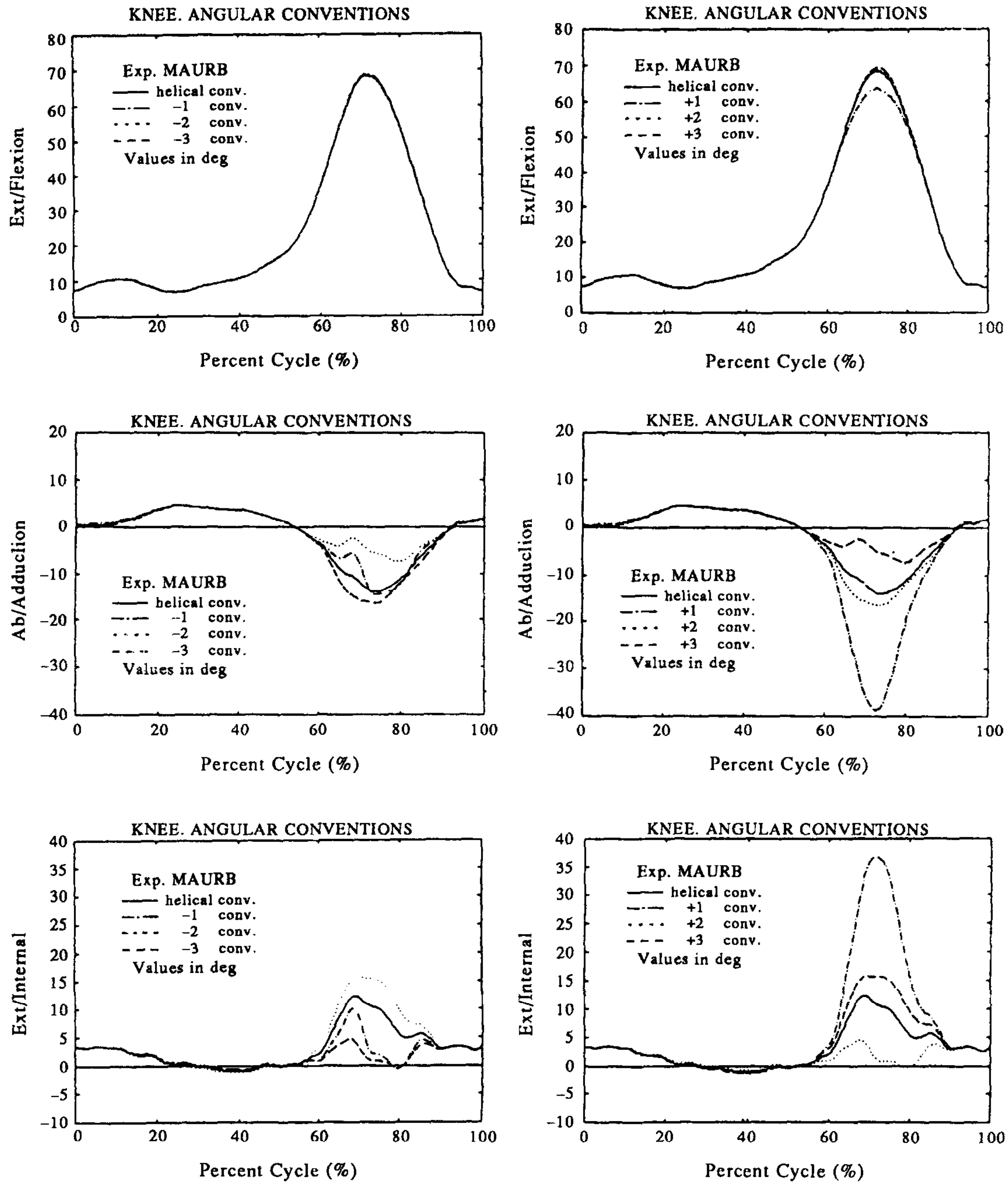

Fig. 6.

DISCUSSION AND IMPLEMENTATION

It is sometimes suggested that the interpretation of the Cardanic representation as a physically meaningful, ordered rotation sequence about the axes of a designated, Cartesian coordinate system or about the axes of an electrogoniometric linkage system is useful from a didactic point of view. Be that as it may, decompositions of 3-D translations, forces and moments are usually not explained in this way, but simply in terms of their (pseudo) vectorial nature. The point of relevance is (or should be) adequate representation rather than physically interpretable reconstruction of 3-D segment and joint angles. The challenge is with the 'engineers, mathematicians and physicists' of Sutherland *et al.* (1988) to explain and demonstrate the usefulness of appropriate angular representations to the clinical community. It may be useful to remark that it is difficult to identify the Cardanic convention's floating axis with a physiologically or mechanically meaningful, anatomical axis, unlike the other two segment-imbedded axes in the case of joint motion.

At first sight, the error propagation sensitivity issue may seem a purely technical argument which, for sufficiently accurate equipment and for attitudes sufficiently far away from gimbal-lock, may not be important. However, the error sensitivity provides immediate insight into the effect of small and orthogonal attitude changes of a moving segment or joint, because of the chosen error model given by equation (11). From a *representational* point of view, it seems important to require that the angular measures used to depict such changes should exhibit minimal, differential sensitivity to these changes. This is certainly the case of position changes, where all measures are equally sensitive and uncorrelated, since $\partial\hat{\mathbf{p}}/\partial\Delta\mathbf{p}=I$ in $\hat{\mathbf{p}}=\mathbf{p}+\Delta\mathbf{p}$; comparison between equations (14) and (19) shows that the attitude vector is better behaved in this respect than Cardanic angles unless the floating. Cardanic angle is small or larger than 140°. Even then, error sensitivity differences between the equivalent attitude vector and the Cardanic angle are rather small, as apparent from Fig. 2.

The present error propagation model can also be used to analyse the effect of uncertainties in the attitudes of the global and local coordinate systems: the definition of a particular rotation convention does not preclude the need to define adequate coordinate systems in the body segments and in the global reference frame. Thus, the rigid-body model represented by equation (1) can be expanded as

$$\mathbf{y}=\{I+\mathscr{A}(\Delta\boldsymbol{\psi}_y)\}R\{I+\mathscr{A}\{\Delta\boldsymbol{\psi}_x)\}'\mathbf{x}+\mathbf{p}+\Delta\mathbf{p}_y+R\Delta\mathbf{p}_x \tag{24}$$

with covariance matrices similar to equations (14) and (19). For example, if the angular errors $\Delta\boldsymbol{\psi}_y$ and $\Delta\boldsymbol{\psi}_x$ are thought to be uncorrelated, with covariance matrices $\sigma^2_{\psi_y}I$ and $\sigma^2_{\psi_x}I$, respectively, σ^2_{ψ} in equations (14) and (19) is replaced by $\sigma^2_{\phi x}+\sigma^2_{\phi y}$. Thus, angular uncertainties in the global and segment-imbedded, local coordinate systems will have a strong influence on the terminal, Cardanic angles whenever the floating angle is not small, while this effect is less prominent on the attitude vector.

The use of a Cardanic convention has been advocated as providing a true, albeit generally 'oblique', joint coordinate system, with the terminal axes each affixed in one of the adjacent segments that together comprise the joint, and with one 'floating' axis between these segments. From this point of view, the question is avoided with respect to what segment-imbedded coordinate system one should decompose other vectorial entities such as translations, forces and moments; thus, Cappozzo (1984) has proposed that decomposing the 'intersegmental couple vector' in this manner is useful for analysis of muscle and ligament function (which takes place across the joint). However, Cappozzo (1986) recommends the use of anatomical segment axes when emphasis is on structural analysis of a body segment. If the floating angle is large, the decomposition along the oblique axes of the 'joint coordinate system' will result in large correlations between the terminal components, e.g. flexion/extension and endo/exorotation in the neck or shoulder if ab/adduction is defined as the floating angle.

Apart from its sign, the attitude vector is identical in either segment's imbedded coordinate system; by a similar argument as for the Cardanic angles above, this follows from transposing the attitude matrix given by equation (15), where $R'(\boldsymbol{\theta})=R(-\boldsymbol{\theta})$. The components of (pseudo) vectorial entities such as velocities, forces and moments are orthogonal in either coordinate system, and related via the general coordinate transformation model. While this entails that joint forces, moments, rotation velocities, etc., will generally have different components when decomposed in either segment's coordinate system, this seems a lesser evil than increasing their interdependence by decomposing them along the axes of a generally oblique, non-Cartesian joint coordinate system with reduced metricality. At any rate, standardization proposals seem advisable for kinetic representations, too. Of course, scalar entities such as joint power should be the same when calculated from vectors in any coordinate system.

A FORTRAN-77 routine PRP FORTRAN implementing the Cardanic and attitude vector conventions (both forward and inverse) is available via BIOMCH-L, an electronic mail discussion forum on Biomechanics and Movement Science operated under LISTSERV@HEARN.BITNET or LISTSERV@NIC.SURFNET.NL at the University of Nijmegen, The Netherlands (van den Bogert *et al.*, 1992). *Retrievers of BIOMCH-L data are encouraged to discuss their findings on the BIOMCH-L forum, as a precursor to formal publications.

CONCLUSION

It seems reasonable to conclude that the attitude vector $\boldsymbol{\theta}$ is generally more suited for representing 3-D attitudes and rotations than Cardanic/Eulerian angles, by virtue of the orthogonality of the (anatomical) axes with respect to which its components are defined, of their global nature, and of their metrical properties at arbitrary attitudes. Only if the floating

*Subscription and file retrieval are free of charge, and file retrieval is for BIOMCH-L subscribers only in order to encourage discussion via BIOMCH-L on the results obtained with this and similar material. Send the requests *subscribe Biomch-L first-name last-name* for subscription, and *send prp fortran* for file retrieval (one request per line, *Subject*: line is irrelevant) to LISTSERV@HEARN.BITNET or LISTSERV@NIC.SURFNET.NL. Archive files for BIOMCH-L and other data are available by electronic mail; however, their availability cannot be guaranteed for indefinite periods. In October 1992, the subscriber count was 650 world-wide.

angle under a chosen Cardanic convention is always small, there is no significant improvement with respect to Cardanic angle. However, it is to be recommended that the same angular convention be used for all segments and joints within and between investigations; it is, therefore, proposed that the balance of these arguments biases towards the attitude vector. The slightly abstract nature of its components is not relevant from a metrical, representational point of view, but of a purely transient, didactic nature; besides, changes in these components can be easily interpreted in terms of physical rotations in a floating coordinate system approximately 'halfway' between the two imbedded, Cartesian coordinate systems needed to define both the attitude vector and the Cardanic approach.

Some recent publications are especially concerned with the definition of segment-imbedded coordinate systems (Kadaba *et al.*, 1990; Pennock and Clark, 1990). It would be useful if future studies relate their work to the representational issues of the current paper, in order to see which aspect is the more important one for establishing unambiguous and general standards for representing segment and joint movement including those cases where large 'floating angles' are found. Examples of the latter are segment movement in sports and joint movement in neck and shoulder under the conventional, Cardanic definition of Chao (1980), Grood and Suntay (1983), and others. Such studies might provide additional, objective arguments in support of the endeavours of the ISB Standardization and Terminology Committee.

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A precursor to this paper was presented at the *Fourth International Biomechanics Seminar* on 27–28 April 1990 in Gothenburgh, Sweden. Partial presentations of this material were given at the *First IOC World Congress on Sport Science* on 28 October–3 November 1989 (Woltring and Fioretti, 1989), at the *First International Symposium on 3-D Analysis of Human Movement* on 28–31 July 1991 in Montréal, Canada (Woltring, 1991a), and in Woltring (1991b).

APPENDIX A
INTRODUCTION IN THE CLINICAL FIELD

It may seem that the mathematical complexity of this paper might hinder the proposed convention to become accepted in clinical and ambulatory practice. Fortunately, there are a number of qualitative and practical considerations that may alleviate this educational task.

In the first place, there does not yet seem to exist an accepted standard on how to represent 3-D angulation; it is the responsibility of the biomechanical community, in consultation with the application field, to generalize currently accepted definitions of purely planar movement to compound 3-D movements. While one might opt for segment- and joint-specific definitions, these will be prone to confusion and may not allow straightforward correlation of movement data between joints and segments (e.g. the use of different Cardanic definitions for movement of a joint and for movement of the segments comprising that joint so as to avoid the pitfalls of gimbal-lock). It is hoped, therefore, that the present study may help in the current debate on standardization of kinematic data reporting (Cavanagh and Woo, 1992; ISB Standardization and Terminology Committee, 1992)—as opposed to kinematic data calculus.

Secondly, the notion of an attitude vector allows direct generalization from other vectorial entities. The attitude vector has a direction and length, just like any other vector, and may, therefore, be decomposed along the axes of any chosen coordinate system.

It is customary and, for reasons of maximal independence between the components, appropriate to choose a Cartesian coordinate system for such purposes, and not an oblique system as in the Cardanic linkage representation. While clinicians may sometimes have difficulties to understand the notion of vectors and of vectorial decomposition, there is a real need, with the onset of general 3-D movement analysis, that vectorial representations be understood in the field of clinical practice. This does not only obtain for positions and angles, but also for velocities, accelerations, forces and moments. Except for special cases, the complexities of 3-D kinematics and kinetics simply do not allow a straightforward generalization from three planar views.

Thirdly, the attitude vector convention allows decomposition along three defined anatomical axes in each of the two segments that comprise a joint; in contrast, the Cardanic convention allows decomposition along only two arbitrarily chosen, anatomical segment axes, one for each segment, plus one along a third, 'floating' axis which generally has no direct anatomical meaning. Also, the attitude vector's 'angles' are pairwise identical in the two segments, apart from their signs. While one might prefer conventions in which the longitudinal axis of oblong body segments is treated differently from the other two axes (e.g. Legnani and Faglia, 1991), there is no good biomechanical reason for such an approach, considering the complex interrelationships in 3-D kinematics and kinetics.

The use of measurement-theoretical considerations such as the error propagation and differential sensitivity arguments above may seem complex. One of these is the global coordinate system of longitude and latitude, where a position change of 1 km along the equator has a completely different effect on the longitudinal coordinate when compared with a similar change along a parallel near either pole. At the poles themselves, the notion of longitude has no meaning, similar to the situation at gimbal-lock in the Cardanic case, nor have the directions East and West. In planar representations of the global surface, the distortion in conventional maps due to scale changes near the two poles is well-known. As long as we stay away from the poles, these problems can be avoided.

The 'need' for physical reconstructability of joint angles by means of a Cardanic representation can be counterargued in a similar manner. The use of longitudes and latitudes does not have to be explained in terms of a physical displacement along the equator followed by a displacement along a meridian, or conversely by a displacement along a meridian followed by a displacement along a parallel: the representational aspect of a longitudinal/latitudinal pair of coordinates can be easily understood without such physical interpretations.

Of course, the above considerations are of a largely theoretical nature; the acceptance of any convention in practice can only take place after sufficient experimentation. It is, therefore, fortunate that a number of vendors have expressed an interest to implement the helical convention in their systems if there is sufficient interest from the market. This would allow the user to try out the various methods which, especially in a database context, will be highly useful. There is an increasing interest to compare individual movement data against normative curves, and to establish their deviations. For example, Gage (1991) states in his Epilogue:

> "As gait analysis become more widely used in the future, orthopaedic surgery will routinely be based on objective evidence of dysfunction and sound hypotheses. Pattern recognition, which we are already using in the treatment of hemiplegia (...), will be applied to diplegia as well. Existing statistical pattern-recognition techniques can be used to accomplish this, but very large data bases will be necessary. There will have to be cooperation between different treatment centers, with data pooling and common software formats for retrieval."

For this purpose, ranges of normality must be established, and the independence between selected measures will play an additional, important role. Up to now, normative databases have not addressed correlations between kinematic (and kinetic) movement quantifiers; this is one of their major limitations at the present time. Hopefully, the use of maximally independent joint 'angles' will be one small step into the right direction.

APPENDIX B

SELECTED REFEREE COMMENTS (Dr E. GROOD, CINCINATTI, OH, U.S.A) AND RESPONSE BY Dr WOLTRING ON THE FIRST (UNREVISED) VERSION OF THIS ARTICLE

General comments

In this paper, the author propses that the representation of three-dimensional rigid-body attitudes be standardized. The standard representation he proposes is referred to as the 'attitude vector'. This vector is the product of the screw rotation angle and the unit vector defining the orientation of the finite screw displacement axis.

The material in this paper would probably be easier to accept if the attitude vector was not being put forth as a standard, but simply as a useful way to describe rigid-body attitude. If it has significant benefits over other approaches, the community at large will adopt it and it will become a *de facto* standard. Even if the author decided to drop the standardization proposal, I would want the issues raised below answered prior to publication.

The major problem with the current proposal is the absence of a physical (geometric) interpretation of the three attitude vector *components* $(\theta_x, \theta_y, \theta_z)$ in terms of rigid-body rotations. The magnitude of a rotation is described by an angle between two lines. A rotation also has a definable rotation axis which is oriented normal to the plane formed by the lines which define the magnitude of the rotation. No such description is provided for the attitude vector *components*. Without this description one can only guess at their correct geometric interpretation.

The major problem with the current paper would be eliminated if the author can provide a clear mathematical and geometric description of the rotation axis for each of the three attitude components $(\theta_x, \theta_y, \theta_z)$. These axes would have the following property: given an initial attitude vector, a rotation about one of the three axes would change only the magnitude of one of the attitude vector components. I am confident that such a description exists for each attitude component. This confidence is based on the fact that all orientations and magnitudes of the attitude vector are possible. Thus, there must be two attitude vectors that differ only in one component. The difference between these two vectors will correspond to a rotation about the desired axis. I will refer to such axes as attitude component axes.

In addition to defining the orientation of the three attitude component axes, the author must also discuss their properties. Specifically, is the orientation of an attitude component axis constant, or does it depend on the magnitude of the rotation performed about it (all other components assumed to be constant during the motion)? If it changes, how are we to interpret this in terms of biological joint rotations? It is likely that the attitude component axes will not be along the coordinate directions used to define the attitude components. If this is the actual case, how should we interpret this fact?

Response: I believe that it is an author's prerogative to choose a stimulating title. By sowing wind, I have harvested a whirlwind, and the very choice of the term standardization proposal has had the beneficial effect of making the biomechanical community aware of certain possibilities. If you would prefer a more 'neutral' title, I will concur—with regret. I am not sure whether the community will accept this proposal purely because it is 'better' in some objective manner; there is a lot of inertia and vested interest in other approaches that various systems are based upon.

The manuscript discusses rather elaboratly whether metrical and physical interpretations are advisable and possible. In biomechanics, there are many abstract notions with limited or perhaps no physical meaning. Net joint forces and moments are examples of this. Yet, the fact that they are abstract, free-body entities has not deterred the community from using them, and from basing surgical and/or conservative decisions on them! If we use physically interpretable things like Cardan angles, we have the disadvantage of arbitrariness because of the sequence effect and strong correlations between representers in a varying oblique (affine) coordinate system. Those who disagree are at liberty to do so, and to write about this with a counterproposal for standardization. However, they should argue why they think that routine representability is less important than transient, didactic interpretability, and why interpretability is so difficult in some cases. Is not this a testimonium paupertatis in didactic quality?

The interpretation of changes in attitude vector components is now provided in the manuscript, following a recent debate on BIOMCH-L with the Referee.

Comment: I believe the standard terms for joint motions in the English language include internal and external rotation, not endo and exorotation. A paper on standardization should use standard terminology.

Response: I am not sure whether endo/exorotation is *not* common parlance in U.K./U.S./Canadian/Australian/etc. biomechanics. If I should write internal/external rotation, should I not also write medial/lateral rotation and forward/backward rotation? What do you think?

Comment: The sequence effect referred to comes from a general misunderstanding of Cardanic/Eulerian angles. I will first provide a background for my comments by discussing particle displacements in Cartesian coordinate systems. I will then use this background to analyse the rotational displacements described in the text of the paper.

Particle displacements

The position of a particle is described by a vector **r** in a fixed coordinate system and by the vector **r**′ in a second rotated coordinate system. For later reference let the x direction of the fixed system be identified with the medial–lateral direction of some biological joint. Note that each system has three degrees of freedom but the significance of the degrees of freedom are different in each system. Since the second system is rotated none of its axes correspond to the medial–lateral direction.

If both systems have the same origin then $\mathbf{r}' = \mathbf{R}\mathbf{r}$, where **R** is the orthonormal rotation matrix comprised of direction cosines between the axes of the two systems. The position coordinates of the particle are different in each system even though the same physical location is being described.

Consider now a displacement of the particle which is described by the vector **p** in the fixed system. The same physical displacement is described by $\mathbf{p}' = \mathbf{R}\mathbf{p}$ in the rotated system. Clearly, the components of the displacement are also different in each system even though the physical displacement represented is the same. Within each coordinate system the component displacements commute. That is, the final displcement can be achieved by sequentially displacing the particle along each coordinate direction in any order.

Rigid-body rotations

In the text the author presents two different rotational triples $\boldsymbol{\phi}$ and $\boldsymbol{\phi}'$ that correspond to the same rotation matrix **R**. Thus, the two triples describe the same physical rotational attitude. One difference between the triples is that one set corresponds to three planar rotations performed in a 123 sequence about the axes of the moving Cartesian system while the second set corresponds to a 231 sequence about the axes of the same Cartesian system.

The question I pose is how should we properly interpret these two attitude triples? Are their components described with respect to the different or the same rotational degrees of freedom? The first of these alternatives is that the components describe rotations about different degrees of freedom (i.e. different coordinate systems), and comparing them would be like comparing the components of **p** and **p**′ which also describe the same displacement, but with respect to different coordinate systems. In this case we would not expect the components to have the same values and the comparison made in the paper would not make sense. Specifically, we could not identify a single rotation, like internal/external, with coordinates of both triples any more than we could identify medial–lateral translation with the x components of both **p** and **p**′.

The second alternative is that the rotations are expressed with respect to the same degrees of freedom (i.e. the same coordinate system). In this case, the components of the two triples really do describe the same physical motions and identifying internal/external rotation with one of the components in both sets makes sense. To decide between these two views we must determine if the degrees of freedom are the same for the components of the two rotation triples $\boldsymbol{\phi}$ and $\boldsymbol{\phi}'$. The degrees of freedom for each component can be determined by identifying the rotation axes that includes rotation to change just that one component at a time. If the axes are different for the two triples we adopt the first view, i.e. the two triples are from different systems. If the axes are the same we adopt the second view, i.e. the triples are from the same system.

The analysis to determine the rotation axis which corresponds to each degree of freedom should be able to be applied at any position. Since the axes of the two Cartesian systems initially coincide, we apply the criteria in the displaced position to avoid the ambiguity of knowing whether the axis is part of the fixed system, the moving system, or neither.

When this analysis is complete it becomes clear that the axes which define each degree of freedom are different for the two triples. Thus, the triples correspond to different systems and it is not proper to identify the same joint rotation with components from both triples. These components are really geometrically different definitions of the joint rotations and should not be expected to have the same value even with the same joint position.

Response: This seems more a topic for a Letter to the Editor and/or for a counterpublication once (if...) the manuscript has been published. In essence, the Referee is constructive, while I am analytical: the Referee wants to get to B from A, I merely want to represent B with respect to A. Also, I would hope that the Referee will abandon his use of the term degrees of freedom.

Comment: It is not correct to describe the axes of the joint coordinate system presented by Fred Suntay and me as an "... ordered sequence *ijk* of the axes *i*, *j*, and *k*, of an electrogoniometric system." The three axes are not chosen in any ordered sequence any more than the axes of an orthogonal Cartesian system and the reference to an ordered sequence here is misleading and inappropriate. Further, the geometric definition of the axes is not in any way hard on an electrogoniometric system. They are simply three axes as described later on in the same paragraph, one fixed in each system and their common perpendicular.

Response: I beg to disagree. There is a sequence of a geometric nature: upstream/downstream or proximal/distal. One axis is selected in one-segment-imbedded (or global) coordinate system, another in the other (or local) segment's coordinate system, and the floating axis is defined as perpendicular to these two imbedded axes. The Referee may desire to confine the use of the term sequence to a temporal order, but it is my prerogative to generalize this notion in a geometrical sense, especially since these interpretations map onto each other in terms of equation (3). See the revised manuscript on path differences under the two interpretations.

Comment: The '... four-component redundancy...' noted is really an insignificant point. Many descriptions of three-dimensional rotations have redundancies. I have heard this described as an advantage.

Response: Occam's razor: From the point of view of parsimoniousness, four components *are* awkward, and even more so nine components if one were to plot all nine attitude matrix terms as a function of time. Without a reference provided in support of 'I have heard this described as an advantage', I cannot respond any further to this comment. I seem to recall that, in some of Sutherland's early work in San Diego, all nine elements of the attitude matrix for some joints were plotted as a function of time. In Shiavi's work, the unit direction vector components become very noisy whenever the total attitude angle is close to zero; in the attitude vector approach, this problem does not occur.