A Refined Technique to Calculate Finite Helical Axes From Rigid Body Trackers

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Finite helical axes (FHAs) are a potentially effective tool for joint kinematic analysis. Unfortunately, no straightforward guidelines exist for calculating accurate FHAs using prepackaged six degree-of-freedom (6DOF) rigid body trackers. Thus, this study aimed to: (1) describe a protocol for calculating FHA parameters from 6DOF rigid body trackers using the screw matrix and (2) to maximize the number of accurate FHAs generated from a given data set using a moving window analysis. Four Optotrak® *Smart Markers were used as the rigid body trackers, two moving and two fixed, at different distances from the hinge joint of a custom-machined jig. 6DOF pose information was generated from 51 static positions of the jig rotated and fixed in 0.5 deg increments up to 25 deg. Output metrics included the FHA direction cosines, the rotation about the FHA, the translation along the axis, and the intercept of the FHA with the plane normal to the jig's hinge joint. FHA metrics were calculated using the relative tracker rotation from the starting position, and using a moving window analysis to define a minimum acceptable rotational displacement between the moving tracker data points. Data analysis found all FHA rotations calculated from the starting position were within 0.15 deg of the prescribed jig rotation. FHA intercepts were most stable when determined using trackers closest to the hinge axis. Increasing the moving window size improved the FHA direction cosines and center of rotation accuracy. Window sizes larger than 2 deg had an intercept deviation of less than 1 mm. Furthermore, compared to the 0 deg window size, the 2 deg window had a 90% improvement in FHA intercept precision while generating almost an equivalent number of FHA axes. This work identified a solution to improve FHA calculations for biomechanical researchers looking to describe changes in 3D joint motion.* [DOI: 10.1115/1.4028413]

Keywords: finite helical axis, center of rotation, screw matrix, moving window analysis

1 Introduction

Kinematic algorithms are essential to understanding joint motion, as well as assessing the effects of pathologies and their related treatments on joint function and stability. One such technique, known as the FHA or screw displacement axis, defines the pose of an object in terms of a unique axis vector, coupled with a rotation about and a translation along the axis. These FHA parameters have been widely used to describe motion in the knee [1], spine [2], elbow [3], ankle [4], and wrist [5]. Most reported techniques to determine the FHA require a set of noncollinear markers affixed to an object, or a set of observable features on the object; however, these require complex vector algorithms and significant mathematical efforts to determine the optimal FHA parameters [5–8]. These methods are categorized as "vector observation" algorithms [9], which ultimately develop a screw [*S*] matrix from which the FHA parameters are extracted [6,10].

Over the last decade, there has been an increasing prevalence of tracking systems using prepackaged rigid body trackers that natively output 6DOF pose information (i.e., position and orientation) in the form of a 4×4 transformation [*T*] matrix. Generally, tracker pose is output relative to the tracker's global co-ordinate frame; however, with simple matrix multiplications, the [*T*] matrices of sequential poses can be transformed to represent the displacement of a tracker relative to itself. This [*T*] matrix is consistent with the [*S*] matrix produced by an FHA algorithm [10]. The nature of the [*S*] matrix and its simple derivation from [*T*] matrix output are salient concepts that have not been elucidated in the biomechanics literature. Further, the most effective tracker setup and analysis technique to determine FHA parameters from Optotrak (NDI, Waterloo, ON) rigid body trackers has not been previously investigated.

Challenges also still remain in determining accurate FHAs for investigations of joint kinematics due to inherent tracker error and vibration [8]. One available technique to improve the calculated FHAs is the use of a moving window analysis [11,12]. Crawford [11] suggested that an arbitrary window size of ± 10 data points spanning each FHA would reduce error; however, this window size may be ineffective for rapid movements or those that have inconsistent angular velocity. A more effective solution may be to traverse the collected data points, while evaluating the rotational displacement between the current tracker position and each subsequent position, until a prescribed minimum FHA rotation is achieved and only then accepting the calculated FHA as valid [12]. This method guarantees the desired rotational displacement (i.e., window size) and also maximizes the number of FHAs created, thus increasing measurement detail.

With an overall aim of producing a simple guideline for FHA implementation, the specific objectives of this study were: (1) to describe a protocol for calculating the screw matrix directly from a setup of 6DOF rigid body trackers and (2) to investigate a moving window analysis technique to generate the largest possible number of FHAs that accurately characterize the motion.

2 Materials and Methods

2.1 Mathematical Concepts. The matrix algebra syntax in this work follows the notation of Craig [13], where the leading sub- and superscripts indicate, respectively, the co-ordinate frame of an object with respect to a frame of reference, while the trailing subscript indicates qualifying information (i.e., description, time point, matrix dimensions, etc.). The position and orientation of an object *Body1* at an instant in time with respect to a reference frame *Body2* can be defined by a 4×4 [*T*] matrix, which is made up of a 3×3 direction cosine rotation [*R*] matrix and a 3×1 *x–y–z* position vector [*P*] (Eq. (1))

$$ {}^{Body2}_{Body1}[T]_{4\times4} = \begin{bmatrix} {}^{Body2}_{Body1}[R]_{3\times3} & {}^{Body2}[P]_{Body1,origin} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} \tag{1} $$

As *Body1* moves, a sequence of [*T*] matrices is defined. However, if the interest is in defining a series of FHAs, then displacement of *Body1* through two time points is required. Though not widely reported in the literature, Beggs [10] defines the relationship between the [*S*] and [*T*] matrices as

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$$ {}^{Body2}_{Body1}[T]_{time2} = {}^{Body2}_{Body1}[T]_{time1}[S]^{-1} \tag{2} $$

or rearranged for $[S]^{-1}$ as

$$ [S]^{-1} = {}^{Body1}_{Body2}[T]_{time1}\,{}^{Body2}_{Body1}[T]_{time2} = {}^{Body1,time1}_{Body1,time2}[T] \tag{3} $$

Now following a matrix inversion, and without any complex numerical algorithms, the $[S]$ matrix can be determined by simple matrix multiplication of two $[T]$ matrices. While both $[S]$ and $[T]$ use the direction cosine matrix to describe orientation, a key difference between the two matrices is the last column, which in $[T]$ defines the position of the object whereas in $[S]$ this is the displacement of the object. The FHA parameters relative to *Body1* are then calculated using the formulas described in the freely available ISB MATLAB code using the $[S]$ matrix as its input along with a defined intersection plane [6].

2.2 Experimental Testing. An OPTOTRAK CERTUS® motion capture system was used. The rigid body trackers were the OPTOTRAK Smart Markers, which contain a triad of noncollinear infrared light emitting diodes (reported accuracy of 0.1 mm). Data were captured using OPTOTRAK's First Principles™ software, recording the 6DOF information of the rigid bodies in $[T]$ matrix format.

A custom jig (CNC machined, 0.001 in. tolerance) that was capable of fixed planar rotations as small as 0.5 deg about a hinge joint (considered the Z axis) was used for this study. Two trackers were rigidly attached to the rotating portion of the jig and two more to a fixed portion (Fig. 1). The two moving trackers (referred to as *Body1_close* and *Body1_far*) were positioned approximately 6 cm and 10 cm away from the hinge axis, respectively. The fixed trackers were also positioned so that one was closer to the hinge axis (i.e., *Body2_close* and *Body2_far*). The camera was rigidly mounted to the wall during testing, approximately 4 m away from the trackers. Tracker data were recorded at 60 Hz and averaged over 2 s in 51 different static positions, generated as the jig planar rotated from 0 deg (neutral) to 25 deg in 0.5 deg increments. In each position, three different sets of $[T]$ matrices were generated for each of the two moving trackers: the moving tracker relative to each of the fixed trackers and to the camera (i.e., global reference frame).

2.3 Data Analysis. The $[S]$ matrices were calculated directly from $[T]$ matrices. From these, the FHA direction cosines, rotation about the FHA (which should match the induced rotation), the translation along the FHA (which should be zero for a pure rotation), and the location of the center of rotation (as measured by the X–Y intercept of the FHA with the $Z=0$ plane) were calculated. No data filtering was performed.

FHA metrics from the relative tracker rotation were initially evaluated for each rotational position as a displacement away from the neutral 0 deg position (i.e., 0–0.5 deg, 0–1 deg, etc.). With the understanding that FHA calculations are error-prone for small rotations, a second technique (moving window analysis) was evaluated as a means to maximize the number of acceptable FHAs from the motion dataset. This technique defined a minimum rotational displacement of the tracker, based on the calculated FHA rotation between an initial position data point and a subsequent position data point, which needed to be achieved before the calculated FHA was considered acceptable. For example, if the minimum rotation was set to 5 deg, then the FHA between 0 deg and 5 deg of displacement would be the first accepted. The initial data point would then be incremented to the next row of data and the process repeated in a loop, such that, for the 0.5 deg separation between data points in this study, the next FHA generated would correspond to 0.5 deg and 5.5 deg. The effect of window size was evaluated for minimum rotations of 0 deg to 10 deg (1 deg increments), where a 0 deg window size would calculate FHAs between adjacent tracker position data points regardless of the rotation between them.

3 Results

Based on displacement away from the neutral position, the rotations calculated about the FHA were within 0.15 deg of the prescribed rotation (0–25 deg in 0.5 deg increments) for all 50 rotations, with an average absolute difference between the calculated and prescribed rotations of 0.06 ± 0.04 deg. The calculated translation along the FHA for this method reached a maximum of

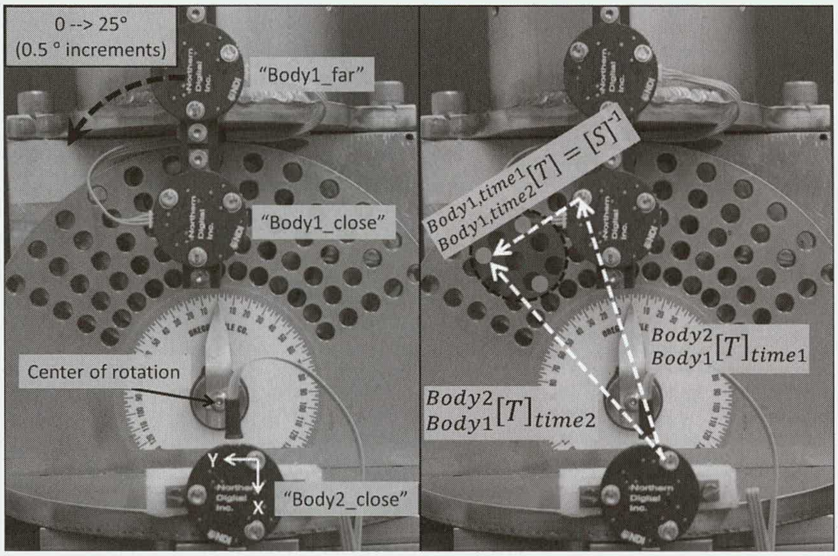

Fig. 1 Left: A custom machined jig that was capable of incremental planar rotations of 0.5 deg about a fixed hinge joint was used. Four OPTOTRAK Smart Markers (rigid body trackers) were attached, two to the moving portion and two to the fixed portion (*Body2_far* not shown). Right: Transformation $[t]$ matrices of a moving tracker *Body1* with respect a reference *Body2* (either a fixed tracker or the camera) were determined by the OPTOTRAK software. The screw $[s]$ matrix was then calculated for varying displacements (i.e., 1 $\rightarrow$ 2).

Table 1 ***X–Y* intercept stability based on tracker combination for the 0 deg window size**

Co-ordinate frames		Intercept standard deviation (mm)	
Reference	Moving	*X*	*Y*
Body2_far	*Body1_close*	6.6	24.9
Body2_far	*Body1_far*	12.5	26.5
Body2_close	*Body1_close*	8.4	7.9
Body2_close	*Body1_far*	12.3	11.7
Camera	*Body1_close*	5.9	29.1
Camera	*Body1_far*	9.5	30.9

0.73 mm in the final tracker position. The center of rotation and axis direction (which should be constant) demonstrated large fluctuations in the first few small displacements before eventually reaching a steady-state.

Analysis of the most effective tracker combination was based on the most stable (i.e., smallest standard deviation) FHA *X–Y* intercept for the 0 deg window size. Data analysis revealed that the smallest standard deviation was obtained using both the fixed and moving trackers positioned closer to the axis of rotation (Table 1). This combination outperformed trackers positioned further from the axis of rotation, as well as using the camera as the reference frame (i.e., precluding the fixed tracker).

Moving window analysis improved the axis direction and center of rotation accuracy with increasing window size (Table 2). For FHAs calculated using the "close" trackers, a window size of 2 deg or greater decreased standard deviation (<1 mm) and revealed an average center of rotation at 20.5, 65.8 mm in the *X–Y* plane, which matched the steady-state value identified in the displacement from neutral method. Further increases in the window size did not change or improve the average location, but did further shrink the standard deviation (i.e., increased precision). Root mean square error of the *X* and *Y* intercept was also evaluated based on a difference from the steady-state value calculated in the displacement from neutral (Fig. 2). Similar improvements were also seen in the direction cosines, where larger window sizes revealed the FHA was nearly identical to the *Z* axis (Table 2). In terms of an ability to maximize the number of accurate FHAs generated, comparing the 0 deg and 2 deg window sizes found that the 2 deg window had a 90% improvement in the FHA intercept precision while still generating 92% of the number of FHAs (46 versus 50). Rotation and translation FHA metrics for the moving window analysis, however, were a function of the window size, where the rotation about the FHA reported was equivalent to the minimum rotational displacement chosen. Calculated FHA translations were less than 0.2 mm for all window sizes examined.

4 Discussion

FHAs are a valuable tool in joint kinematic analysis. This study presented a refined technique to calculate accurate FHAs, and is useful as a guide to expedite the work of investigators using 6DOF rigid body trackers. Using the screw matrix and a common FHA parameter extraction technique, the OPTOTRAK Smart Markers were very effective for determining FHA rotations as small as 0.5 deg to within 0.15 deg [6]. When calculating the center of rotation, small rotations were still error-prone, but improved with application of the moving window analysis, such that a standard deviation of less than 1 mm for a minimum rotation of 2 deg or larger was achieved. As this window size was increased, there was a limited benefit to the accuracy, suggesting that a window size of 2–5 deg or higher would be appropriate for most biomechanical investigations, though joints with significantly larger ranges of motion could explore increased rotational displacement window sizes. The caveat here is that too large of a minimum rotation may cause for relevant kinematic data to be skipped. Ultimately, the take away point should be to use the smallest rotational displacement window that the data allows in order to obtain the most information about the motion pathway. For applications where the total range of motion is less than 2 deg, the current data suggests it would be challenging to recommend the FHA as a suitable technique.

A clear advantage of the moving window analysis evident in this work was that an almost equivalent number of FHAs, and with much greater accuracy, were created for the 2 deg window compared to the 0 deg window. While these results considered only the analysis of 51 static positions, applied to a continuously recorded joint motion data set, this could represent a significant improvement to the accuracy of a far greater number of calculated FHAs. This would be especially valuable in describing joints with small ranges of motion.

In regard to tracker setup, this study found that positioning the trackers closer to the center of rotation, for both the moving and fixed trackers, improved the intercept stability, which agrees with previous investigations [8]. This improvement may result from small errors in rotational position leading to exaggerated errors in center of rotation position at larger distances. Furthermore, the generated FHAs from the data set were more stable when calculated relative to a body reference tracker, as opposed to the camera. This is opposite to the findings of Duck et al. [14] who, using the FLOCK OF BIRDS® system (Ascension Technology, Milton, VT), concluded that measurement error stack-up was reduced by avoiding a reference tracker, and instead fixing the reference body segment to the tracker's global co-ordinate frame (i.e., electromagnetic transmitter). An equivalent setup is not practical with the OPTOTRAK CERTUS since its working volume begins at 1.5 m from the camera. Over this distance, vibration and building sway

Table 2 **Minimum moving window size effect on average *X–Y* intercept and the direction cosines of the FHA (±SD)**

Moving window size (deg)	FHAs created	Average *X* intercept (mm)	Average *Y* intercept (mm)	Average *x* direction cosine	Average *Y* direction cosine	Average *Z* direction cosine
0	50	20.2 ± 8.4	63.3 ± 7.9	0.019 ± 0.365	−0.005 ± 0.279	−0.895 ± 0.076
1	48	20.4 ± 1.6	65.0 ± 1.8	0.008 ± 0.130	0.000 ± 0.082	−0.987 ± 0.014
2	46	20.6 ± 0.9	65.5 ± 0.9	0.018 ± 0.088	−0.005 ± 0.052	−0.994 ± 0.006
3	44	20.4 ± 0.7	65.7 ± 0.5	0.029 ± 0.055	−0.005 ± 0.043	−0.997 ± 0.003
4	43	20.5 ± 0.5	65.8 ± 0.5	0.027 ± 0.045	−0.008 ± 0.034	−0.998 ± 0.002
5	40	20.5 ± 0.4	65.8 ± 0.4	0.028 ± 0.037	−0.005 ± 0.026	−0.999 ± 0.001
6	38	20.5 ± 0.4	65.8 ± 0.3	0.027 ± 0.027	−0.010 ± 0.017	−0.999 ± 0.001
7	36	20.5 ± 0.3	65.8 ± 0.2	0.028 ± 0.026	−0.009 ± 0.014	−0.999 ± 0.001
8	34	20.5 ± 0.3	65.8 ± 0.2	0.027 ± 0.023	−0.008 ± 0.015	−0.999 ± 0.001
9	33	20.5 ± 0.3	65.8 ± 0.2	0.027 ± 0.019	−0.007 ± 0.014	−0.999 ± 0.001
10	30	20.5 ± 0.2	65.8 ± 0.2	0.027 ± 0.020	−0.005 ± 0.013	−0.999 ± 0.001

Note: (1) The "moving window size" represents the minimum rotation that must be observed prior to a FHA being generated. In the case of 0 deg, this means that no minimum is imposed and thus all potential FHAs are considered. (2) All data are reported relative to the *Body1_close* tracker, where the standard deviation reported is based on the number of FHAs generated.

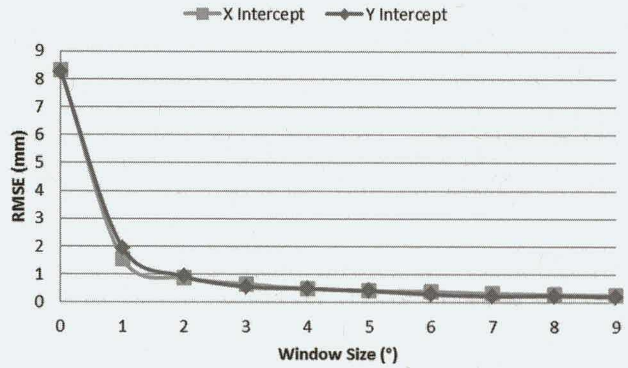

Fig. 2 Root mean square error (RMSE) of the *X–Y* intercept position in millimeters versus the moving window size (in degrees)

introduce relative movements between the camera and trackers on the specimen [15]. Although the base of the jig used in this study was fixed to ground and relative to the camera, the algorithm is compatible with a joint system that moves relative to the camera. Use of the reference tracker on the specimen cancels out gross movements of the specimen relative to the camera, thus isolating the joint motion. This was achieved by a simple co-ordinate transformation to set the reference tracker as the *Body1* frame for all the pose [T] matrices.

As the intended application of this work is for joint kinematics, further treatment is required to calculate FHAs relative to an anatomic frame. Bone co-ordinate frames need to be defined relative to their body-fixed trackers. This is usually done through digitization—measuring the locations of bony landmarks relative to the tracker's frame with a calibrated stylus. Next, all tracker poses from the motion must be transformed to their corresponding bone co-ordinate frame. The data are now ready to be input into Eq. (3), with *Body1* set as this bone frame.

The topic of filtering was not included in this investigation, as the focus of this work was to take the reader quickly from native tracker output to final joint FHA parameters. Filtering kinematic data has been shown by some to improve FHA accuracy for joint biomechanics and should be considered as an additional technique to reduce FHA error [14,16]. Furthermore, a limitation of this study was that only static measurements were made. However, this was done to control known tracker positions and the number of time points. The hinge motion of the jig was also likely not representative of most biomechanics studies, yet for this investigation, provided a reliable and stable axis of rotation. Finally, while the moving window analysis was very effective for determining center of rotation and axis direction, this method would only report a FHA rotation equivalent to the defined window size, so studies concerned with reporting the true rotation angle away from a defined neutral position should consider additional calculations for these specific displacements.

In conclusion, this work presented a simple but effective starting point for researchers looking to readily calculate FHAs from rigid body trackers. Furthermore, the accuracy of the FHA parameters produced can be improved with moving window analysis.

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