

1. Write the following monomials from $\mathbb{C}[X_1, X_2, X_3, X_4]$ increasingly with respect to the lexicographic order:

$$X_1^5 X_2^4 X_3^3, X_1^4 X_2^5 X_3^3, X_1^3 X_2^4 X_3^5, X_1^5 X_3^7, X_1^5 X_2^4 X_3^3.$$

2. Decide whether the following polynomials are symmetric:

- a) $X_1^2 X_2^2 + X_2^2 X_3^2 + X_3^2 X_1^2 \in \mathbb{Q}[X_1, X_2, X_3, X_4]$
- b) $(X_1^2 - X_2^2)(X_2^2 - X_3^2)(X_3^2 - X_1^2) \in \mathbb{R}[X_1, X_2, X_3]$
- c) $(X_1 - X_2)^6 (X_2 - X_3)^6 (X_3 - X_1)^6 \in \mathbb{C}[X_1, X_2, X_3]$

3. Write the polynomial

$$X^3 + Y^3 + Z^3 + T^3 + U^3 \in \mathbb{Q}[X, Y, Z, T, U]$$

as a polynomial of the fundamental symmetric polynomials.

4. Organize $\mathcal{M}_{2,3}(\mathbb{R})$ as a vector space over \mathbb{R} .

5. Is it possible to organize $(\mathbb{Z}[i], +)$ as a vector space over \mathbb{Q} ?

6. Show that:

- a) $\{(x, y, z, t) \in \mathbb{C}^4 \mid 3x = iz\} \leq_{\mathbb{C}} \mathbb{C}^4$.
- b) $\{A \in \mathcal{M}_n(\mathbb{R}) \mid A = A^t\} \leq_{\mathbb{R}} M_n(\mathbb{R})$.