Combinatorics is the mathematical field that concerns with counting all possible arrangements of a group of objects, given certain constraints. For example: “In how many possible ways can a deck of cards be shuffled?”. To which the answer is 52!. One could add the constraint “… if the first card is an ace of spades.”, after which the answer changes to 51!. These counting problems are closely related to probability, but have discrete solutions, rather than percentages.

Humans are quite good at solving combinatorics problems as they recognize commonly reoccurring structures in these problems for which there exist enclosed formulas.

Before continuing, let’s discuss a general framework for characterizing the 12 most common enumerative problems between 2 finite sets, known as “The Twelvefold Way”. Each problem can be calculated using a mathematical formula. Between the mentioned sets exists a function f: X -> Y, which can be injective, surjective or neither. To understand this intuitively, let’s think of the domain as a bag of balls and the image as a collection of boxes. Then, a function is equivalent with putting every ball in one of the boxes. If every box must contain at least 1 ball, the function is surjective. If each box may only have at most 1 ball , the function is injective. When none of these criteria is met, the function is neither.

Next, we can also categorize the problems of The Twelvefold Way by distinguishably. The sets X and Y can be both distinguishable (**≠)**, only X can be distinguishable (and X indistinguishable) (**=X**), only Y can be distinguishable(**=Y**), or they can be both indistinguishable (**=**). This grouping can again be explained using the balls & boxes example. If a set is said to be distinguishable, then the balls (X) or the boxes (Y) have properties that make them different from the rest. An easy way is to visualize the balls or boxes to have different colours. This way, we need to count a separate case for putting a ball in a red box, versus putting it in a blue box. If a set is indistinguishable, however, then each ball (X) or box (Y) has the same monotone color and putting ball A in box B is the same case as putting ball A in box C.

**Glossary**

**Reasoning Algorithm:** generates conclusions from known facts by logical techniques (induction, deduction, …).

**Lifted Reasoning = Lifted Inference:** exploiting symmetries (redundancies) to speed up reasoning algorithms.

https://en.wikipedia.org/wiki/Combinatorics

https://en.wikipedia.org/wiki/Twelvefold\_way