CS5340 : Assignment 2

Due on 6:30pm Wednesday 26 Oct. Turn in the hardcopy at the begining of the class, as well as ellectronically at IVLE.

Problem 1

(30/100)

1. Evaluate the entropy of a multivariate Gaussian $\mathcal{N}(x|\mu, \Sigma)$.
2. Evaluate the KL divergence (relative entropy) between two Gaussians $p(x) = \mathcal{N}(x|\mu_p, \Sigma_p)$ and $q(x) = \mathcal{N}(x|\mu_q, \Sigma_q)$.
3. In class we know that the exponential family is of the form

$$p_w(x) = \exp\left\{w^T \Phi(x) - A(w)\right\}.$$

In particular, the log-partition Function $A(w)$ is a cumulant Generating Function of the sufficient statistics $\Phi(x)$. Show that by taking its first and second derivatives, we obtain the first and second order moments (i.e. expectation and covariance).

Problem 2

(20/100) Consider two discrete variables x and y with each having three possible states: for example, $x, y \in \{0, 1, 2\}$. **Construct** a joint distribution $p(x, y)$ over these variables having the property that the value x^* that maximizes the marginal $p(x)$, along with the value y^* that maximizes the marginal $p(y)$, together have probability zero under the joint distribution, i.e., $p(x^*, y^*) = 0$.

Problem 3

(50/100) Implement the sum-product algorithm for an arbitrary tree-structured graph. Please document and hand in your code, including brief descriptions of the data structures that you define and steps involved. Run your algorithm on the tree in the following figure, using the edge potential functions

$$\psi_{st}(x_s, x_t) = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$

for all edges (s, t), and the node potential functions

$$\psi_s(x_s) = \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} \quad \text{for } s = 1, 3, 5 \text{ and } \psi_s(x_s) = \begin{pmatrix} 0.1 \\ 0.9 \end{pmatrix} \quad \text{otherwise.}$$

Report the values of the single node marginals for all nodes.