DIGITAL ELECTRONICS AND LOGIC

INTRODUCTION

Digital electronics evolved from the principle that transistor circuitry could easily be designed and fabricated to output one of the two voltage levels based on the levels placed at its input. The two distinct levels (usually 5V and 0V) can be represented by 1 and 0.

Numerical Representation

There are two ways of representing the numerical values of quantities: analog and digital

(i) Analog Representation

- It's a numerical representation in which a quantity is represented by a voltage, current, or meter movement that is proportion to the value of that quantity e.g. car speedometer, the electric iron thermostat, audio microphone.

(ii) Digital Representation

- It's a representation in which a quantity is represented by the symbols called digits e.g. digital watch. That is, digital representation represents discrete quantities or changes in discrete steps.

Advantages of digital techniques

Some of the reasons for shifting from analog to digital are:

a) **Digital systems are easier to design** – this is due the fact that circuits which are used in digital systems are switching circuits thus easy to design.
b) **Storage of information is easy** – this is accomplished by special switching circuits that can latch onto information and hold it for a time as long as necessary.
c) **Greater accuracy and precision** – these systems have a capability to handle as many digits of precision as needed by simply adding more switching circuits.
d) **Operation can be controlled by a program** – it's easy to design digital systems whose operation is controlled by set of stored instruction called a program.
e) **Digital systems are less affected by noise** – unwanted fluctuations (or noise) in voltage are not as critical in digital as it is in analog systems. This is because in digital systems, the exact value of the voltage is not important.
f) More digital circuitry can be fabricated on IC devices

Disadvantages of Digital techniques

There is only one major drawback with the digital techniques, that the real-world problems are analog. Thus to take advantage of digital techniques when handling analog quantities, the following three steps are used:-

a) Convert the real-world inputs to digital form
b) Process the digital information
c) Convert the digital outputs back to real-world analog form

As an illustration, a flow rate measurement and control system is as shown in fig. 1.

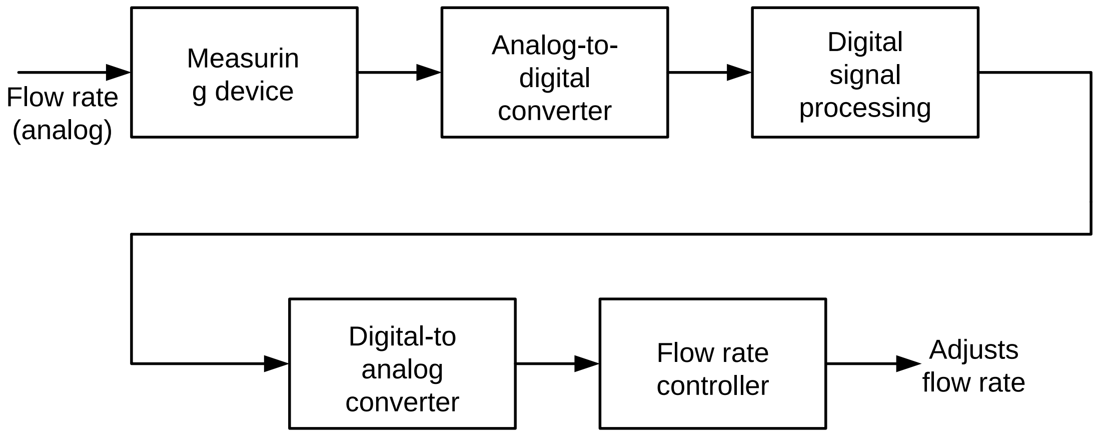

Fig. 1 illustrating a flow measurement and control system

Examples of Digital Systems

Some of digital systems used in day-to-day life are:-

a) In communication systems
b) Business transactions
c) Traffic control
d) Medical treatment
e) Industrial applications
f) Digital telephones
g) Digital TV and disks
h) Digital cameras
i) Weather monitoring
j) Digital computers

NUMBER SYSTEM AND CODES

- Number system can be defined as the representation of values using special symbols.
- There are several number systems but most commonly used ones in digital electronics are
  a) Decimal number system
  b) Binary number system
  c) Octal number system
  d) Hexadecimal number system

Decimal Number System

- This number system has a radix or base of 10 that is, it contains ten unique symbols (or digits) use to represent numbers. These are: 0.1,2,3,4,5,6,7,8,9. Any of these may be use in each position of the number. (Radix is defined as the number of different digits which can occur in each position in the number system)
- Also this number system is a position-value system which means that the value of a digit depends on its position. The absolute value of each digit is fixed but its position value (or place value or weight) is determined by its position in the overall number. For example,

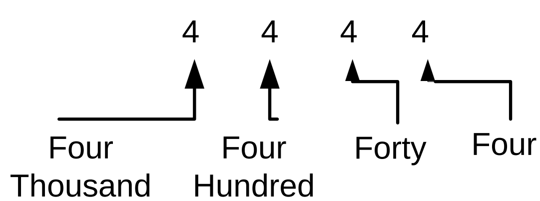

Fig 2

- The weights of each position can be expressed a powers of 10. For example, the number 3547.216 can be represented as follows:-

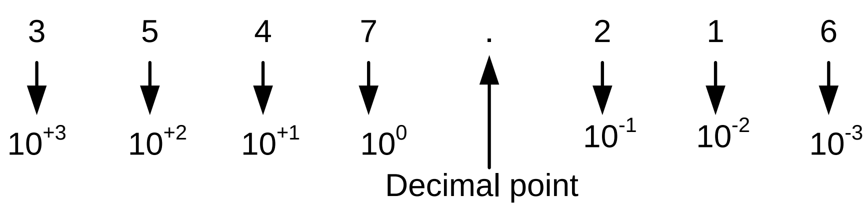

Fig. 3 Illustrating position values as powers of 10 in a decimal number system

- The powers are numbered to the left of the decimal point starting with 0 and to the right of the decimal point starting with -1.
- To count beyond 9, two digit formations are used. That is, the second digit followed by first (10), second followed by second (11), second followed

by third (12), etc. thus the numbers are 10, 11, 12, ...19, 20, 21, 22, ... 29, 30, 31, 32, ...

Binary Number System

- It has a radix of 2 that is; it uses only two digits 0 and 1 to represent numbers. Thus, all binary numbers consist of a string of 0s and 1s e.g. 10, 101 and 1011.To avoid confusion with decimal numbers; a subscript of 10 is used for decimal and 2 for binary e.g.
  10_{10}, 1011_{10}, 5742_{10} - decimal numbers and
  10_2, 101_2, 1100001_2 – binary numbers
- Like decimal number system, binary system is also positionally-weighted. However, the position value of each bit corresponds to some power of 2 as illustrated fig.4.
- The position values of different bits are given by ascending powers of 2 to the left of binary point and by descending power of 2 to the right of binary point

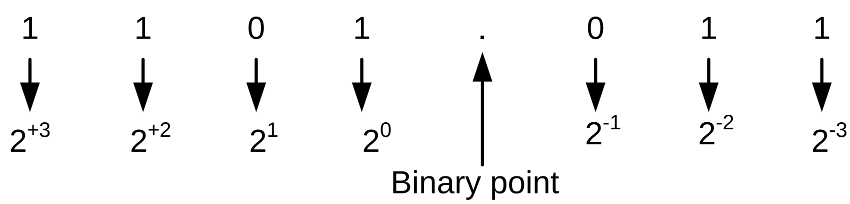

Fig. 4 Illustrating position values as powers of 2 in a binary number system

- To count beyond 1, the same process is used as in decimal. That is, second followed by firs (10), second followed by second (11), second followed by first followed by first (100), etc. Hence the numbers after 1 are 10, 11, 100, 101, 110, 111 ...
- Table 1 below shows a binary count from 0 through 15 and their decimal equivalent.

Table 1 Counting in Binary

Decimal number	**Binary number**			
	2^{+3}	**2^{+2}**	**2^{+1}**	**2^{0}**
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0

7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

- It may be noted from the table that four bits (or digits) are needed to count from 0 to 15 (i.e. 2^4-1). In general with n bits, we can count up to a number equal to 2^n- 1. Thus,

$$\text{Largest decimal number} = 2^n - 1$$

For example, if n = 5, then we can count from 0 to (2^5- 1) = 31.

Exercise

What is the largest decimal value that can be represented by (a) a 8-bit binary, (b) a 16-bit binary number?

Integer Binary-to-Decimal Conversion

- The following procedure is used to convert a given binary integer (whole number) in its equivalent decimal number.

(i) Write the binary number i.e. all bits in a row
(ii) Directly under the bits, write 1, 2, 4,8,16 … starting form right to left
(iii) Cross out the decimal weight which lie under 0 bits.
(iv) Add the remaining weights to get the decimal equivalent

Example 1 *Convert 11001 to its equivalent decimal number*

Step 1	1	1	0	0	1
Step 2	16	8	4	2	1
Step 3	16	8	$\cancel{4}$	$\cancel{2}$	1
Step 4	16 + 8 + 1 = 25				

Hence, $11001_2 = 25_{10}$

Fractional Binary-to-Decimal Conversion

- The procedure is the same as for binary integers except that the weights after binary point have negative powers.

Example 2 *Convert the fraction 0.101 into its decimal equivalent.*

Step 1	.	1	0	1
Step 2		2^{-1}	2^{-2}	2^{-3}
Step 3		2^{-1}	$\cancel{2^{-2}}$	2^{-3}

Step 4 $\frac{1}{2} + \frac{1}{8} = 0.625$

$\therefore\ 0.101_2 = 0.625_{10}$

Example 3 *Convert the binary number 101.101 into its decimal equivalent.*

1	0	1	.	1	0	1
4	$\cancel{2}$	1		2^{-1}	$\cancel{2^{-2}}$	2^{-3}

4+1+0.5+0.125

$101.101_2 = 5.626_{10}$

Decimal-to-Binary Conversion

- There are two methods of decimal-to-binary conversion;

(i) Sum-of-weights method and

(ii) Repeated division by-2 method

(i) Sum-of-weights method for Integer Decimal-to-Binary Conversion

- It uses the binary weights. To find the binary number that is equivalent to a given decimal number is to determine the set of binary weights whose sum is equal to the decimal number. For example, a list of the first eight binary weights from right to left is: 128, 64, 32, 16, 8, 4, 2, 1.

Example 4 *Convert 9 to binary equivalent*

$9 = 8 + 1$ or $9 = 2^3 + 2^0$

Placing 1s in the appropriate weight positions, 2^3 and 2^0 and 0s in the 2^2 and 2^1 positions, we can write the binary number for decimal 9 as

$$\begin{array}{cccc} 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 0 & 1 \end{array}$$

$\therefore\ 9_{10} = 1001_2$

Example 5 Convert each of the following decimals numbers to their binary equivalent using sum-of-weights methods:

a) 17
b) 24
c) 61
d) 93

a) Given the decimal number 17

$$17 = 16 + 1 = 2^4 + 2^0$$

Placing 1s in the appropriate weight positions, 2^4 and 2^0 and 0s in the 2^3, 2^2 and 2^1 positions, we can write the binary number for decimal 17 as

$$\begin{array}{ccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 0 & 0 & 1 \end{array}$$

$$\therefore\ 17_{10} = 10001_2$$

b) Given the decimal number 24

$$24 = 16 + 8 = 2^4 + 2^3$$

Placing 1s in the appropriate weight positions, 2^4 and 2^3 and 0s in the 2^2, 2^1 and 2^0 positions, we can write the binary number for decimal 24 as

$$\begin{array}{ccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$\therefore\ 24_{10} = 11000_2$$

c) Given the decimal number 61

$$61 = 32 + 16 + 8 + 4 + 1 = 2^5 + 2^4 + 2^3 + 2^2 + 2^0$$

Placing 1s in the appropriate weight positions, 2^5, 2^4, 2^3, 2^0 and 0s in the 2^1 positions, we can write the binary number for decimal 61 as

$$\begin{array}{cccccc} 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 1 & 1 & 0 & 1 \end{array}$$

$$\therefore 61_{10} = 111101_2$$

d) Given the decimal number 93

$$93 = 64 + 16 + 8 + 4 + 1 = 2^6 + 2^4 + 2^3 + 2^2 + 2^0$$

Placing 1s in the appropriate weight positions, 2^6, 2^4, 2^3, 2^2, 2^0 and 0s in the 2^5 and 2^1 positions, we can write the binary number for decimal 93 as

$$\begin{array}{ccccccc} 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}$$

(ii) Sum-of-weights method for Conversion of Fractional Decimal-to-Binary

- In order to find a binary fractional number that is equivalent to a given decimal fractional number, a set of binary weights whose sum is equal to the decimal number have to be determined. For example, the first four fractional binary weights are: 0.5, 0.25, 0.125 and 0.0625.

Example 6 *Convert the decimal fraction 0.625 to its equivalent binary.*

$$0.625 = 0.5 + 0.125 = 2^{-1} + 2^{-3}$$
$$= 0.101_2$$

Example 7 *Convert the decimal fraction 0.375 by using sum-of-weights method to its equivalent binary fraction.*

$$0.375 = 0.25 + 0.125 = 2^{-2} + 2^{-3}$$
$$= 0.011_2$$

(iii) Division-by-2 method for Integer Decimal-to-Binary conversion

- The procedure is as follows:-

a) Begin by dividing the given decimal number by 2
b) Divide each resulting quotient by 2 until there is a 0 whole number quotient.

c) The remainders generated by each division form the binary number. These remainders are taken in the reverse order (from bottom to top).

Example 8 Convert the decimal integer 10 to its binary equivalent.

$10 \div 2 = 5$	With a remainder of	0	Top ↑
$5 \div 2 = 2$	With a remainder of	1	
$2 \div 2 = 1$	With a remainder of	0	
$1 \div 2 = 0$	With a remainder of	1	Bottom

$$\therefore \quad 10_{10} = 1010_2$$

Exercise: Convert each of the following decimal numbers using division-by-2 to its binary equivalent

a) 19
b) 68

(iv) Multiplication-by-2 for Fractional Decimal to Binary Conversion

- The procedure is as follows:-

a) Begin by multiplying the given decimal fraction by 2 and then multiplying each resulting fractional part of the product by 2.
b) Repeat step a) until the fractional product is zero or until the desired number of decimal places is reached.
c) The carried bits or carries generated by the multiplication produce the binary number. These carries are taken in the forward direction.

Example 9 *Convert the fractional decimal 0.3125 to its equivalent binary fraction.*

$0.3125 \times 2 = 0.625$	with a carry	0	Top
$0.625 \times 2 = 1.25 = 0.25$	with a carry	1	
$0.25 \times 2 = 0.50 = 0.50$	with a carry	0	
$0.50 \times 2 = 1.00 = 0$	with a carry	1	↓ Bottom

Exercise *Convert the decimal fractions below to their equivalent binary fraction (up to 4 binary places) using multiplication-by-2 method.*

a) 0.9028
b) 0.777
c) 0.6667

Octal Number System

- It has a radix of 8 i.e. it's composed of eight digits 0,1,2,3,4,5,6, and 7. To count beyond 7, two digit combinations in the same way as in decimal and binary is formed. Thus beyond 7, we count as 10, 11, 12,13,14,15, 16, 17, 20, 21 …
- With n octal digits, we can count from 0 to (8^n-1). For example, with 2 octal digits positions we can count for 00_8 to 77_8.
- The position value for each digit is given by different powers of 8 as shown below:

$$8^{+4} \quad 8^{+3} \quad 8^{+2} \quad 8^{+1} \quad 8^{0} \quad . \quad 8^{-1} \quad 8^{-2} \quad 8^{-3}$$

Octal point

- Table 2 shows the binary and octal numbers corresponding to the first ten decimal numbers.

Table 2: Binary and Octal Numbers corresponding to the first ten decimal numbers

Decimal	Binary	Octal
0	000	0
1	001	1
2	010	2
3	011	3
4	100	4
5	101	5
6	110	6
7	111	7
8	1000	10
9	1001	11
10	1010	12

NB: A subscript of 8 or the letter O is used to signify an octal number 17_8 or $17O$.

(i) Octal-to-Decimal Conversion

- The procedure is the same that for binary-to-decimal except that s digit of 8 is used instead of 2.

Example 10 *Convert 206.104_8 into its decimal equivalent number.*

$$\begin{array}{ccccccc} 2 & \cancel{0} & 6 & \cdot & 1 & \cancel{0} & 4 \\ 8^2 & \cancel{8^1} & 8^0 & & 8^{-1} & \cancel{8^{-2}} & 8^{-3} \end{array}$$

$$\begin{aligned} \therefore\ 206.104_8 &= 2\times 8^2 + 6\times 8^0 + 1\times 8^{-1} + 4\times 8^{-3} \\ &= 128 + 6 + \frac{1}{8} + \frac{4}{512} \\ &= 134\left(\frac{17}{128}\right)_{10} \end{aligned}$$

(ii) Decimal-to-Octal Conversion

- The double-dabble method is used with 8 acting as the multiplying factor for integers and the dividing factor for fractions.

Example 11 *Convert* 175_{10} *into its octal equivalent*

175	÷ 8	= 21	With a remainder of	7	↑	Top
21	÷ 8	= 2	With a remainder of	5		
2	÷ 8	= 0	With a remainder of	2		Bottom

Taking the remainders in the reverse order, we get

$$\therefore\ 175_{10} = 257_8$$

Example 12 *Convert* 0.15_{10} *into its octal equivalent.*

0.15	×	8	= 1.20	= 0.20	with a carry	1	Top
0.20	×	8	= 1.60	= 0.60	with a carry	1	
0.60	×	8	= 4.80	= 0.80	with a carry	4	↓ Bottom

Taking the remainders in the forward order, we get

$$\therefore\ 0.15_{10} \cong 0.114_8$$

Example 13 *Convert* 0.3125_{10} *into its octal equivalent*

$0.3125 \times 8 = 2.50 = 0.50$ with a carry 2 Top

$0.50 \times 8 = 4.00 = 0.00$ with a carry 4 Bottom

Taking the remainders in the forward order

$$\therefore \quad 0.3128_{10} = 0.24_8$$

(iii) Octal-to Binary Conversion

- This is achieved by converting each octal digit into its 3-bit binary equivalent. Table 3 shows the 3-bit binary equivalent for the octal digits 0, 1, 2,... 7.

Table 3 Octal-to-binary conversion

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

- Thus using table 3 octal-to-binary conversion can be achieved.

Examples 14 *Convert* 13_8 *into its binary equivalent.*

$$\begin{array}{cc} 1 & 3 \\ \downarrow & \downarrow \\ 001 & 011 \end{array}$$

$$\therefore 13_8 = 001011_2 \text{ or simply } 1011_2$$

Exercise *Convert the following octal numbers to their binary equivalent*

a) 321
b) 4653
c) 13274

(iv) Binary-to Octal Conversion

- This is the reverse of octal-to-binary conversion.

Example 15 *Convert 10111010 to octal equivalent*

$$\begin{array}{ccc} 010 & 111 & 010 \\ \downarrow & \downarrow & \downarrow \\ 2 & 7 & 2 \end{array}$$

$$10111010_2 = 272_8$$

Example 16 *Convert 110101111.1001001 to octal equivalent.*

$$\begin{array}{ccccccc} 110 & 101 & 111 & \cdot & 100 & 100 & 100 \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow \\ 6 & 5 & 7 & & 4 & 4 & 4 \end{array}$$

$$110101111.1001001_2 = 657.444_8$$

Usefulness of Octal number System

Binary numbers especially with 64 bits are used in computers to represent some code that convey non numerical information. They might represent:

a) Actual numerical data
b) Numbers corresponding to a location called (address) in memory
c) An instruction code
d) A code representing alphabetic and other non numerical characters
e) A group of bits representing the status of devices internal or external to the computer

When dealing with a large quantity of binary numbers of many bits, its convenient and more efficient to use octal numbers than binary.

Hexadecimal Number System

- It's primarily used as a "shorthand" way of displaying binary numbers because it is very easy to convert between binary and hexadecimal.
- It has a radix of 16 which means it uses 16 distinct symbols to represent numbers. This symbols are : 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Place value (or weights) for each digit is in ascending powers of 16 for integers and descending powers of 16 for fractions.
- To count beyond F, a 2-digit combination is used, that is, take the second digit followed by the first digit, then second followed by second, then second followed by third and so on. The first few numbers and their equivalents are given in table 4.

Table 4 Hexadecimal numbers and their decimal equivalents

Hexadecimal	Decimal	Hexadecimal	Decimal	Hexadecimal	Decimal
0	0	B	11	16	22
1	1	C	12	17	23
2	2	D	13	18	24
3	3	E	14	19	25
4	4	F	15	1A	26
5	5	10	16	1B	27
6	6	11	17	1C	28
7	7	12	18	1D	29
8	8	13	19	1E	30
9	9	14	20	1F	31
A	10	15	21	20	32

(i) Hexadecimal-to Decimal Conversion

- The following is the procedure to convert a hexadecimal number to it equivalent decimal number.
  a) Write the hexadecimal number
  b) Directly under the hexadecimal number, write the position weight of each digit working from right to left
  c) Multiply the decimal value of each hexadecimal digit by its position weight and take sum of the products.

Example 17 *Convert E5 to its equivalent decimal number*

Step a: E ↓ 5 ↓

Step b: 16^{+1} 16^{0}

Step c: $(E \times 16^{+1}) + (5 \times 16^{0})$

$= (E \times 16) + (5 \times 1)$

$= (14 \times 16) + (5 \times 1)$

$= 224 + 5$

$= 229$

$\therefore\ E5_{16} = 229_{10}$

Example 18 *Convert 0.12_{16} to its equivalent decimal fraction.*

Step a   0.   1   2

Step b 16^{-1} 16^{-2}

Step c $(1 \times 16^{-1}) + (2 \times 16^{-2})$

$= 0.0625 + 0.0078$

$= 0.0703$

$\therefore 0.12_{16} = 0.0703_{10}$

(ii) **Decimal-to –Hexadecimal Conversion**

The procedure is as follows;-

a) Begin by dividing the given decimal number by 16
b) Divide each resulting quotient by 16 until there is a zero whole number quotient
c) The remainders generated by each division form the hexadecimal number

Example 19 *Convert 650 to its equivalent hexadecimal number*

			Top ↑
$650 \div 16 = 40.625 = 40$ with a remainder 0.625	→	$0.625 \times 16 = 10$ (=A)	
$40 \div 16 = 2.5 = 2$ with a remainder 0.5	→	$0.5 \times 16 = 8$	
$2 \div 16 = 0.125 = 0$ with a remainder 0.125	→	$0.125 \times 16 = 2$	Bottom

$\therefore \quad 650_{10} = 28A_{16}$

Exercise Convert the following decimal numbers to their hexadecimal equivalents:-

a) 2890
b) 4019.345

(iii) Hexadecimal-to-Binary Conversion

a) The procedure is as follows:-
b) Write the hexadecimal number
c) Write the 4-bit binary equivalent for each hexadecimal digit as shown in table 5
d) Check if there are any zeros on the left most position of the binary number obtained. Drop off the zeros and write the answer as a binary number

Example 20 *Convert $2D6_{16}$ to its equivalent binary number*

Step a	2	D	6
Step b	0010	1101	0110
Step c	001011010110		

$\therefore \quad 2D6_{16} = 1011010110_2$

(iv) Binary-to-Hexadecimal Conversion

The procedure is as follows:-

a) Write the binary number
b) Starting from the right most position, group the binary number into groups of four-bits. If necessary, add zeros at the left-most position.
c) Convert each 4-bit binary to its equivalent hexadecimal digit

Example 21 *Convert 10111 to its equivalent hexadecimal.*

Step a	1	0	1	1	1
Step b	0001			0111	
Step c	1			7	

Three zeros are added to form a four-bit group

$\therefore \quad 10111_2 = 17_{16}$

Exercise *Convert the following binary numbers to their hexadecimal equivalents.*

a) 10100110
b) 1111110000
c) 100110000010

(v) **Hexadecimal-to-Octal Conversion**

There are two methods to accomplish this.

1. Convert the given hexadecimal number to binary equivalent and then from binary to octal
2. Convert the given hexadecimal number to its decimal equivalent and then from decimal to octal.

However the first method is much simpler and convenient. The procedure is as follows:-

a) Write the hexadecimal number
b) Replace each hexadecimal digit by its 4-bit binary equivalent.
c) Form 3-bit combinations by starting from the right-most position. Replace each 3-bit combination by its octal equivalent.

Example 22 *Convert 5C2 to its octal equivalent.*

Step a	5		C	2
Step b	0101		1100	0010
Step c	010	111	000	010
	2	7	0	2

$\therefore \quad 5C2_{16} = 2702_8$

Example 23 *Convert A7.3B$_{16}$ into its octal equivalent.*

A	7		3	B	
1010	0111		0011	1011	
010	100	111	001	110	110
2	4	7	1	6	6

$\therefore \quad A7.3B_{16} = 247.166_8$

(vi) Octal-to-hexadecimal conversion

It's similar to the hexadecimal-to-octal except only in group of the digits.

Example 24 *Convert 3218 to its hexadecimal equivalent.*

Step a	3	2	1
	↓	↓	↓
Step b	011	010	001

Step c 0 1101 0001

$$0\ \underbrace{1101}_{D}\ \underbrace{0001}_{1}$$

$\therefore \quad 321_8 = D1_{16}$

Excise: *Convert the following numbers to hexadecimal*

a) 6475_8
b) 645.67_8
c) 876.45_8

ARITHMETIC OPERATIONS

Binary Addition

The following rules are used

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 10 i.e. 0 with a carry over of 1

Example 25 *Perform the following binary additions*

a) 100101 + 100101
b) 1011.01 + 1001.11
c) 100.011 + 1011.011

Solutions

1 0 0 1 0 1	1 0 1 1 . 0 1	1 1 0 0 . 0 1 1
+1 0 0 1 0 1	+1 0 0 1 . 1 1	+1 0 1 1 . 0 1 1
1 0 0 1 0 1 0	1 0 1 0 1 . 1 0	1 0 1 1 1 . 1 1 0

Binary Subtraction

The following rules are used

0 - 0 = 0
1 - 0 = 1
1 - 1 = 1
10 - 1 = 1

Example 26 *Perform the following binary subtractions*

a) 110.01 – 100.1

b) 11.01111 – 10.01001

Solutions

a)
$$\begin{array}{r} 111.01 \\ -\underline{100.10} \\ \underline{010.11} \end{array}$$

b)
$$\begin{array}{r} 11.01111 \\ -\underline{10.01001} \\ \underline{01.00110} \end{array}$$

Complement of a binary number

Its possible to use the circuits designed for binary addition to perform the binary subtraction if the problem of subtraction is change to that of addition. This can be achieved using complements of binary numbers.

There are two binary complements: 1's complement and 2's complements

1's complement representation

1's complement of a binary number is obtained by changing each '0' to '1' and each '1' to '0'. For example 1's complement of 010111 is 101000 and 1's complement of 1001.1110 is 0110.0001.

2's complement representation

The 2's complement of a binary number is obtained by adding 1 to its 1's complement i.e.

2's complement = 1's complement + 1

It also called true complement. For example, the 2's complement of 1011_2 is obtained as

1's complement of 1011 is 0100

2's complement of 1011 is
$$\begin{array}{r} 0100 \\ \underline{+\ 1} \\ 0101 \end{array}$$

$\therefore$ 2's complement of 1011_2 is 0101_2

1's complement subtraction

The rules are

(i) Compute the 1's complement of the subtrahend

(ii) Add this complement to the minuend

(iii) Perform the end –round carry of the last 1 or 0

(iv) If there is no end-around carry (i.e. 0 carry), then the answer must recomplemented and a a negative sign attached to it.

(v) If the end-around carry is 1, no recomplementing is necessary.

Example 29 *Subtract 101_2 from 111_2*

$$\begin{array}{cccccl}
 & 1 & 1 & 1 & & \\
+ & 0 & 1 & 0 & \longleftarrow & \text{1's complement of subtrahend 101} \\
\hline
\boxed{1} & 0 & 0 & 1 & & \\
 & & & 1 & \longleftarrow & \text{end-around carry} \\
\hline
 & 0 & 1 & 0 & &
\end{array}$$

Example 30 *Subtract 1101_2 from 1010_2*

$$\begin{array}{cccccll}
 & 1 & 0 & 1 & 0 & & \\
+ & 0 & 0 & 1 & 0 & \longleftarrow & \text{1's complement of subtrahend 1101} \\
\hline
\boxed{} & 1 & 1 & 0 & 0 & &
\end{array}$$

No carry

$\therefore$ recomplementing the answer and attaching a $-$ve sign have -0011_2

2's complement subtraction

The procedure is:-

(i) Find the 2's complement of the subtrahend

(ii) Add this complement to the minuend

(iii) Drop the final carry

(iv) If the carry is 1, the answer is positive and needs no recomplementing

(v) If the is no carry, recomplement the answer and attach minus sign.

Example 31 *Use 2's complement to subtract 1010_2 from 1101_2*

$$\begin{array}{cccccll}
 & 1 & 1 & 0 & 1 & & \\
+ & 0 & 1 & 1 & 0 & \longleftarrow & \text{2's complement of subtrahend 1010} \\
\hline
\boxed{1} & 0 & 0 & 1 & 1 & &
\end{array}$$

Drop carry

Example 32 *Use 2's complement to subtract 1101_2 from 1010_2*

$$\begin{array}{cccccll}
 & 1 & 0 & 1 & 0 & & \\
+ & 0 & 0 & 1 & 1 & \longleftarrow & \text{2's complement of subtrahend 1101} \\
\hline
\boxed{} & 1 & 1 & 0 & 1 & &
\end{array}$$

No carry

Since there is no carry, the answer must be recomplemented. First we must subtract 1 from it to get 1100 and recomplement to get 0011. After attaching the minus sign, the final answer is - 0011_2.

Exercise

Perform the following binary subtraction using 2's complement arithmetic.

(i) 11011 – 11001
(ii) 11011.00 – 1100.00
(iii) 0.01111 – 0.01001
(iv) 111.01 – 10.111
(v) 111.01 – 110.11
(vi) 10111.1 – 10011.1

Binary Multiplication and Division

Both are simple and is done just like decimal multiplication and division.

Example 27 *Perform the following binary multiplication*

a) 1.01× 10.1
b) 101.01 × 11

Solutions

a)
$$\begin{array}{r} 1\;.\;0\;1 \\ \times\,1\;0\;.\;1 \\ \hline 1\;0\;1 \\ 0\;\;\;0\;0\;\; \\ 1\;1\;\;\;1\;\;\;\;\;\; \\ \hline 1\;1\;.\;0\;0\;1 \end{array}$$

b)
$$\begin{array}{r} 1\;0\;1\;.\;0\;1 \\ \times\;\;\;\;\;\;\;\;1\;1 \\ \hline 1\;0\;1\;0\;1 \\ 1\;0\;1\;0\;1\;\;\; \\ \hline 1\;1\;1\;1.1\;1 \end{array}$$

Example 28 *Convert the following binary divisions*

a) 1111001 ÷ 1001
b) 11.11 ÷ 0.101

Solutions

a)

```
        1101.0111
1001 | 1111001
       1001
        1100
        1001
        01101
         1001
         10000
          1001
          01110
           1001
            1010
            1001
              01
```

b)

```
         101
0.101 | 11.110
        101
         101
         101
          00
```

Exercise

(i) Convert 10100110_2 to hexadecimal

(ii) Convert 6800_{10} to octal

(iii) Find decimal equivalent of binary number 0.0111

(iv) Convert the following decimal numbers to their equivalent octal numbers

a) 4429.625
b) 791.125
c) 11.9375

CODES

- Codes have been used for security reasons so that others may not be able to read the message even if it is intercepted.
- In modern digital equipments, they are used to represent and process numerical information.
- The choice of codes depends on the function it has to serve. They are used :-
  a) to perform arithmetic operations.
  b) to store and transmit information because of their high efficiency i.e. the give more information using fewer bits.
  c) to simplify and reduce the circuitry required to process information.
  d) to detect and correct errors in digital circuits.
- Some of these are:-

1) Binary Coded Decimal (BCD) Code

- Each decimal digit is represented by a group of four bits. Since the right-to-left weighing of the 4-bit position is 8-4-2-1, it is also called an 8421 code.
- The coding is as shown in the table.

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- Thus, 10_{10} is represented by 0001 0000 in BCD while 17_{10} is represented by 0001 0111 in BCD.
- Note that with four-bits, sixteen numbers $\left(2^4\right)$ can be represented although in the BCD code only ten of these are used. Thus, the following six combinations are invalid: 1010, 1011, 1100, 1101, 1110 and 1111.

Example 1: Write the decimal number 369 in BCD.
Solution: $369_{10} = 001101101001_{BCD}$

Example 2: Typically digital thermometers use BCD to drive their digital displays. How many BCD bits are required to drive a 3-digit thermometer display? What 12 bits are sent to display for a temperature of 157 degrees.
Solution:
There are 12 BCD bits required to drive a 3-digit thermometer display because each BCD digit is represented by a group of four bits.
$157_{10} = 000101010111_{BCD}$

Example 3: Find the equivalent decimal value for the BCD code nu\][Pmber 0001010001110101
Solution:
$0001010001110101_{BCD} = 1475_{10}$

BCD Addition

In this addition, three cases normally occurs

1) Sum is equal to or less than 9 and carry is 0
2) Sum is greater than 9 and carry is 0
3) Sum is less than or equal to 9 but carry is 1

1) Sum is equal to or less than 9 and carry is 0

Example 1: Add 2_{10} to 6_{10} in BCD

$$\begin{array}{l} 2 \rightarrow 0\,0\,1\,0 \\ \underline{6 \rightarrow 0\,1\,1\,0} \\ \quad\;\; 1\,0\,0\,0 \end{array}$$

2) Sum is greater than 9 and carry is 0

Example 2: Add 7_{10} to 6_{10} in BCD

$$\begin{array}{lll} & 7 \rightarrow 0111 & \\ & \underline{6 \rightarrow 0110} & \\ & \quad 1101 & \leftarrow \text{Invalid BCD number with 0 carry} \\ & \quad \underline{0110} & \leftarrow \text{We add 6 for correction} \\ 1 & \quad 0011 & \leftarrow \text{ valid BCD number with carry} = 1 \\ \downarrow & \quad \downarrow & \\ 1 & \quad 3 & \end{array}$$

Hence,

$7_{10} + 6_{10} = 13_{10}$

3) Sum is less than or equal to 9 but carry is 1

Example 3: Add 9_{10} to 8_{10} in BCD

$$\begin{array}{ll} 9 \rightarrow 1001 & \\ \underline{8 \rightarrow 1000} & \\ \;\; 1 \;\; 0001 & \leftarrow \text{Sum is valid BCD number and carry} = 1 \\ \;\; \downarrow \quad \downarrow & \\ 0001\;0001 & \leftarrow \text{Incorrect BCD result} \\ \underline{0000\;0110} & \leftarrow \text{We add 6 for correction} \\ 0001\;0111 & \leftarrow \text{ corrent BCD result} \\ \downarrow \qquad \downarrow & \\ 1 \qquad 7 & \end{array}$$

Hence,

$9_{10} + 8_{10} = 17_{10}$

Example 4: Add 57_{10} to 26_{10} in BCD

$$
\begin{array}{lll}
57 & \rightarrow 0101 & 0111 \\
26 & \rightarrow 0010 & 0110 \\
\hline
83 & 0111 & 1101 \leftarrow \text{Sum is valid BCD number and carry} = 0 \\
 & \overset{\textit{valid BCD}}{\downarrow} & \overset{\textit{Invalid BCD}}{\downarrow} \\
 & 0111 & 1101 \leftarrow \text{Incorrect BCD result} \\
 & 0000 & 0110 \leftarrow \text{We add 6 for correction} \\
\hline
 & 1000 & 0011 \leftarrow \text{ corrent BCD result} \\
 & \downarrow & \downarrow \\
 & 8 & 3
\end{array}
$$

Hence,

$57_{10} + 26_{10} = 83_{10}$

Example 5: Add 83_{10} to 34_{10} in BCD

$$
\begin{array}{llll}
83 & & \rightarrow 1000 & 0011 \\
34 & & \rightarrow 0011 & 0100 \\
\hline
117 & & 1011 & 0111 \\
 & & \overset{\textit{Invalid BCD}}{\downarrow} & \overset{\textit{valid BCD}}{\downarrow} \\
 & & 1011 & 0111 \leftarrow \text{Incorrect BCD result} \\
 & & 0110 & 0000 \leftarrow \text{We add 6 for correction} \\
\hline
 & 0001 & 0001 & 0111 \leftarrow \text{ corrent BCD result} \\
 & \downarrow & \downarrow & \downarrow \\
 & 1 & 1 & 7
\end{array}
$$

Hence,

$83_{10} + 34_{10} = 117_{10}$

BCD Subtraction

This can be performed using one of the following
(i) Nine's complement method
(ii) Ten's complement method

(i) BCD subtraction using Nine's complements

The 9's complement of a BCD number is obtained by subtracting it from 9. 9's complements of various numbers are given in the table below.

Decimal digit	0	1	2	3	4	5	6	7	8	9
9's complement	9	8	7	6	5	4	3	2	1	0

This subtraction is similar to 1’s complement subtraction with the steps being (Assume A_{BCD} - B_{BCD}):-

1. Obtain 9’s complement of B
2. Then, add A and 9’s complement of B
3. If a carry is generated, then add it to the sum to obtain the final result.
4. If a carry is not produce then the result is negative and hence we take the 9’s complement of the result.

Example 6: Subtract 3_{10} from 7_{10} in BCD

$$\begin{array}{rl} 7 & \\ +\ \ 6 & \leftarrow \text{9's complement of 3} \\ \hline 1\ \ \ 3 & \\ +\downarrow\rightarrow 1 & \leftarrow \text{add carry to the sum} \\ \hline 4 & \leftarrow \text{Final resuslt is positive an is in the true form} \end{array}$$

Example 7: Perform $4_{10} - 7_{10}$ using 9’s complement.

$$\begin{array}{rrl} 4 & 0100 & \\ +\underline{2} & \underline{0010} & \leftarrow \text{9's complement of 7} \\ & 0\ \ 0110 & \end{array}$$

Since final carry is 0, the result is negative obtained by taking 9's complement of the result

Taking the 9’s complement of the result have:

$$\begin{array}{rr} 9 & 1001 \\ -\ \underline{6} & \underline{0110} \\ 3 & 0011 \end{array}$$

Hence, $4_{10} - 7_{10} = -3_{10}$

Example 8: Perform $83_{10} - 21_{10}$ using 9’s complement

The 9’s complement of the 21_{10} is

$$\begin{array}{rl} 99 & \\ -\underline{21} & \\ 78 & \leftarrow \text{9's complement of 21} \end{array}$$

Add 83_{10} to 78 i.e.

$$
\begin{array}{rll}
83 & \rightarrow 1000 & 0011 \\
78 & \rightarrow 0111 & 1000 \\
\hline
161 & 1111 & 1011 \\
 & \underset{\downarrow}{\textit{Invalid BCD}} & \underset{\downarrow}{\textit{Invalid BCD}} \\
 & 1111 & 1011 \leftarrow \text{Incorrect BCD result} \\
 & 0110 & 0110 \leftarrow \text{We add 6 for correction} \\
\hline
 & 1\ 0110 & 0001 \leftarrow \text{ corrent BCD result} \\
 & & 1 \leftarrow \text{ the end around carry is add to get the final result} \\
\hline
 & 0110 & 0010 \\
 & \downarrow & \downarrow \\
 & 6 & 2
\end{array}
$$

Hence, $83_{10} - 21_{10} = 62_{10}$

Example 9: Perform $52_{10} - 89_{10}$ using 9's complement

The 9's complement of the 89_{10} is

$$
\begin{array}{r}
99 \\
-\underline{89} \\
10 \leftarrow \text{9's complement of 89}
\end{array}
$$

Add 52_{10} to 10 i.e.

$$
\begin{array}{rll}
52 & \rightarrow 0101 & 0010 \\
10 & \rightarrow 0001 & 0000 \\
\hline
62 & 0110 & 0010 \leftarrow \text{Final carry is 0, so the sum is negative}
\end{array}
$$

The 9's complement of 62 is 37. Hence,

$52_{10} - 89_{10} = -37_{10}$

(ii) BCD subtraction using Ten's complements

The 10's complement is obtained by adding 1 to 9's complement. The 10's complement is used to perform BCD subtraction as under:-

1. Obtain the 10's complement of the subtrahend
2. Then, add the minuend to the 10's complement of the subtrahend
3. Discard the carry. If the carry is 1, then the answer is positive and in true form.
4. If a carry is not produced, then the answer is negative obtained by taking the 10's complement of the answer.

Example 10: Perform $9_{10} - 4_{10}$ using 10's complement.

Solution:

The 9's complement of 4 is $9_{10} - 4_{10} = 5_{10}$

The 10's complement of 4 is $9_{10}-4_{10}=5_{10}+1=6_{10}$

Add 9_{10} to the 10's complement of 4_{10}

$$\begin{array}{rrl}
9 & \rightarrow 1001 & \\
6 & \rightarrow \underline{0110} & \\
 & 1111 & \leftarrow \textit{invalid BCD} \\
 & \underline{0110} & \leftarrow \text{add } 0110_2 \text{ for correction} \\
1 & 0101 & \leftarrow \text{discard the final carry}
\end{array}$$

Therefore, $9_{10}-4_{10}=5_{10}$

Example 11: Perform $3_{10}-8_{10}$ using 10's complement.

Solution:

The 9's complement of 8 is $9_{10}-8_{10}=1_{10}$

The 10's complement of 8 is $9_{10}-8_{10}=1_{10}+1=2_{10}$

Add 3_{10} to the 10's complement of 8_{10}

$$\begin{array}{rrl}
3 & \rightarrow 0011 & \\
2 & \rightarrow \underline{0010} & \\
 & 0101 & \leftarrow \text{since there is no carry, then the answer should be negative} \\
 & 5 &
\end{array}$$

Then, the 10's complement of 5_{10} is $9-5=4+1=5$. Hence,

$3_{10}-8_{10}=-5_{10}$

Example 12: Perform $54_{10}-22_{10}$ using 10's complement method.

Solution:

The 9's complement of 22 is $99-22=77$

The 10's complement of 22 is $77+1=78$

Add 54_{10} to the 10's complement of 22_{10}

$$\begin{array}{rrrl}
54 & \rightarrow 0101 & 0100 & \\
\underline{78} & \underline{\rightarrow 0111} & \underline{1000} & \\
161 & 1100 & 1100 & \\
 & \textit{Invalid BCD} & \textit{Invalid BCD} & \\
 & \downarrow & \downarrow & \\
 & 1100 & 1100 & \leftarrow \text{Incorrect BCD result} \\
 & \underline{0110} & \underline{0110} & \leftarrow \text{We add 6 for correction} \\
 & \cancel{1}\ 0011 & 0010 & \leftarrow \text{ the end around carry is add to get the final result} \\
 & \downarrow & \downarrow & \\
 & 3 & 2 &
\end{array}$$

Therefore, $54_{10}-22_{10}=32_{10}$

Example 13: Perform $22_{10} - 54_{10}$ using 10's complement method.

Solution:

The 9's complement of 54 is $99 - 54 = 45$

The 10's complement of 54 is $45 + 1 = 46$

Add 22_{10} to the 10's complement of 54_{10}

$$\begin{array}{lll} 22 & \rightarrow 0010 & 0010 \\ \underline{46} & \underline{\rightarrow 0100} & \underline{0110} \\ & 0110 & 1000 \end{array} \leftarrow \text{since there was no final carry, then the aswer is negative}$$

$$\begin{array}{cc} \overset{valid\ BCD}{\downarrow} & \overset{valid\ BCD}{\downarrow} \\ 6 & 8 \end{array}$$

The 10's complement of 68 is $99 - 68 = 31 + 1 = 32$

Therefore, $22_{10} - 54_{10} = -32_{10}$

ASSIGNMENT I

1. Convert the following numbers to Decimal, Hexadecimal and Octal form:
   a) 101101.1101_2 **[4 marks]**
   b) 11011011.100101_2 **[4 marks]**
2. Convert $2AC5.D_{16}$ to decimal, octal and binary. **[3 marks]**
3. Using 2's complement perform the following: $42_{10} - 68_{10}$. **[4 marks]**
4. Solve $DDCC_{16} + BBAA_{16} = (\ldots)_{16}$. **[2 marks]**
5. Solve $723_8 + 237_8 = (\ldots)_8$. **[2 marks]**
6. Determine the value of base x, if
   a) $193_x = 623_8$ **[2 marks]**
   b) $225_x = 341_8$ **[2 marks]**
7. Perform the following in BCD using 10's complement method.
   a) $36_{10} - 23_{10}$ **[3 marks]**
   b) $233_{10} - 643_{10}$ **[4 marks]**