An Analysis of Moon Crater Diameters in Relation to the Late Heavy Bombardment Theory

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1 Introduction

The moon's craters are a physical record of astronomical events that may have had a hand in the formation of our Solar System. By investigating existing crater data we can hypothesize different events that may have caused these. Using the Robbins Lunar Crater Database, we can infer astronomical events, such as late heavy bombardment, that may have created many of the similarly-sized near-earth objects (NEOs) (Robbins 2019). This analysis would separate possible NEOs associated with current and long existing astronomical features, such as the main asteroid belt, compared to those from instantaneous activity. Discrepancies in expected crater size distributions and actual crater size distributions (Strom et al. 2018) could explain the occurrence of instantaneous astronomical events like late heavy bombardment, a moment in astronomical history in which high collision rates amongst asteroids and planets occurred, driven by post accretion and planetary instability. Although widely accepted, this hypothesis remains difficult to prove conclusively. If real crater size distributions differ significantly from synthetically generated crater distributions data, this may lend credence to the theory of Late Heavy Bombardment (Lowe & Byerly 2018).

From the Robbins Lunar Crater Database, craters have been cataloged using points, position, orientation, ellipse fit, circle fit, and quality of fit. Robbins used the Lunar Reconnaissance Orbiter (LRO) Camera's wide angle camera (WAC), Lunar Orbiter Laser Altimeter (LOLA) and Terrain Camera (TC) on SELENE/Kaguya to gather image data. After image collection, they manually identified craters by picking out and mapping rim points. The Robbins dataset assumes that quasi-circular depression features are impact craters and that quasi-ellipsoidal features with the long axis radial to a much larger crater are impact-caused features (Robbins 2019). At present, the method for comparing size frequency distributions of craters and projectiles operates under the assumption that each crater was formed by a projectile with one impact velocity and one impact angle (Ivanov et al. 2001).

2 Methods

Craters were first identified using the real dataset from the Robbins Lunar Crater Database, looking specifically at the circular diameter of the craters. Under the assumption that all craters used in the simulation for our algorithm are circular, the data was filtered in which diameters greater than 200 kilometers were excluded, based on assumptions from Hughes (2002). Craters less than 5 kilometers in diameter were additionally excluded due to the likelihood of smaller craters being secondary craters caused by debris from primary impact (Robbins 2019). The resulting data was used to generate the actual moon crater diameter distribution shown in Figure 1.

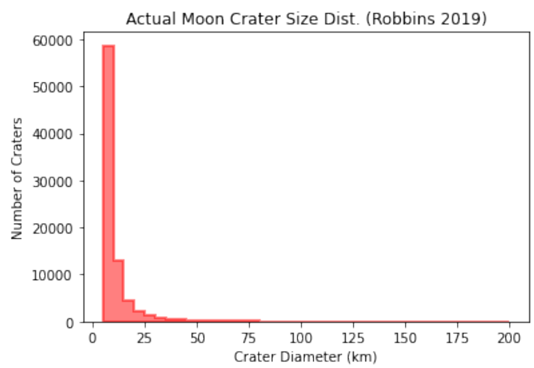

Figure 1: The distribution of actual moon craters with diameter 5 km $\leq D \leq$ 200 km (Robbins 2019). The graph shows a large quantity of craters being around 5km while showing the number of craters less than 5km decreases drastically.

Using the Monte Carlo method and diameter distribution plot of NEOs from Matthias *et al*, a distribution of 83,000 asteroid diameters were generated to act as the synthetic projectile dataset. (Figure 2). Using the D/d ratio in Hughes (2002), in which D=diameter of the crater produced, d=diameter of the asteroid that created the crater, the minimum and maximum asteroids were calculated for the synthetic dataset, using values from the Robbins dataset. According to Hughes (2002), the ratio of smaller craters around 0.88 kilometers in diameter to the diameter of the asteroid that produced them is around 8, while the ratio of larger craters around 200 kilometers in diameter to the asteroid that produced them is around 16. The minimum asteroid diameter d to use in the synthetic dataset was calculated by dividing the minimum crater diameter from the real crater dataset (5km) by 8 to get the minimum asteroid size (0.625km; 625m). The same method was used to calculate the maximum asteroid size; resulting in the maximum asteroid diameter (12.5km; 12,500m).

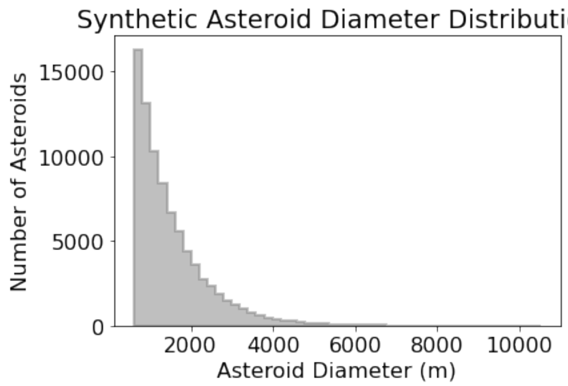

Figure 2: The distribution of synthetic asteroid diameters 625 m $\leq d \leq$ 12500 m. This graph shows a large amount of smaller asteroids in around the 625m range while showing there are a lot fewer large asteroids.

Then, using the equation for energy of asteroid collision that resulted in the formation of crater size D upon impact from Hughes (2002),

$$E = 9.1 * 10^{24} * D^{2.59} \tag{1}$$

where E is the kinetic energy of the impacting projectile in ergs, and D is the diameter of the resulting crater in km.

From equation (1), we want to identify the relationship between the diameter of the impacting projectile and the diameter of the crater on a planetary surface. We converted the units of E in equation (1) from erg to Nm,

$$E = 9.1 * 10^{24} * 10^{-7} * D^{2.59} \tag{2}$$

and then converted D from kilometers to meters,

$$E = 9.1 * 10^{19} * D^{2.59}. \tag{3}$$

Next, we used the equation for kinetic energy,

$$\frac{1}{2}mv^2 = 9.1 * 10^{19} * D^{2.59} \tag{4}$$

and the the equation for mass,

$$m = V\rho, \tag{5}$$

where: m = mass of the asteroid, v = impact velocity of the asteroid, V = volume of the the asteroid and ρ = density. Using equation (4) and (5) to approximate the mass of an asteroid using volume of a sphere V and asteroid diameter d,

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3. \tag{6}$$

We then combined equations (5) and (6) to find the mass of the asteroid,

$$m = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \rho. \tag{7}$$

Next, we substituted m for equation (7) in the energy equation (4) and simplified,

$$\frac{2}{3}\pi\left(\frac{d}{2}\right)^3 \rho^2 = 9.1 * 10^{19} * D^{2.59}. \tag{8}$$

Finally, we rearranged equation (8) to solve for D,

$$D = \sqrt[2.59]{\frac{\frac{2}{3}\pi\left(\frac{d}{2}\right)^3 \rho v^2}{9.1 * 10^{19}}}, \tag{9}$$

to define the relationship between the size of the crater and the size of the asteroid that produced it.

For the sake of simplicity, we used constant values for the velocity and density of the asteroids. Following assumptions made in Hughes (2002), we used a velocity of 20800 m/s and an asteroid density of 3650 kgm^{-3}. Finally, we randomly sampled asteroid diameters from the asteroid diameter distribution data and converted those samples to crater diameters using equation (9). This resulted in a distribution of synthetic crater diameters shown in Figure 3.

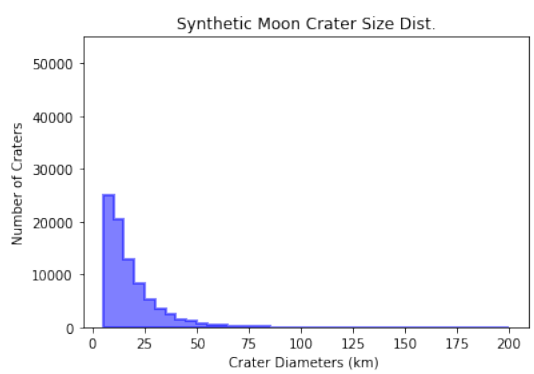

Figure 3: The distribution of synthetic moon craters with diameters 5 km $\leq D \leq$ 200 km.

3 Results

The means and standard deviations of the actual moon crater data r and the simulated moon crater data s are given numerically in equations (10-13) and is shown graphically in Figure 4.

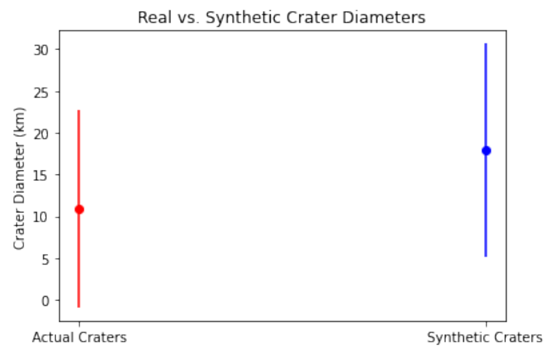

Figure 4: This figure shows the the mean and standard deviation of the real and synthetic crater diameters. As we see in this plot and in equation (10), the mean diameter of actual lunar craters is 10.91 km while the mean diameter of synthetic lunar craters is 17.92. The standard deviations of both are quite similar and the means of either one overlap and fall within the other's standard deviations. This is to say that the distributions of either data set are not that different.

$$\bar{X}_r = 10.91\ km \tag{10}$$

$$\bar{X}_s = 17.92\ km \tag{11}$$

$$\sigma_r = 11.81 \tag{12}$$

$$\sigma_s = 12.74 \tag{13}$$

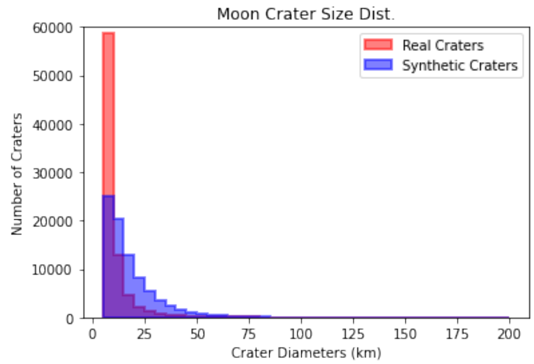

Figure 5: The figure above is a comparison between synthetically generated moon crater diameters and actual moon crater diameters from the Robbins crater database. The range of craters in the Robbins data was limited to craters under 200 km in diameter (Robbins 2019). The simulated crater distribution was created using equations and assumptions from Hughes (2002). The simulated impacting bodies were limited to 12.5 km in size. Both graphs include the same amount of data points of 83,000 craters.

Figure 5 shows a large discrepancy between the two data sets. The synthetic data shows a much smaller number of craters from 5 km to 10 km than the real crater data, as well as a less steep decrease in the numbers of craters at higher size ranges. This difference is also highlighted by the means of each graph being about 10 km apart from each other as identified in equation (10) and (11). With that said, Figure 4 shows that each mean is within one standard of the other, which implies that the simulated data is more similar to the actual crater data than might be inferred from just looking at Figure 5.

4 Conclusion

Discrepancies in moon crater distribution from synthetically generated data compared to the actual data from Robbins, leads us to believe that late heavy bombardment may have occurred. More specifically there are far more small craters on the moon than expected, in addition to there being significantly fewer large craters in existence. For simplicity of the model, all asteroids were assumed to have the same impact velocities and densities. Additionally, the Hughes equation inherently makes assumptions about the constants used. Although our assumptions made while crafting the simulated model may contribute to some of the data discrepancies observed, the behavior of the model is clearly seen.

The greater number of craters seen in the 5 - 10 km range of the Robbin's distribution, compared to that of the simulated dataset generated off of currently existing data, tells us that there was an earlier time in which smaller asteroids existed at a greater density than what we see in the current data today. The synthetically generated dataset fails to demonstrate the high quantity of 5 - 10 km diameter asteroids, telling us that the current state of asteroid data fails to recognize the information that can be calculated from lunar craters. Additionally, it is known that smaller projectiles move at a higher velocity and are more affected by the gravity of large astral bodies in space. This would ultimately lead to a greater likelihood of impacts occurring on the moon by smaller projectiles which is seen in the crater distribution of the Robbin's dataset. The synthetic dataset fails to match the numerical group of smaller projectiles as it does not recognize the possibility of late heavy bombardment generating a rapid growth of smaller-diameter projectiles.

There has been continuous disagreement on the best equation to use since it's very difficult to verify a calculated asteroid size to an actual asteroid. Further exploration into the equation used and the range in which it is applicable could provide greater light into the accuracy of the model demonstrated. In the Hughes paper, he notes that there are other mathematical applications that could be used to visualize the predicted asteroid distribution curve that can be applied to different crater size ranges. Further validation in generated distributions is needed to see if the simulated data accurately matches that of the Robbins Lunar Crater dataset.

This study acts as a step into generalized model generation hinting towards the validity of late heavy bombardment. The synthetically generated dataset references that of currently existing asteroid data which may be overlooking the data that can be found from looking at lunar craters. Due to the synthetic dataset's differences compared to the reverse calculation of the Robbin's dataset, it is possible that late heavy bombardment could have occurred.

5 References

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