Introducción a grafos, DFS, BFS y sus aplicaciones

Miguel Ortiz

Programación competitiva para ICPC

¿Qué es un grafo?

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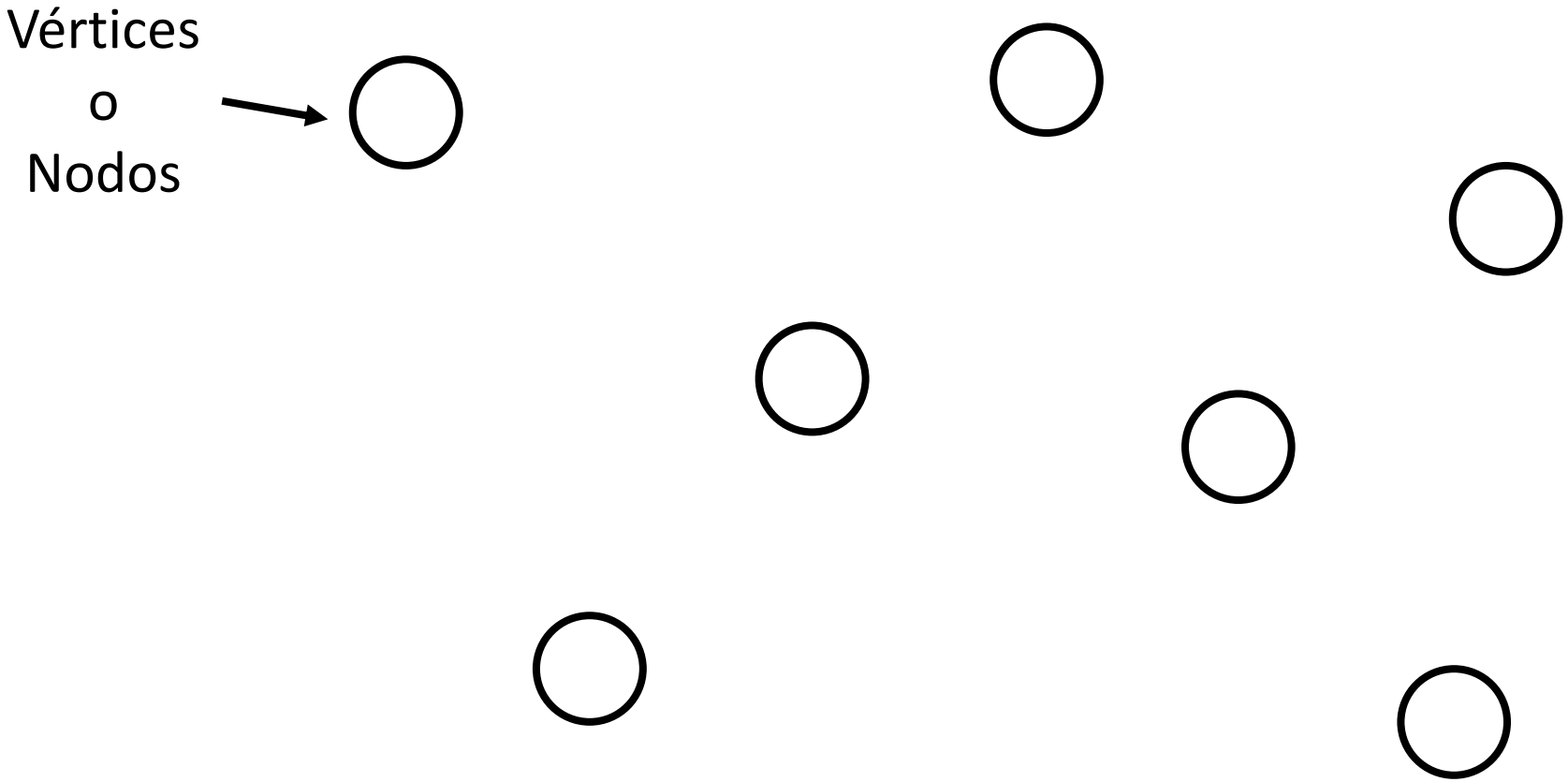
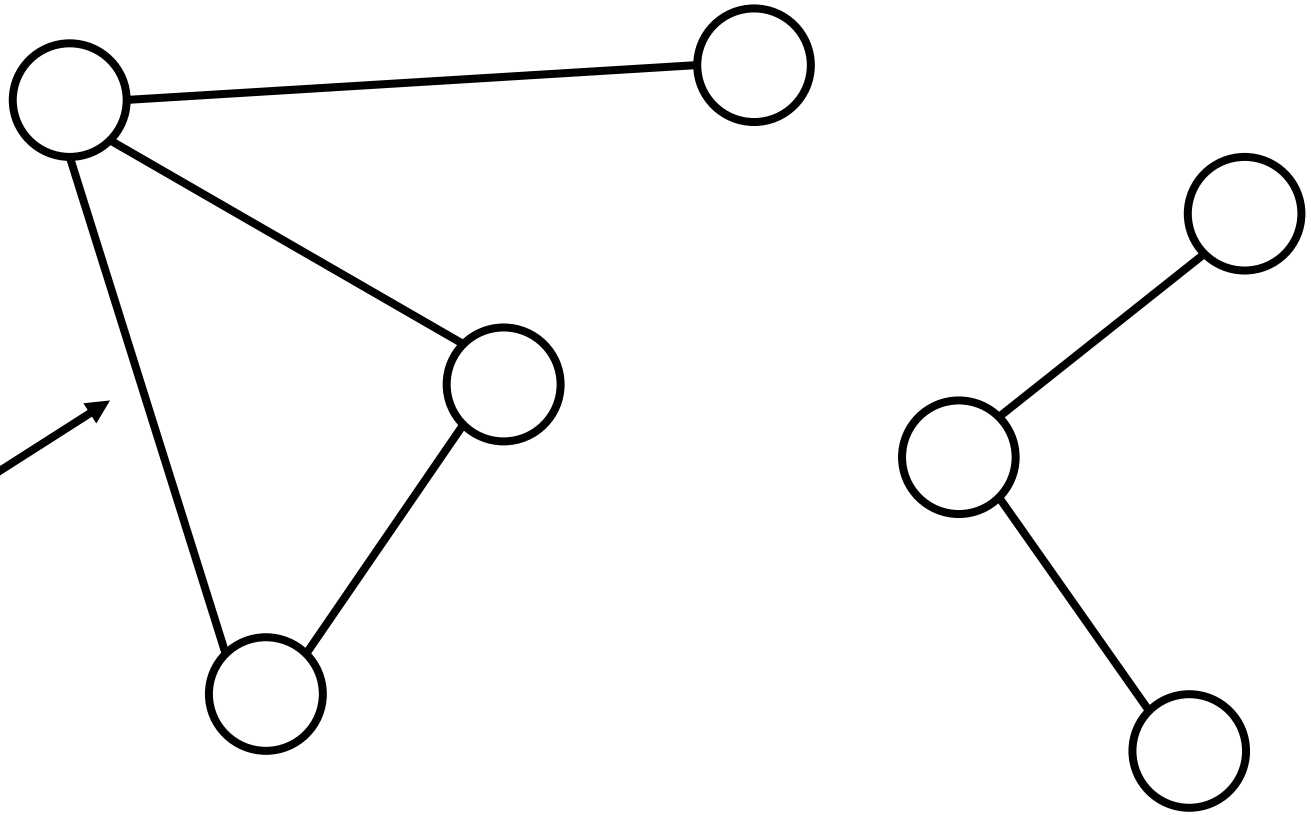


¿Qué es un grafo?

Vértices
o
Nodos

Aristas



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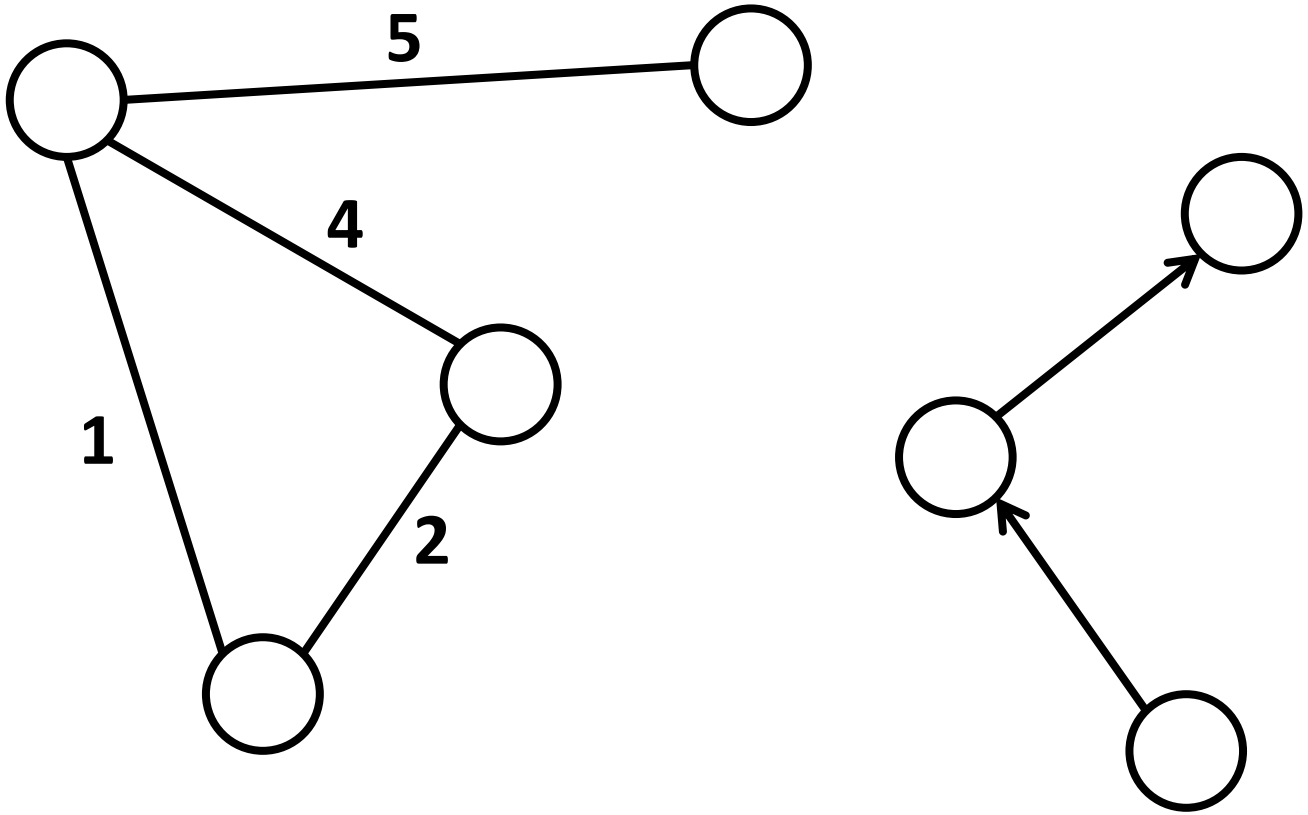

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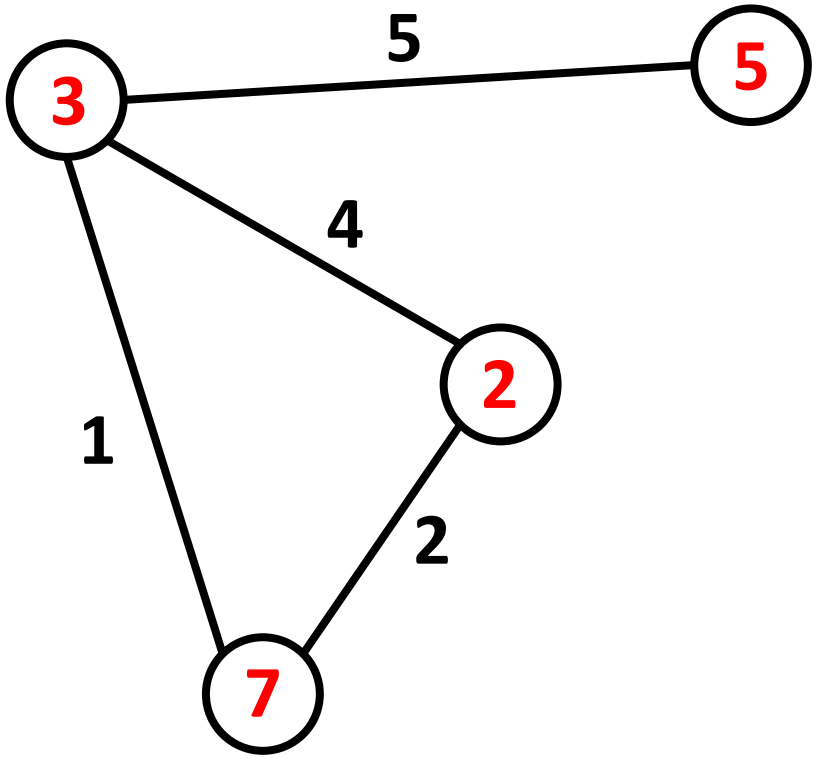
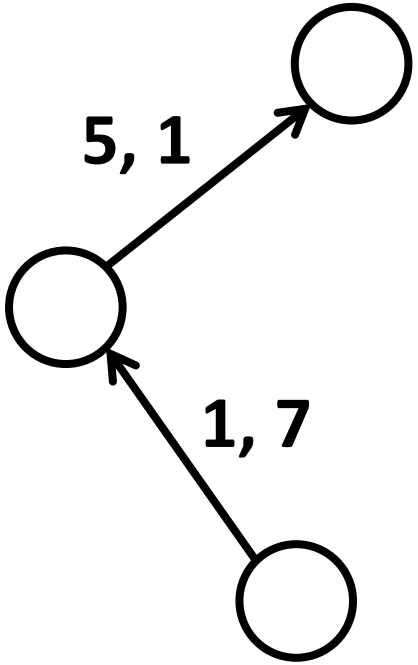
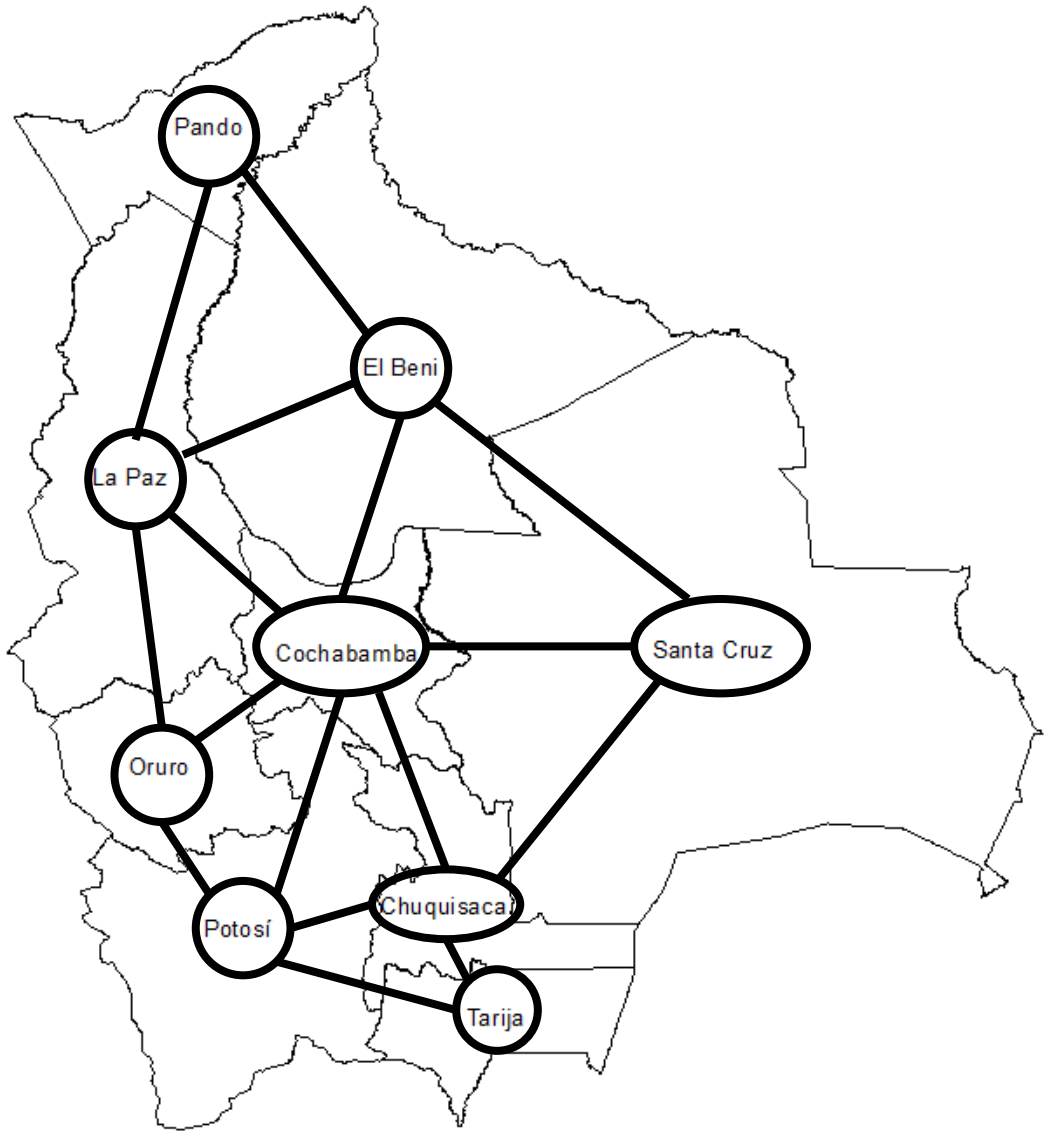
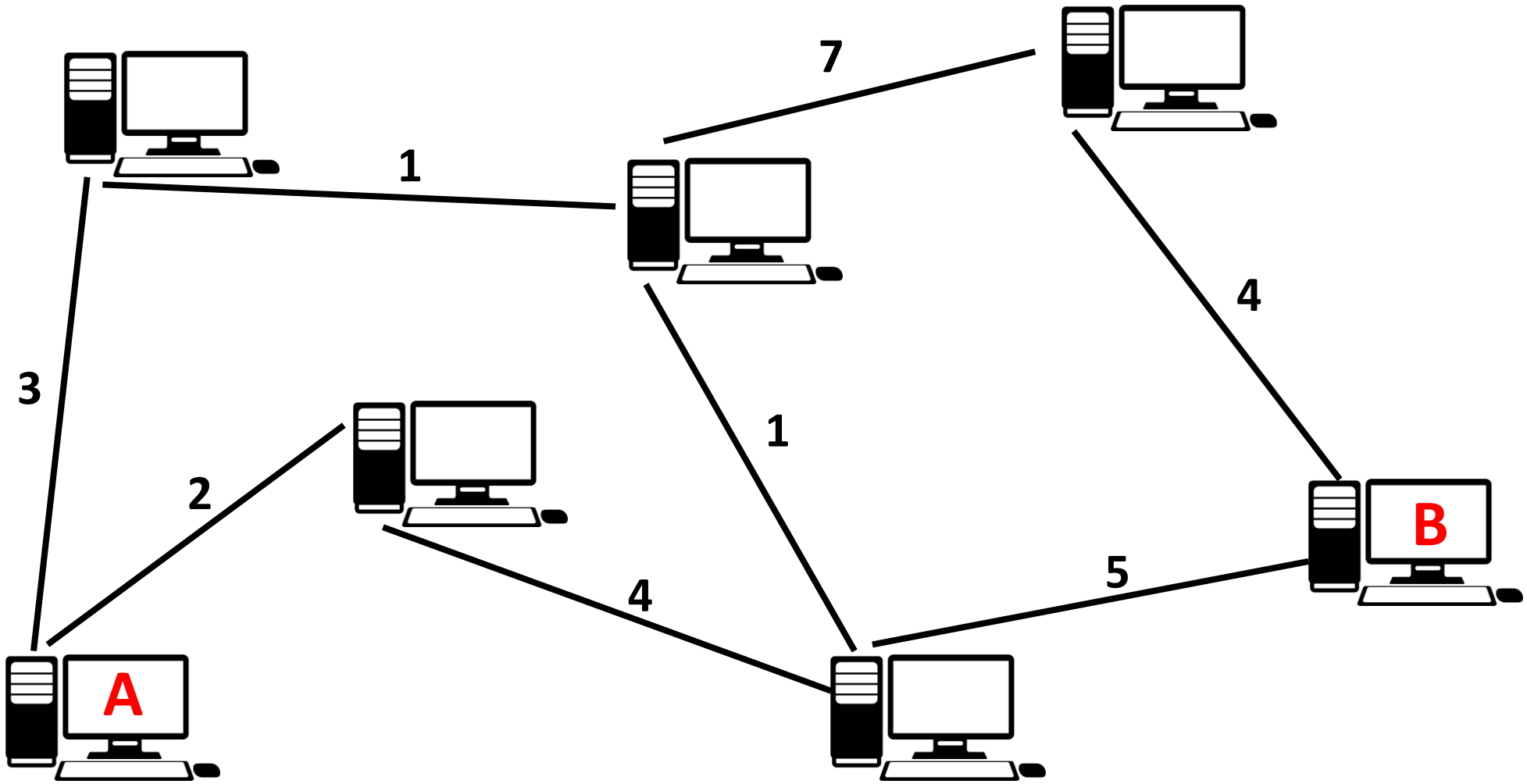




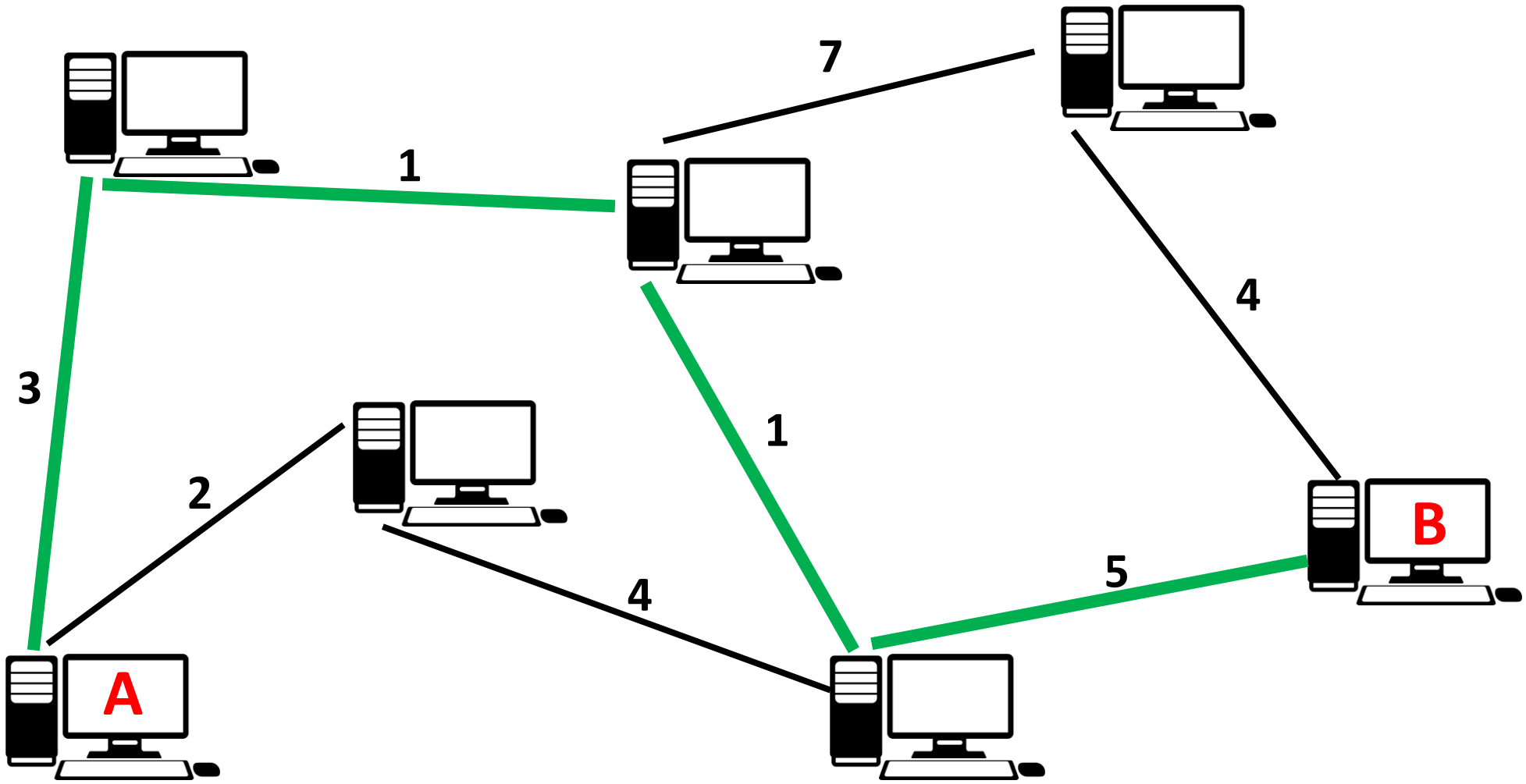
Ejemplos


Pando
El Beni
La Paz
Cochabamba
Santa Cruz
Oruro
Chuquisaca
Potosí
Tarija

Ejemplos



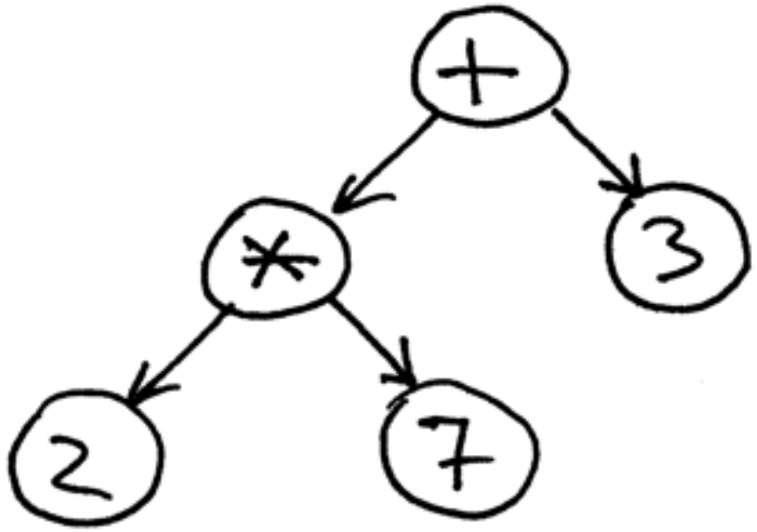
Ejemplos



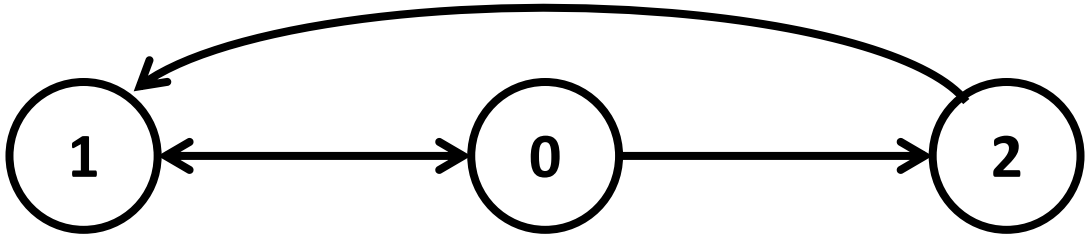
Ejemplos

2 * 7 + 3

AST



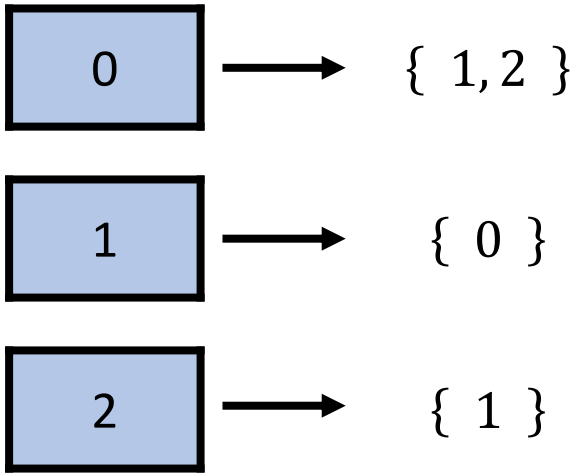
¿Cómo se representan?



Matriz de adyacencia

Nodo de salida \ Nodo de llegada	0	1	2
0	0	1	1
1	1	0	0
2	0	1	0

Listas de adyacencia



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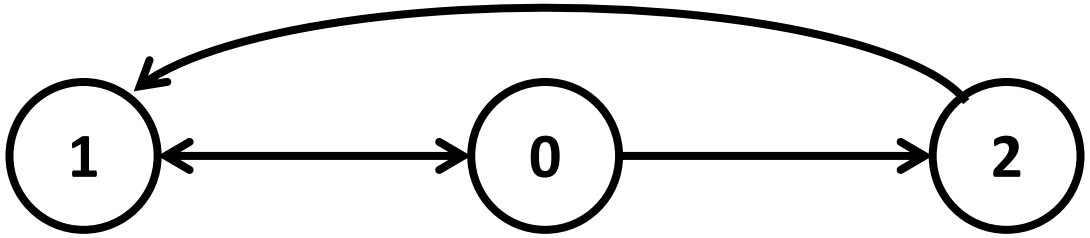


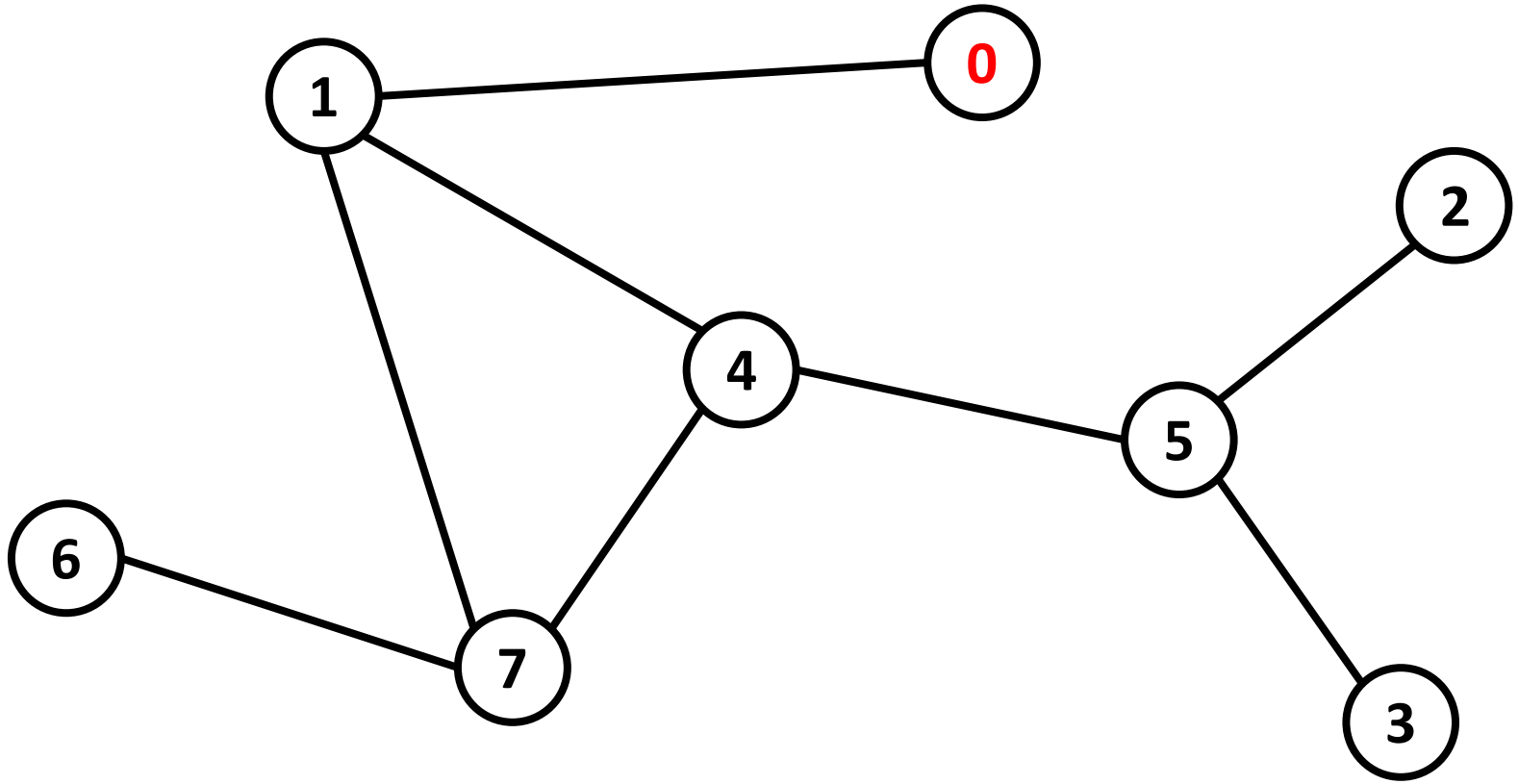

```
// Matriz de adyacencia
int g[n][n];
for (i = 0 -> n-1) {
  for (j = 0 -> n-1) {
    g[i][j] = 0;
  }
}
// Llenado
g[0][1] = 1;
g[0][2] = 1;
g[1][0] = 1;
g[2][1] = 1;
```

```
//Lista de adyacencia
vector<int> g[n];
// vector<vector<int>> g;
// Llenado
g[0].push_back(1);
g[0].push_back(2);
g[1].push_back(0);
g[2].push_back(1);
```

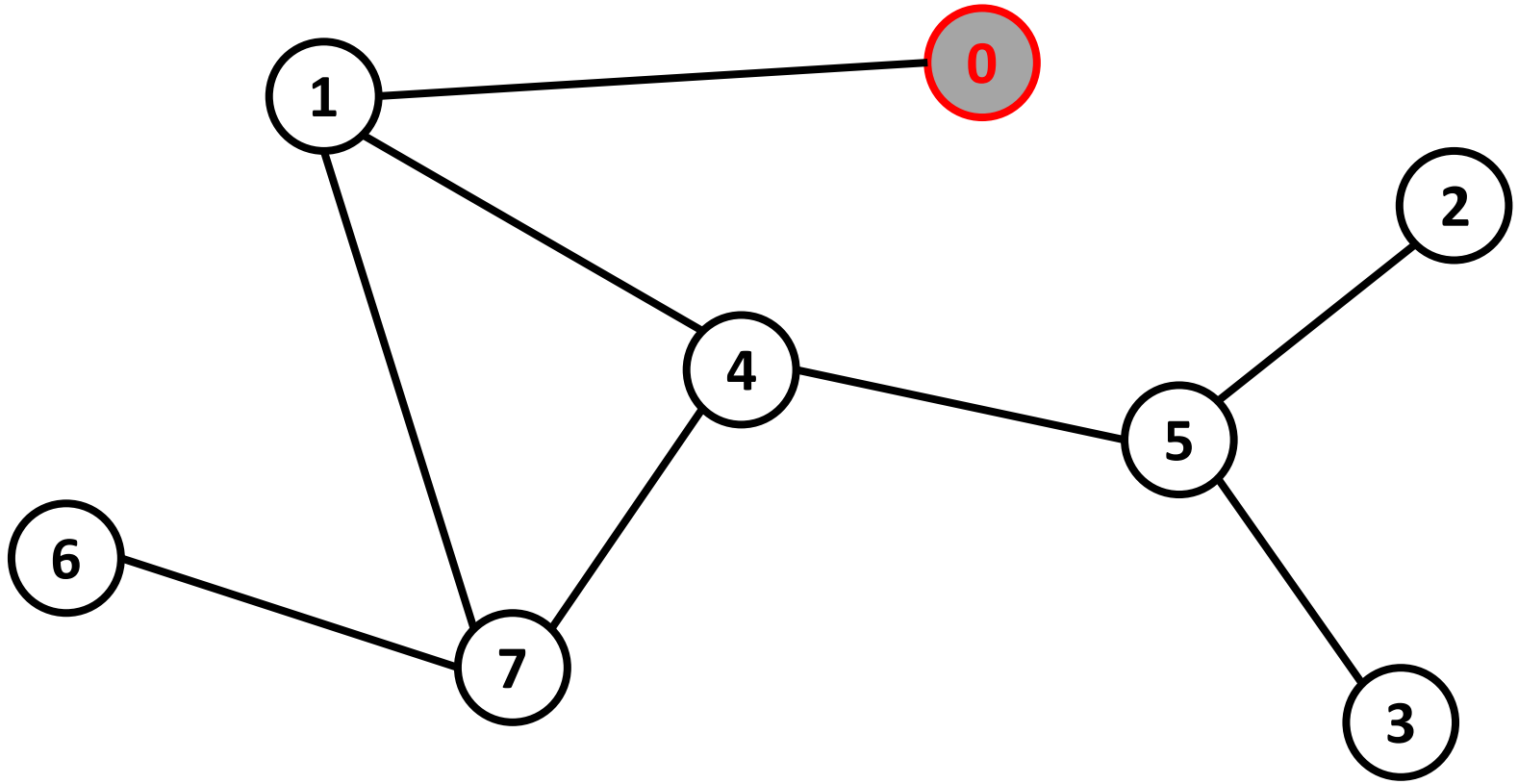
DFS

- Algoritmo de recorrido de grafos
- Empieza en un nodo, lo marca como visitado y se mueve a sus vecinos que aun no han sido visitados
  - Si hay varias opciones para moverse, se mueve a cualquiera
- Cuando no hay vecinos sin marca, vuelve por donde vino
- Hace el mismo procedimiento en cada paso hasta haber visitado todos los nodos posibles
- Sigue un solo camino por el grafo, como caminar por un laberinto con una mano siempre en la pared

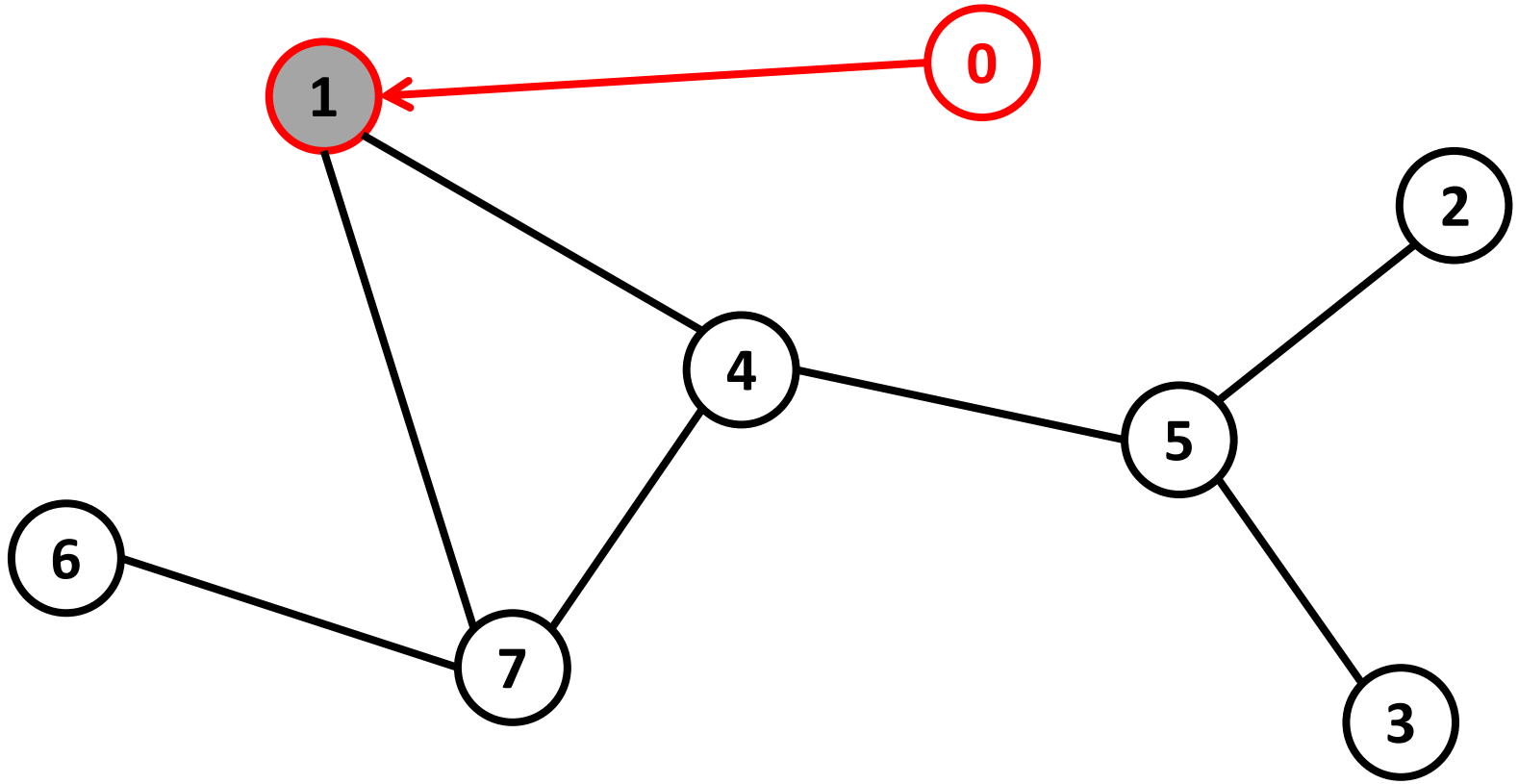
DFS



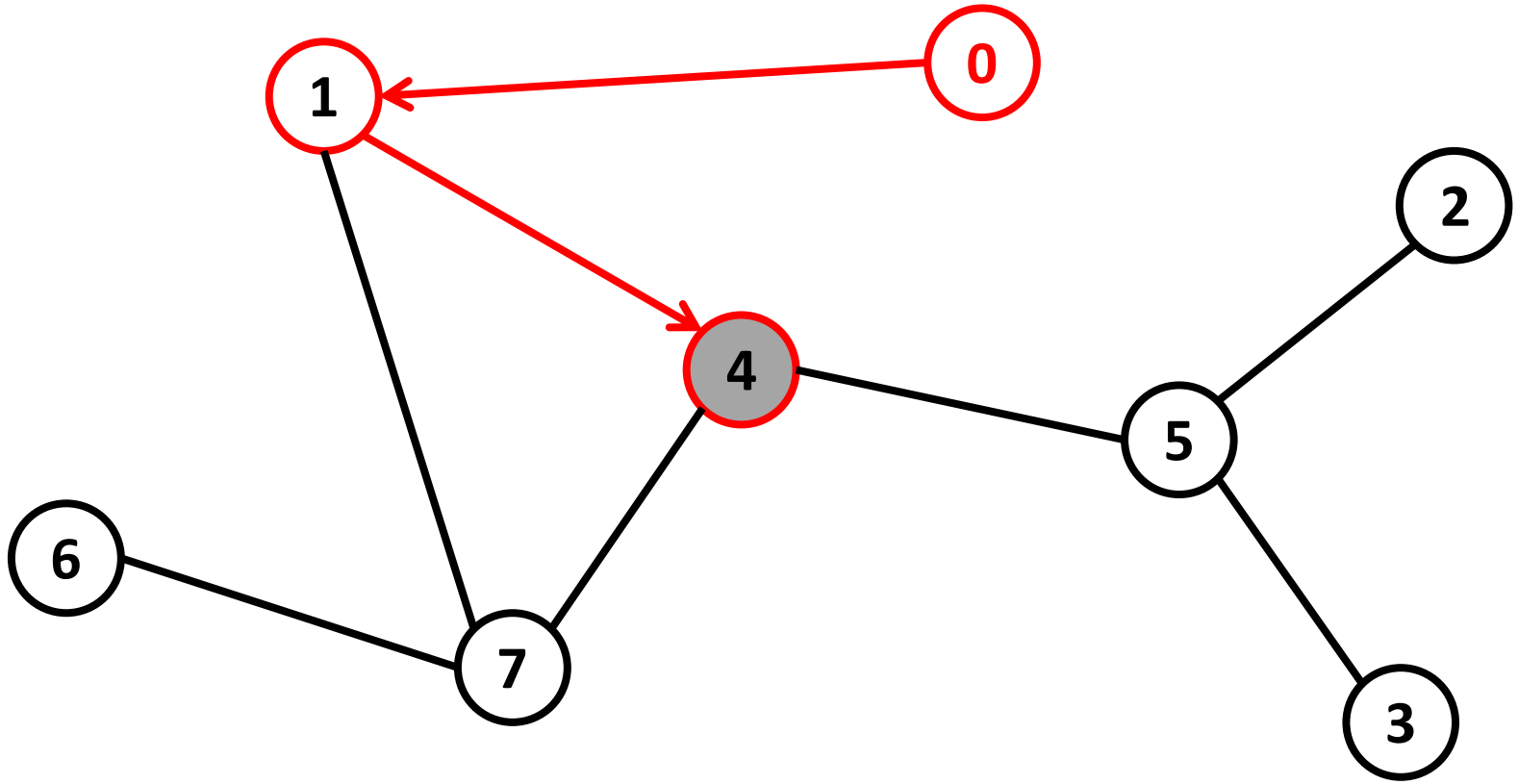
DFS



DFS



DFS



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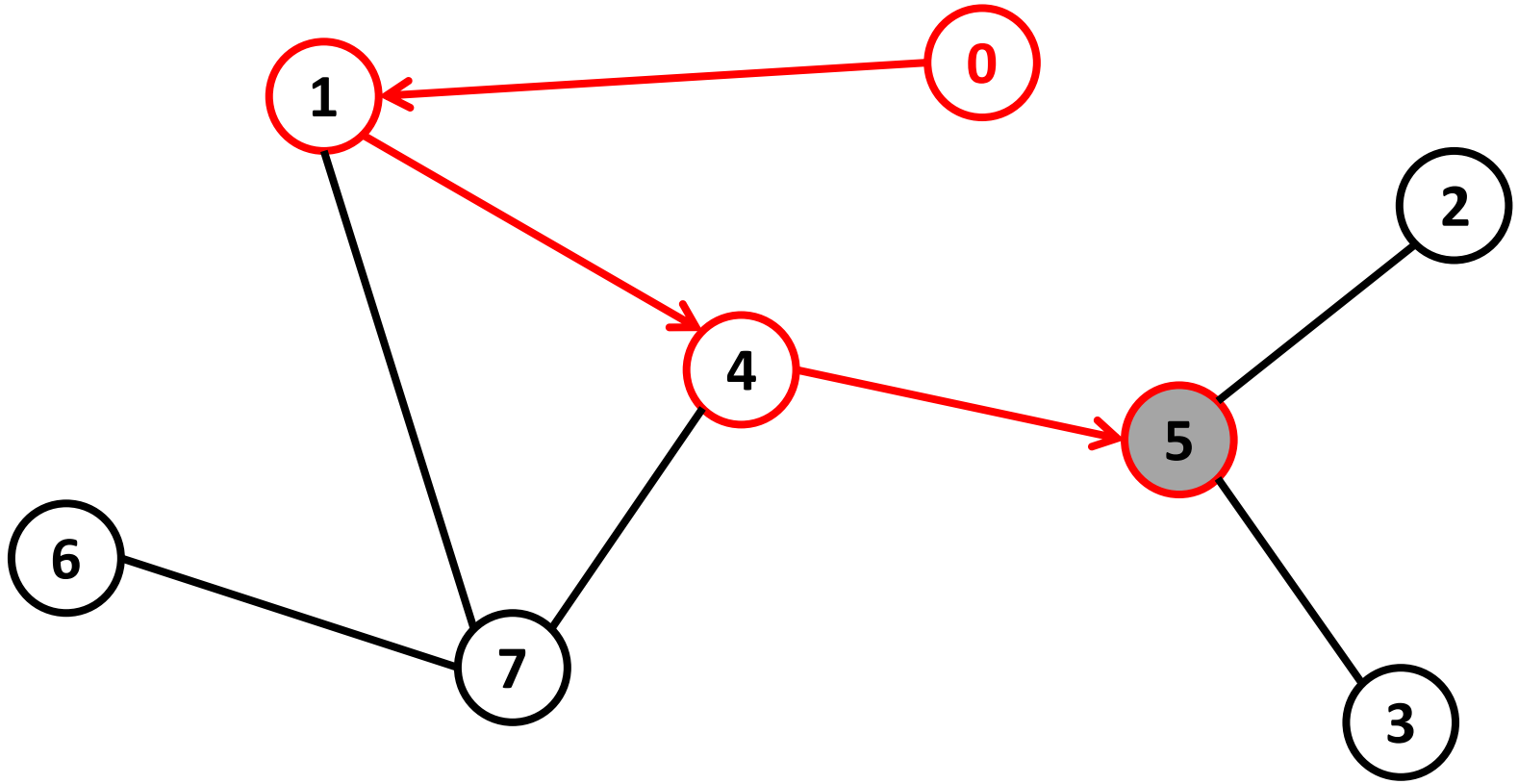

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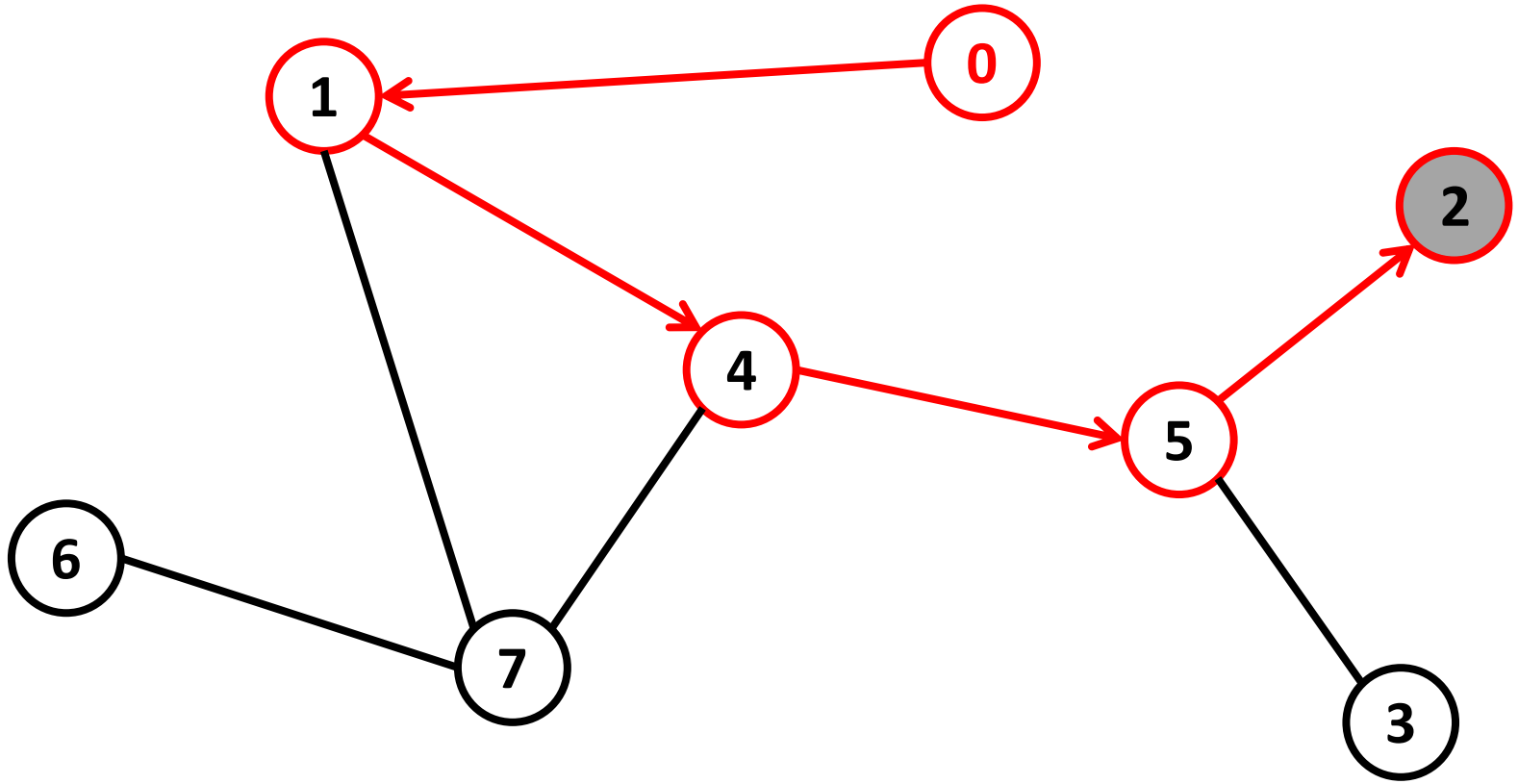

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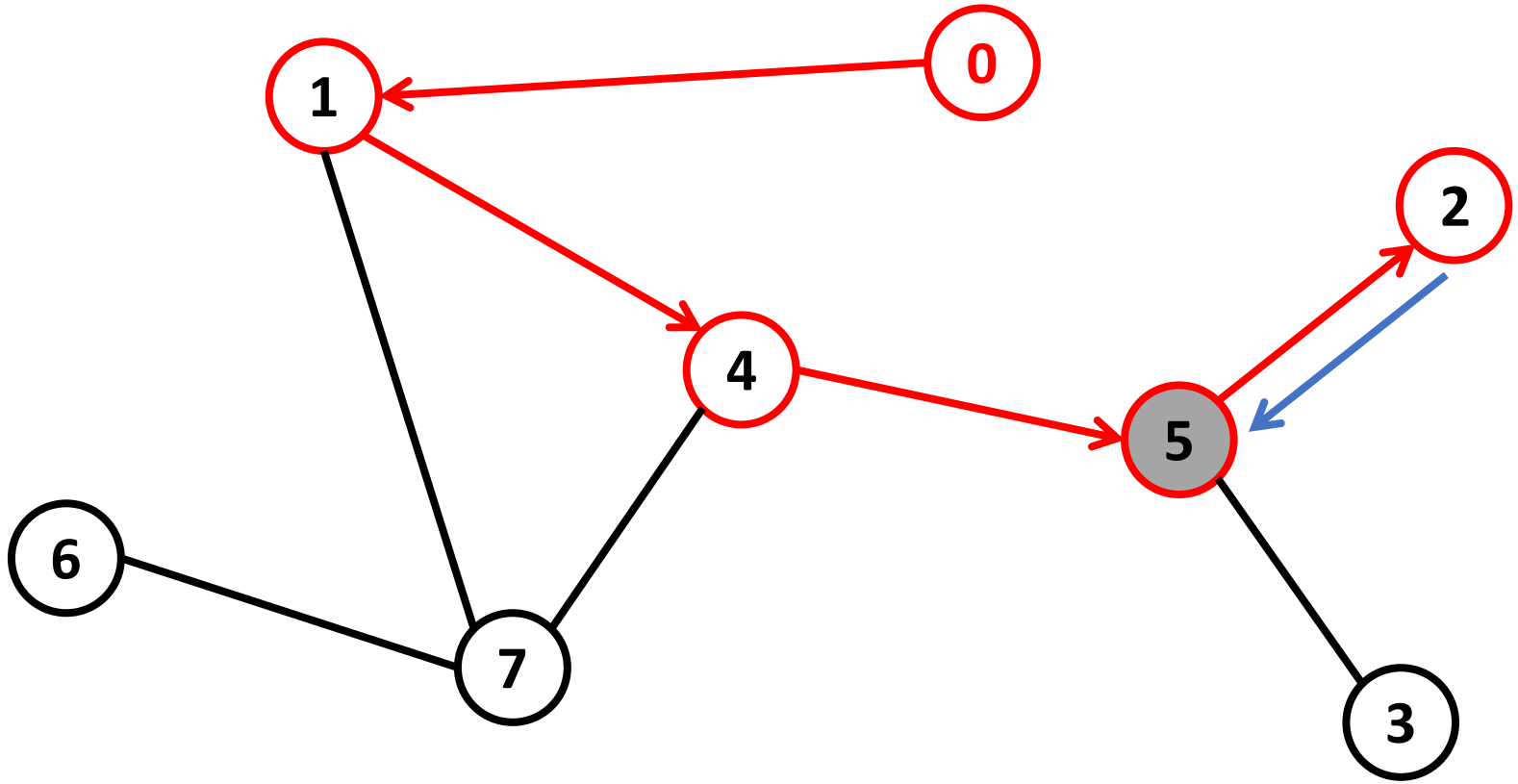

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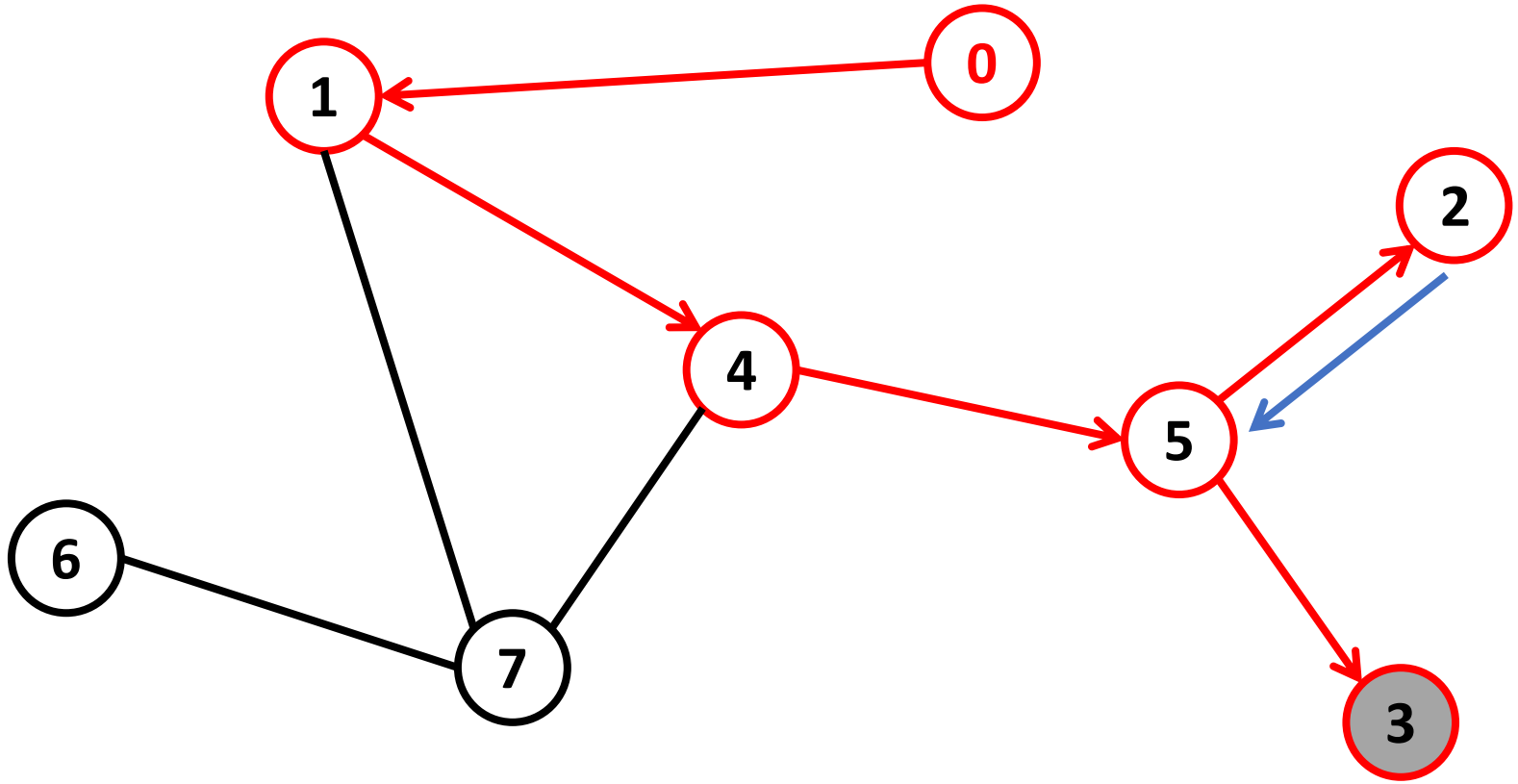

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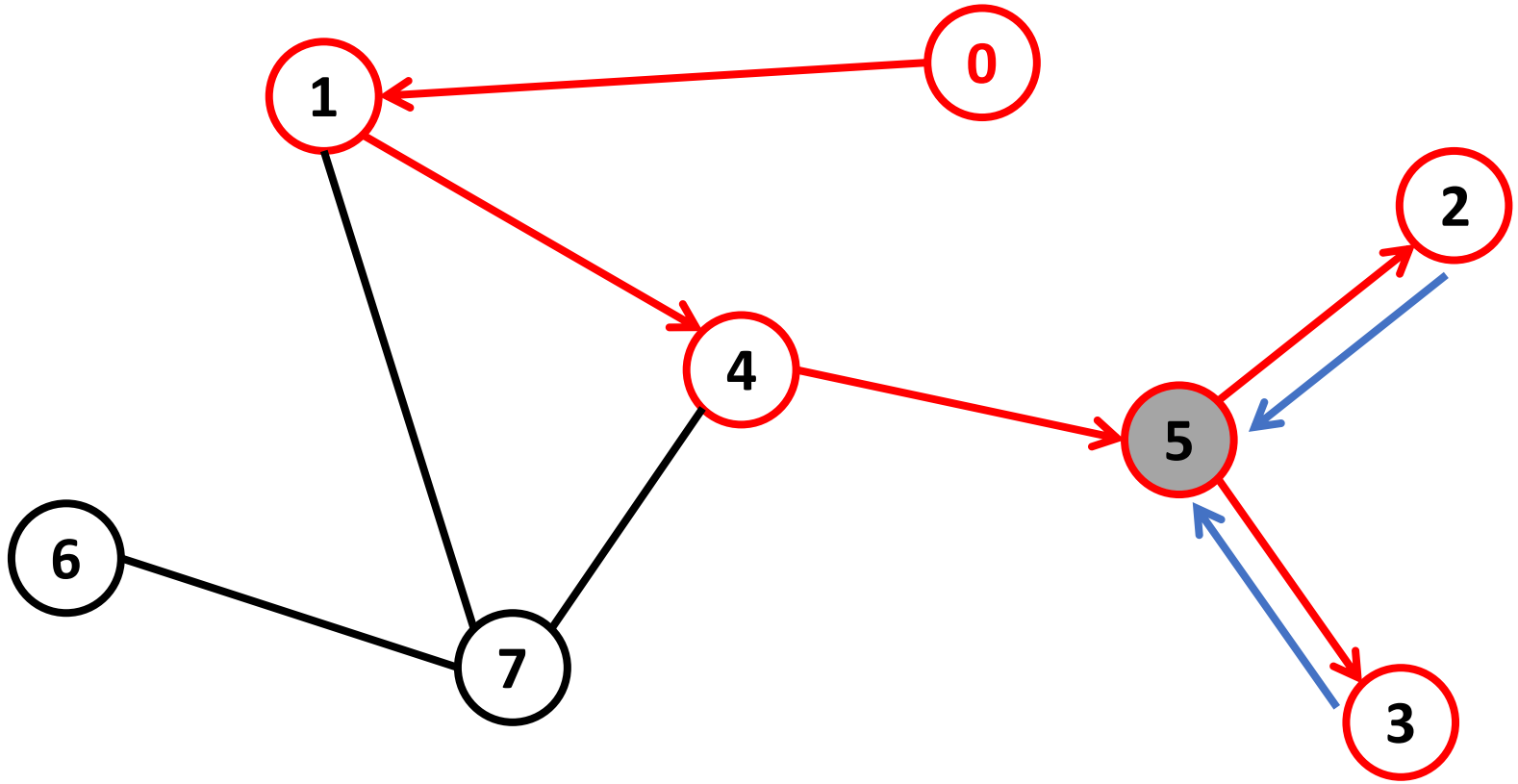

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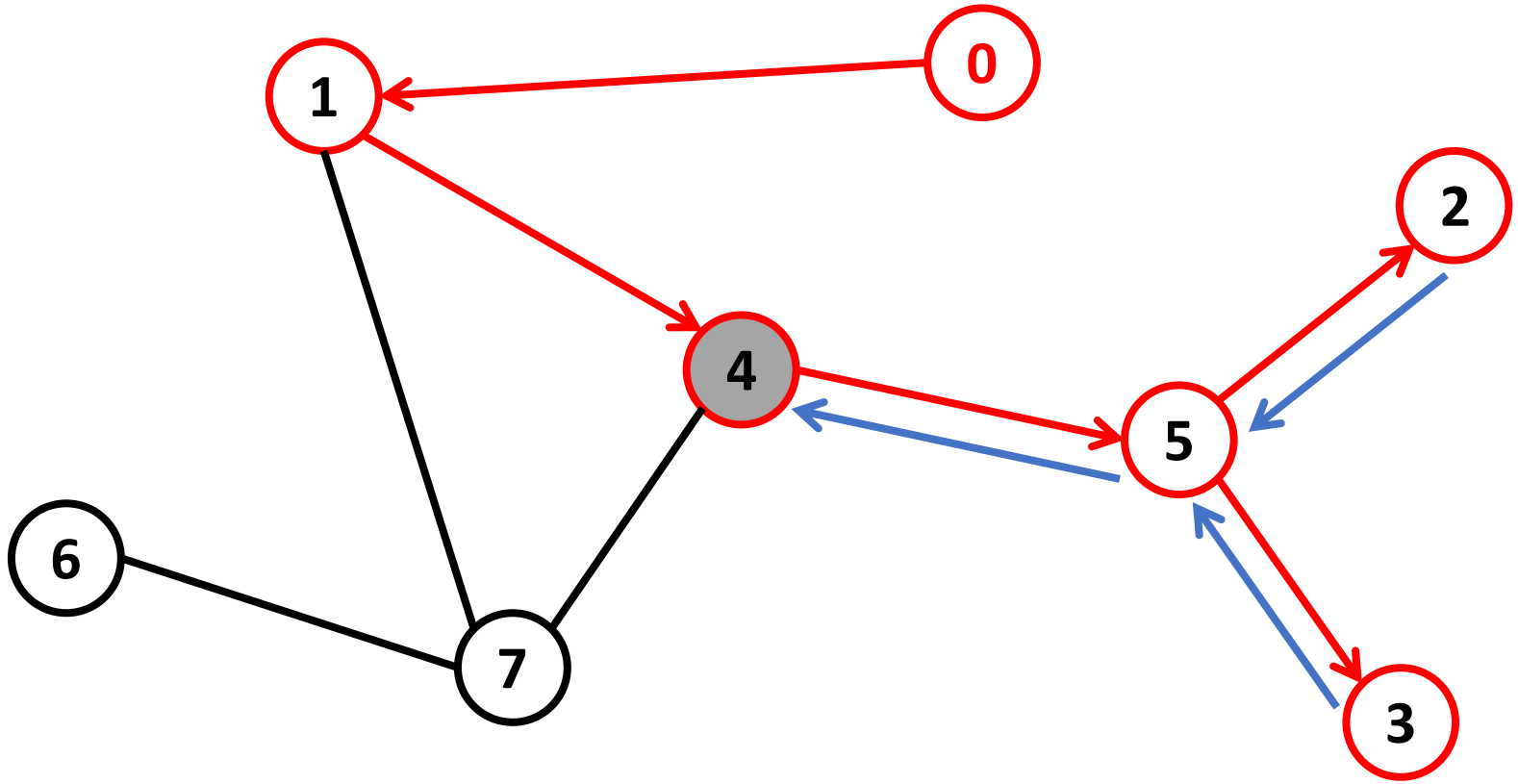

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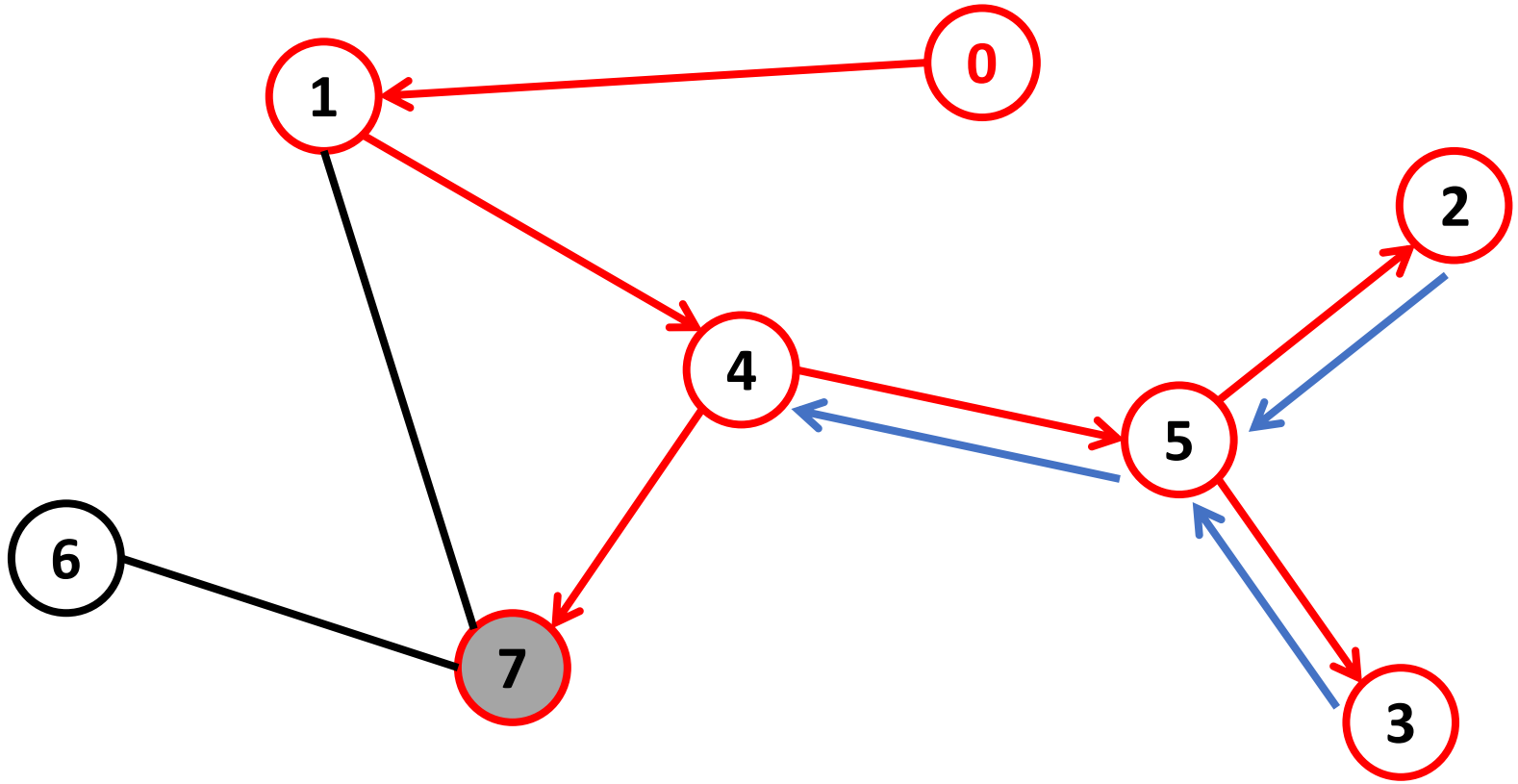

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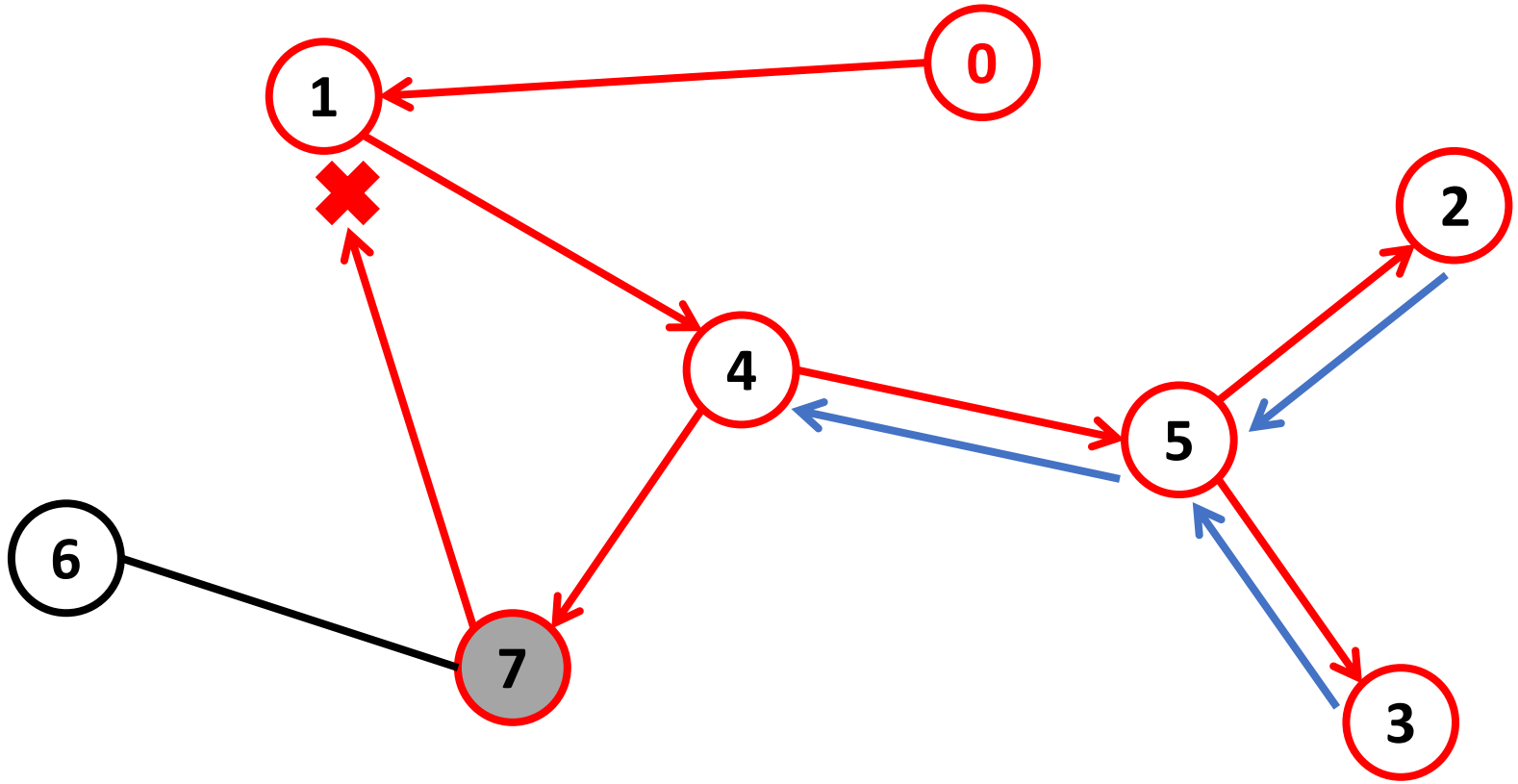

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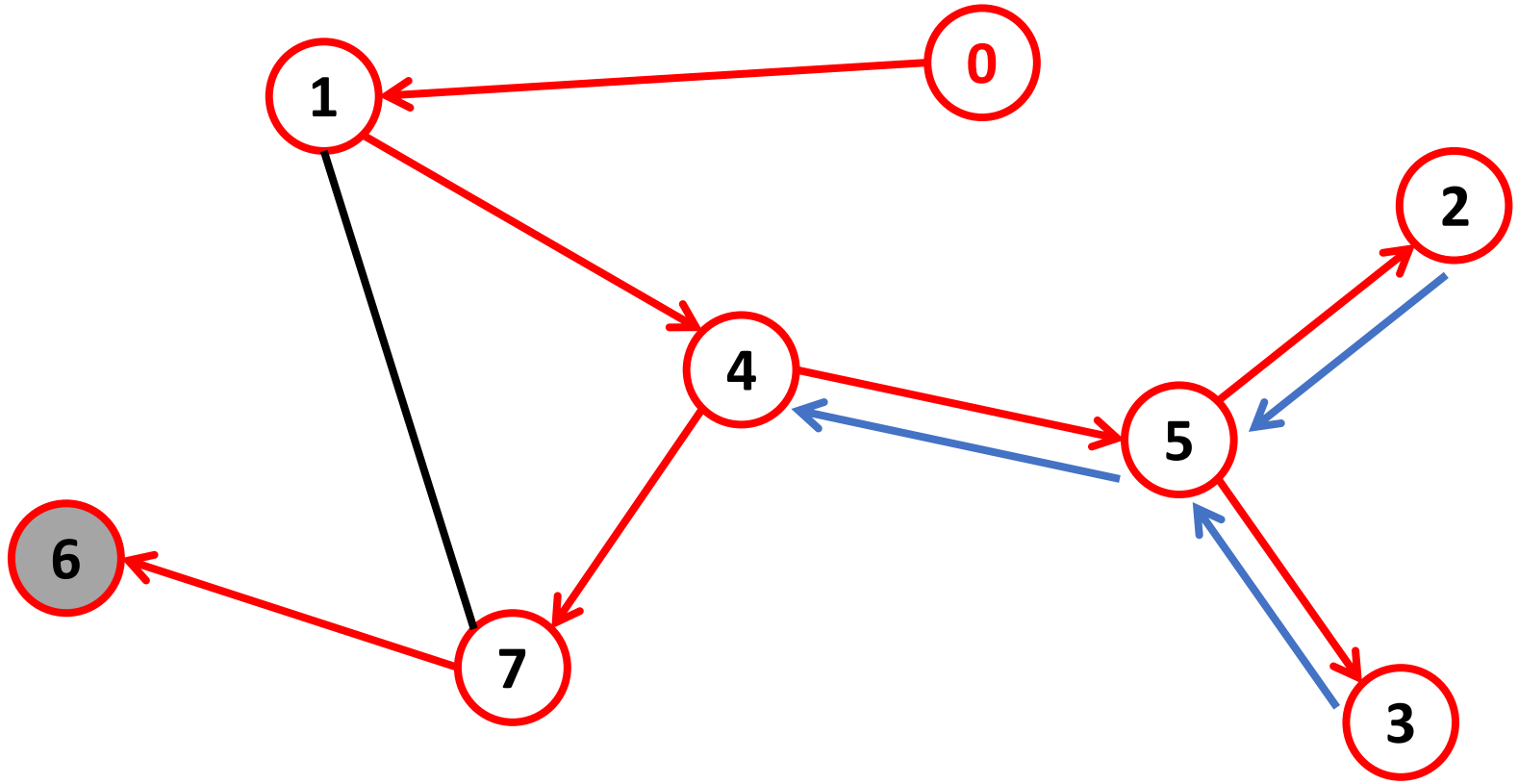

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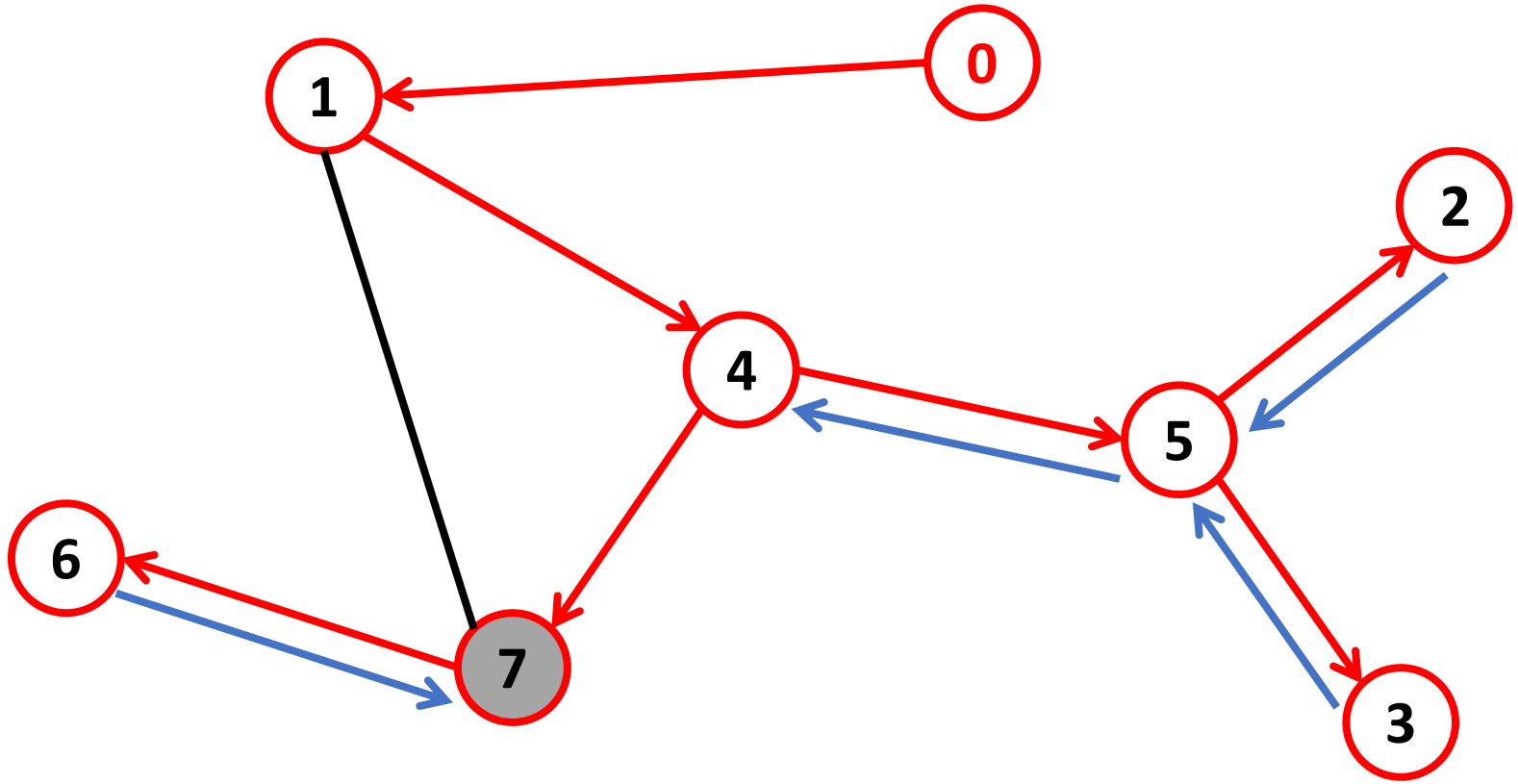

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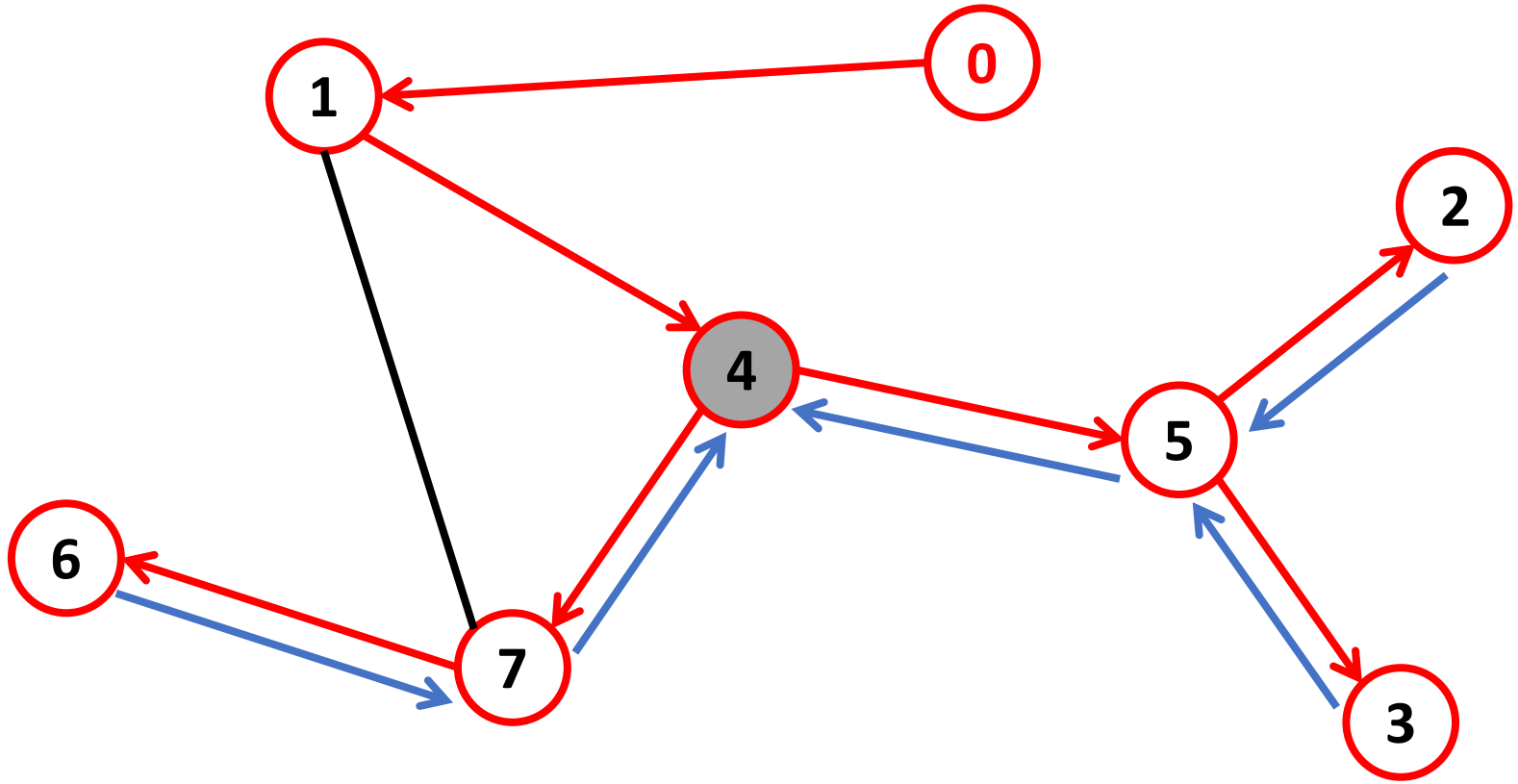

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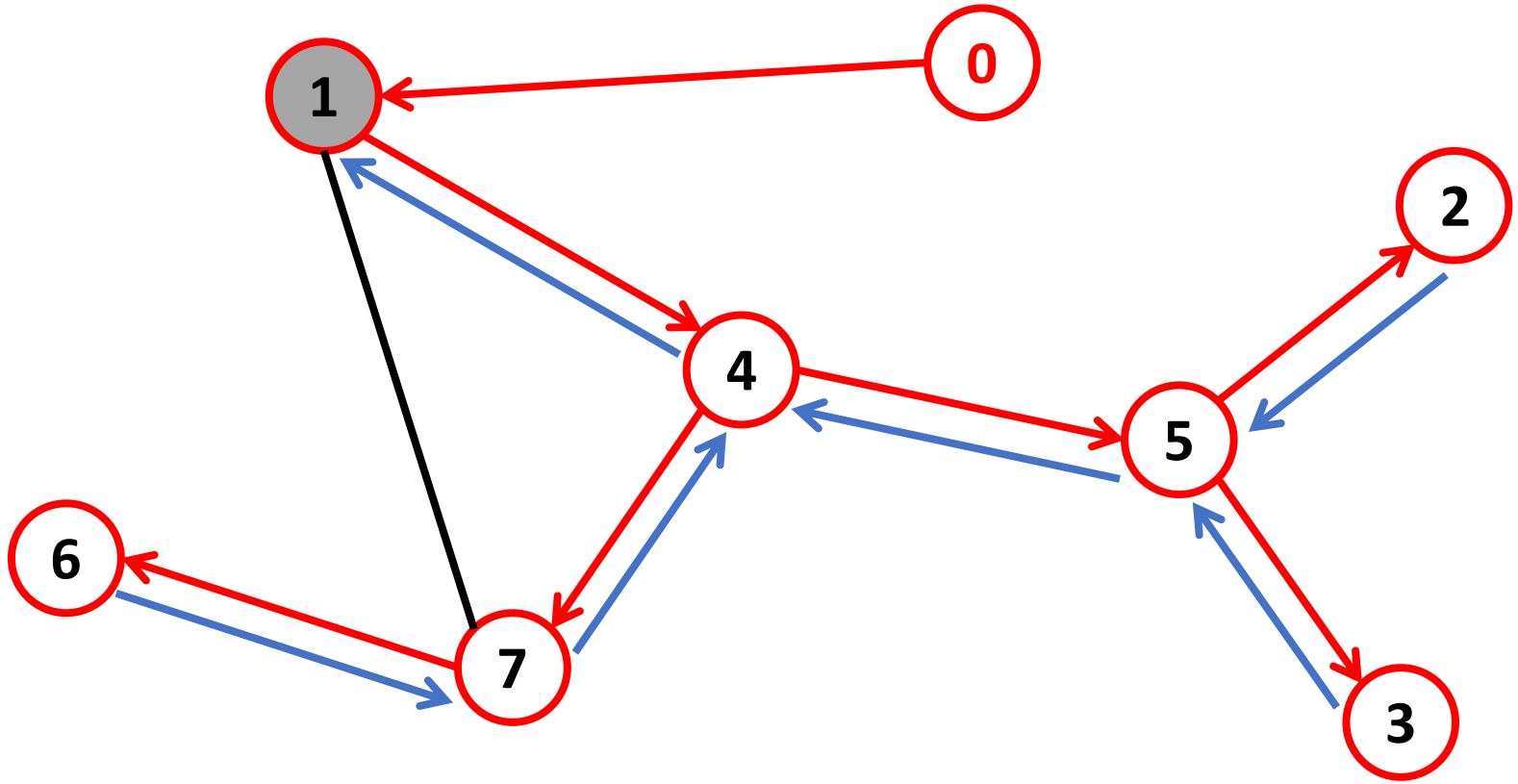

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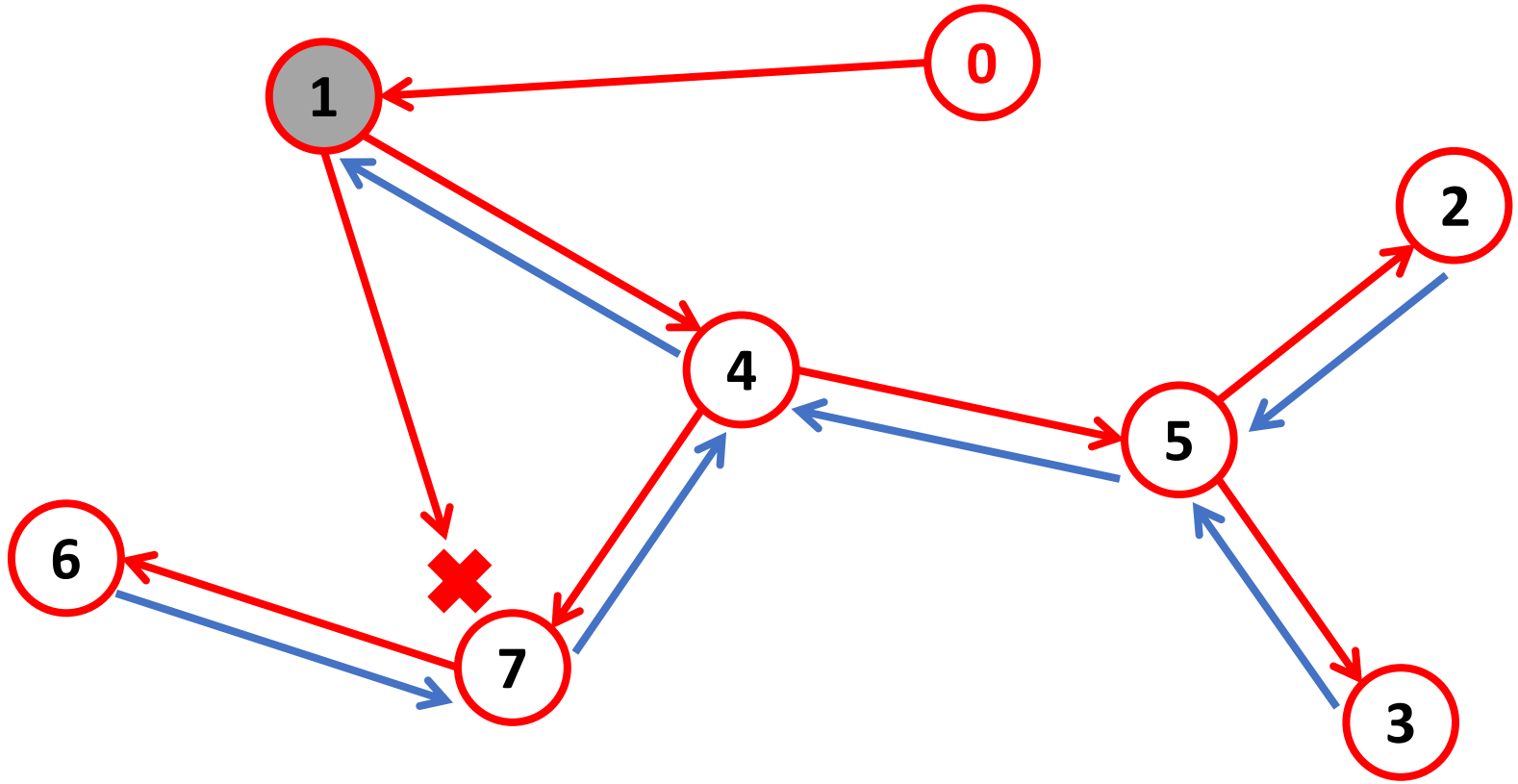

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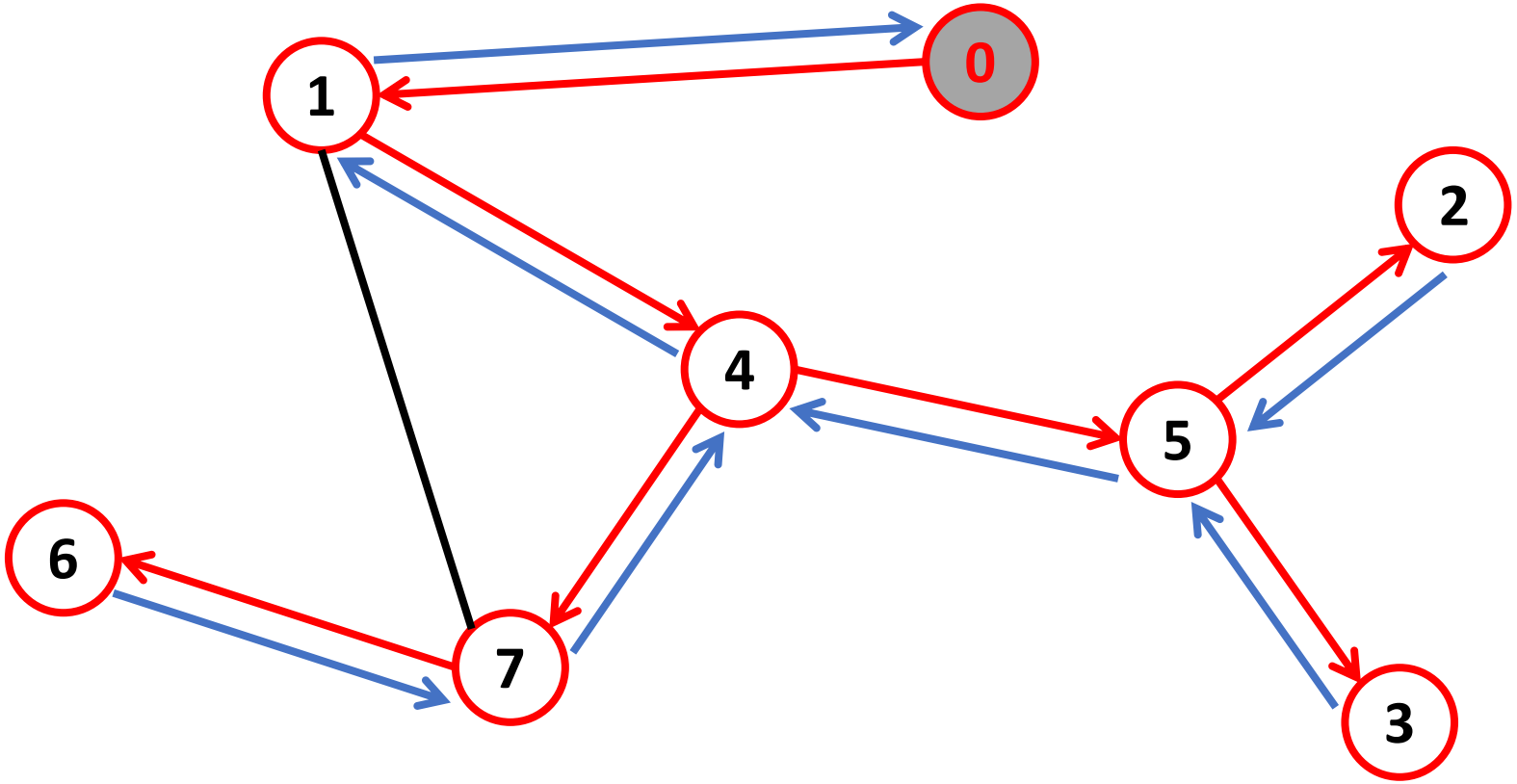

Código

Necesitamos:

```
vector<int> g[n];

bool vis[n];
```

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```
void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      dfs(v);
    }
  }
}
```

Código

Iterativo:

```
stack<int> st;
vector<int> vis(n, false);
st.push(u);
while (st.size() > 0) {
  int u = st.top();
  st.pop();
  vis[u] = true;
  for (int v : adj[u]) {
    if (!vis[v]) {
      st.push(v);
    }
  }
}
```

Recursivo:

```
void dfs(int u) {
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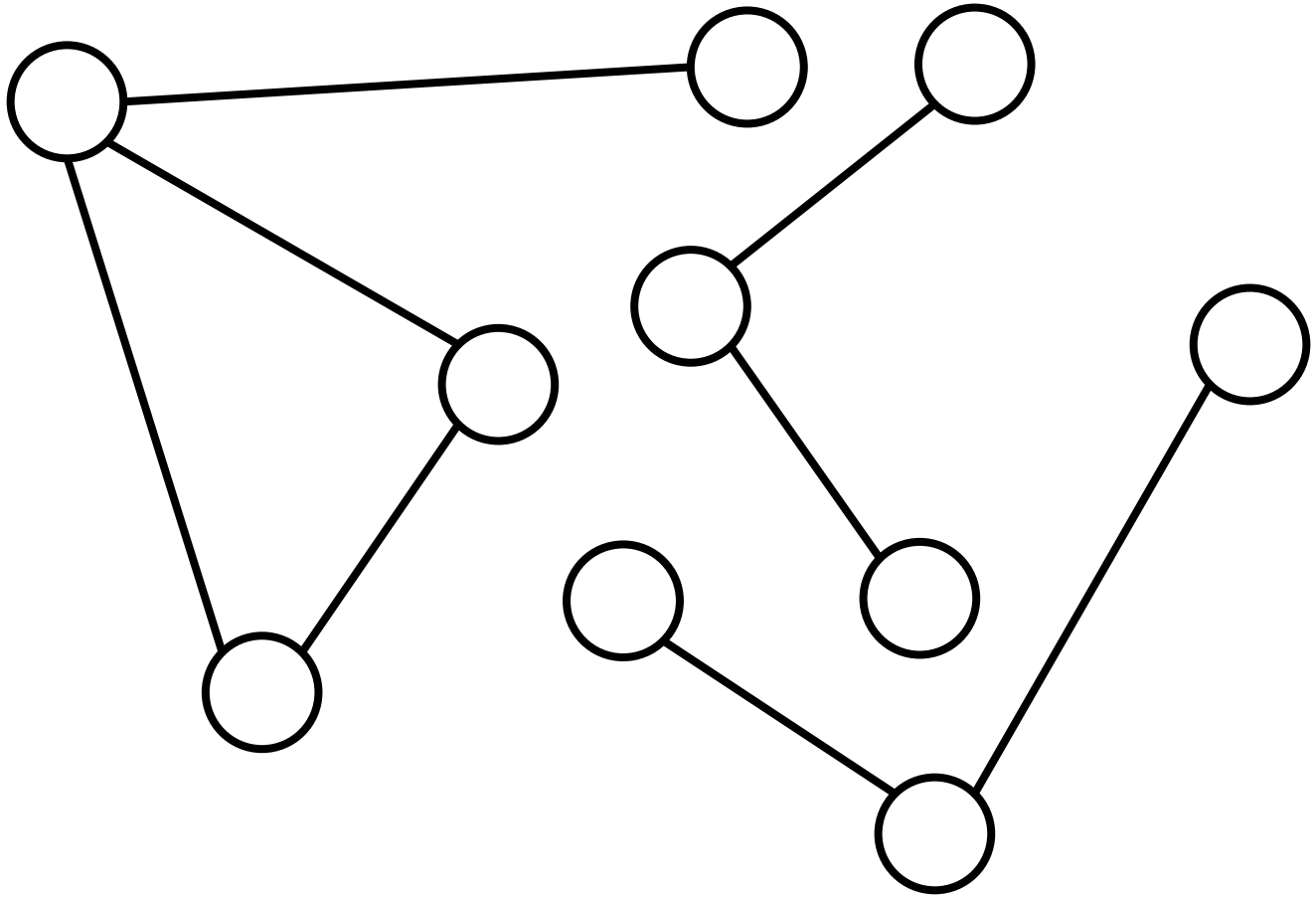
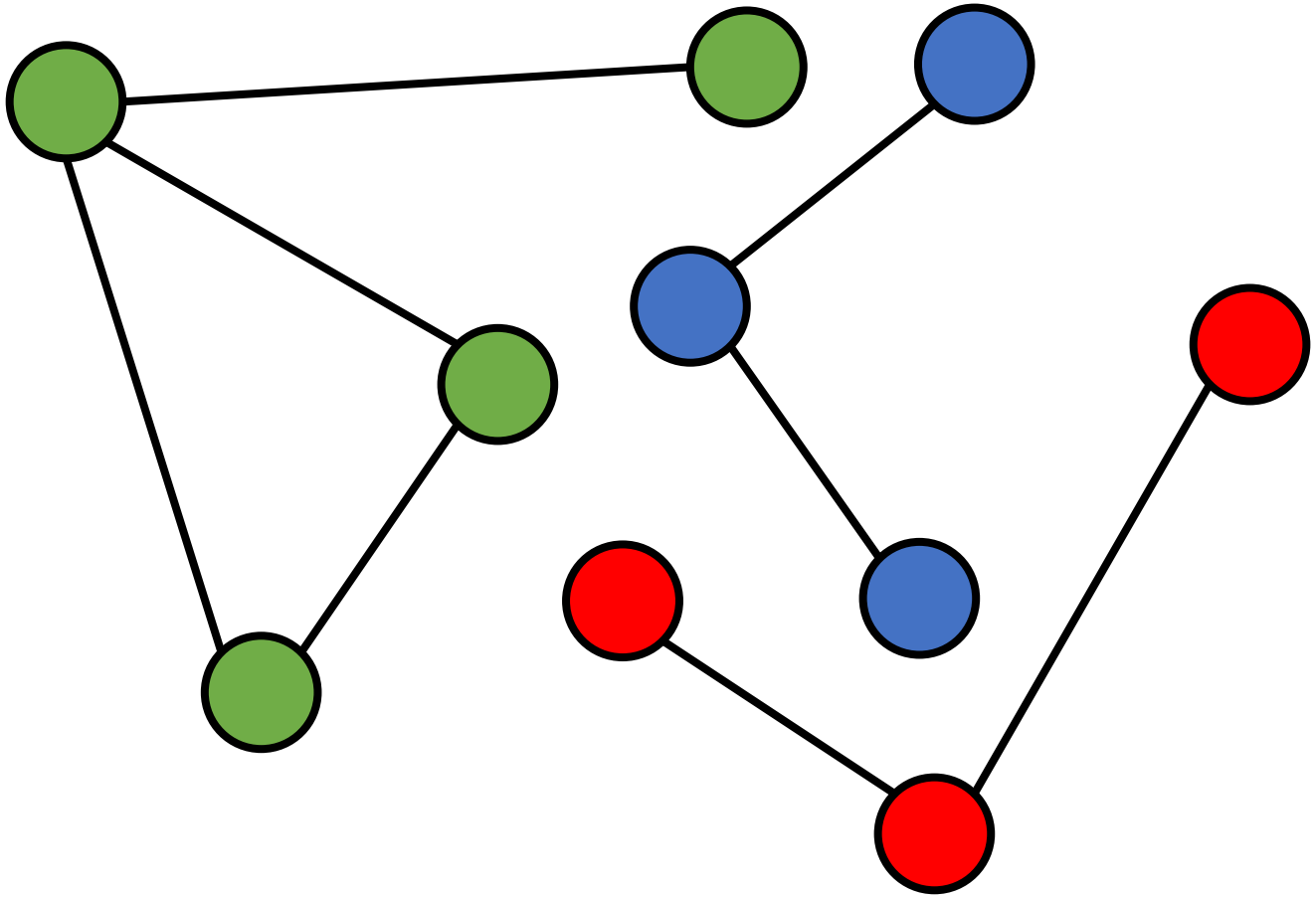
Aplicación – Verificar conectividad

Componentes Conexos



Aplicación – Verificar conectividad

Componentes Conexos



Aplicación – Verificar conectividad

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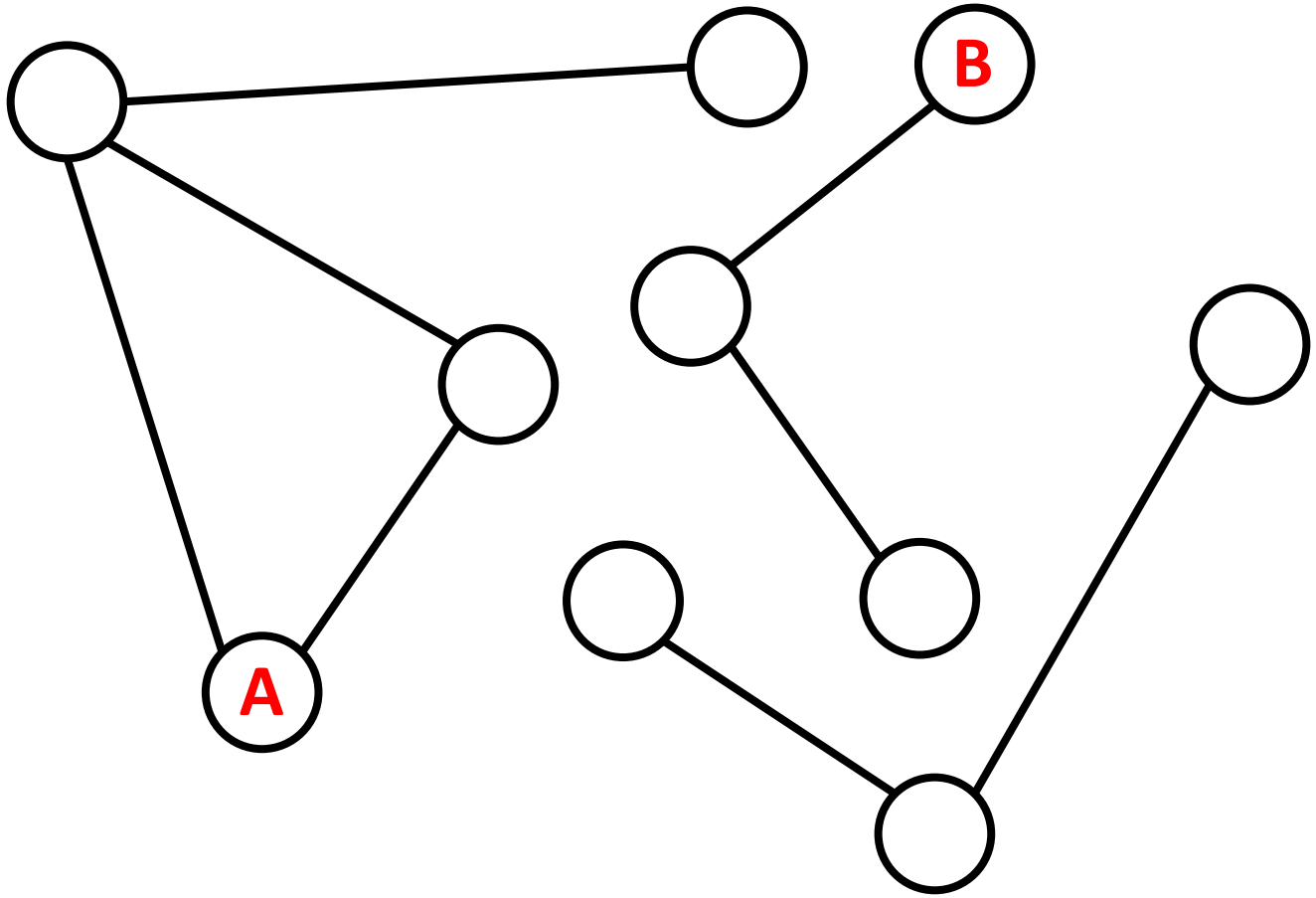
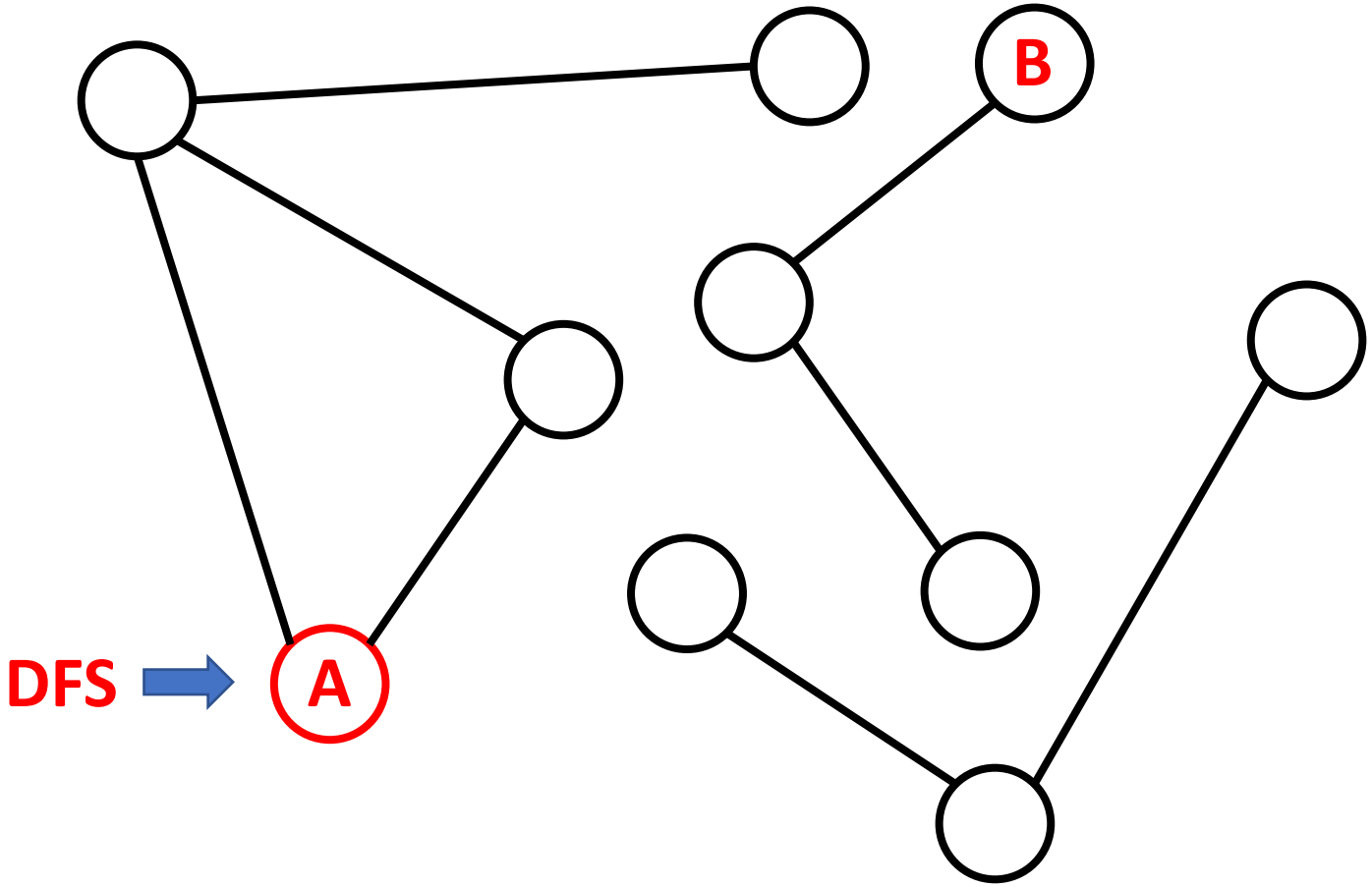


Aplicación – Verificar conectividad

Componentes Conexos



Aplicación – Verificar conectividad

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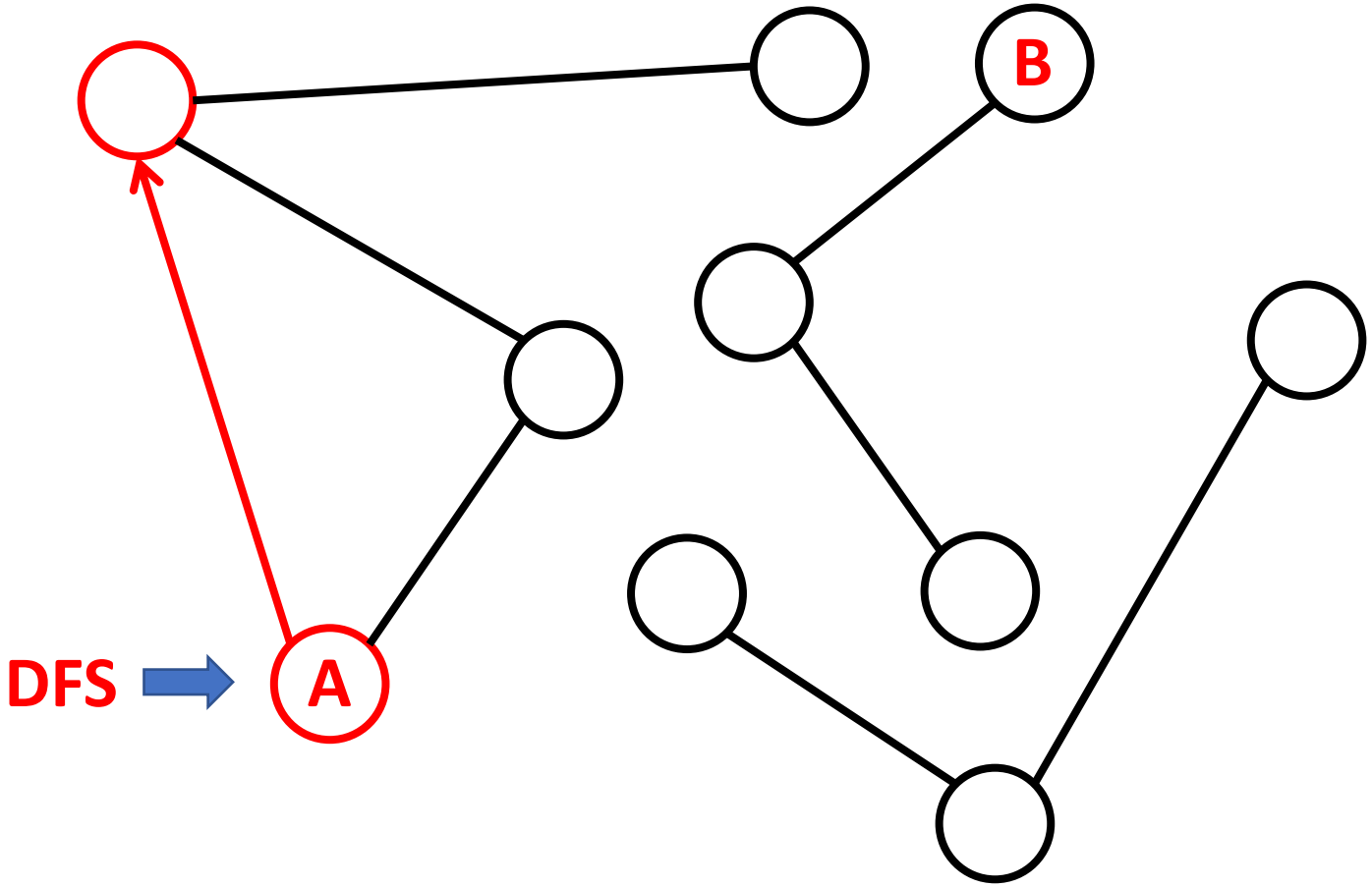

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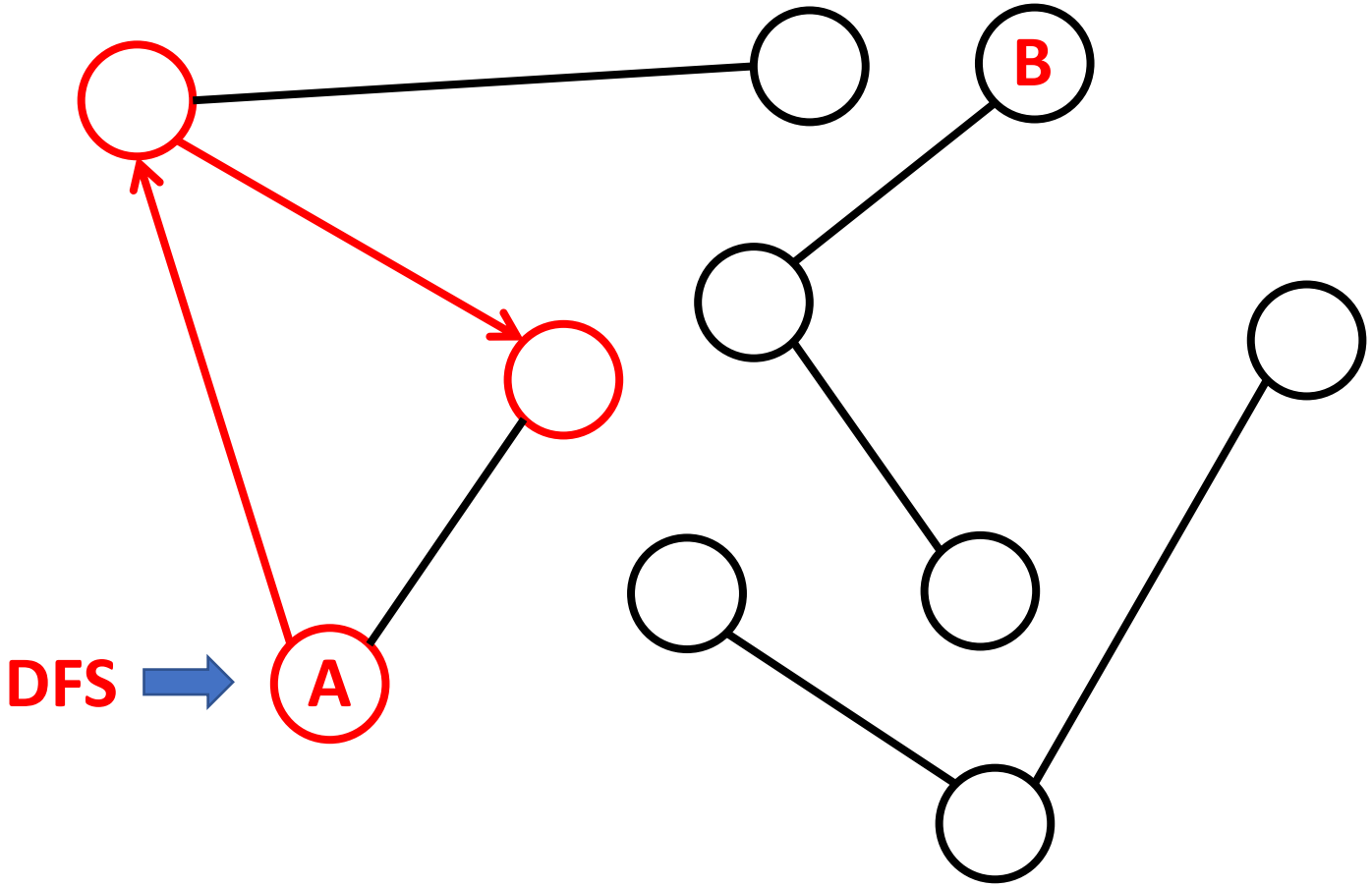

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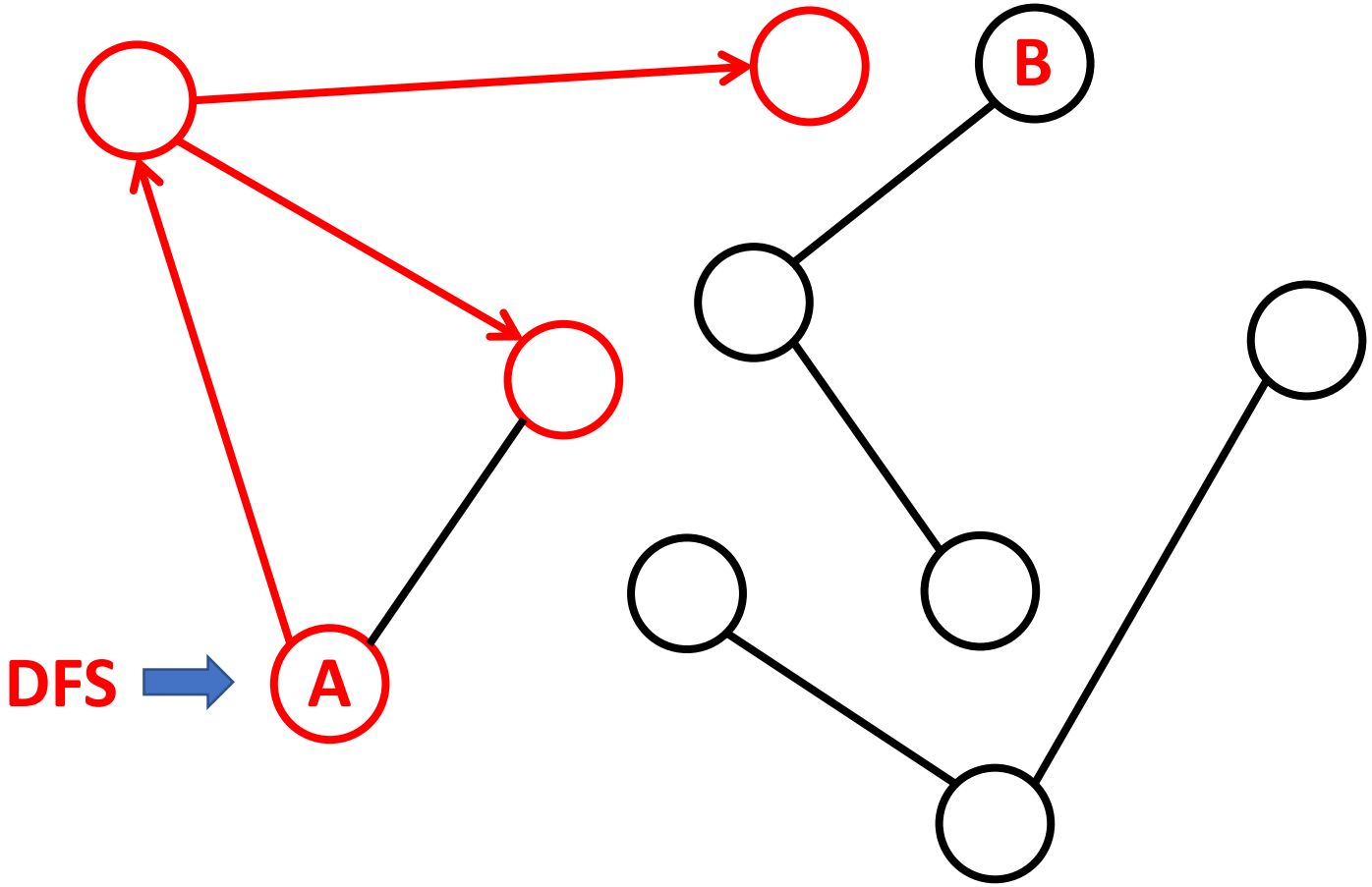

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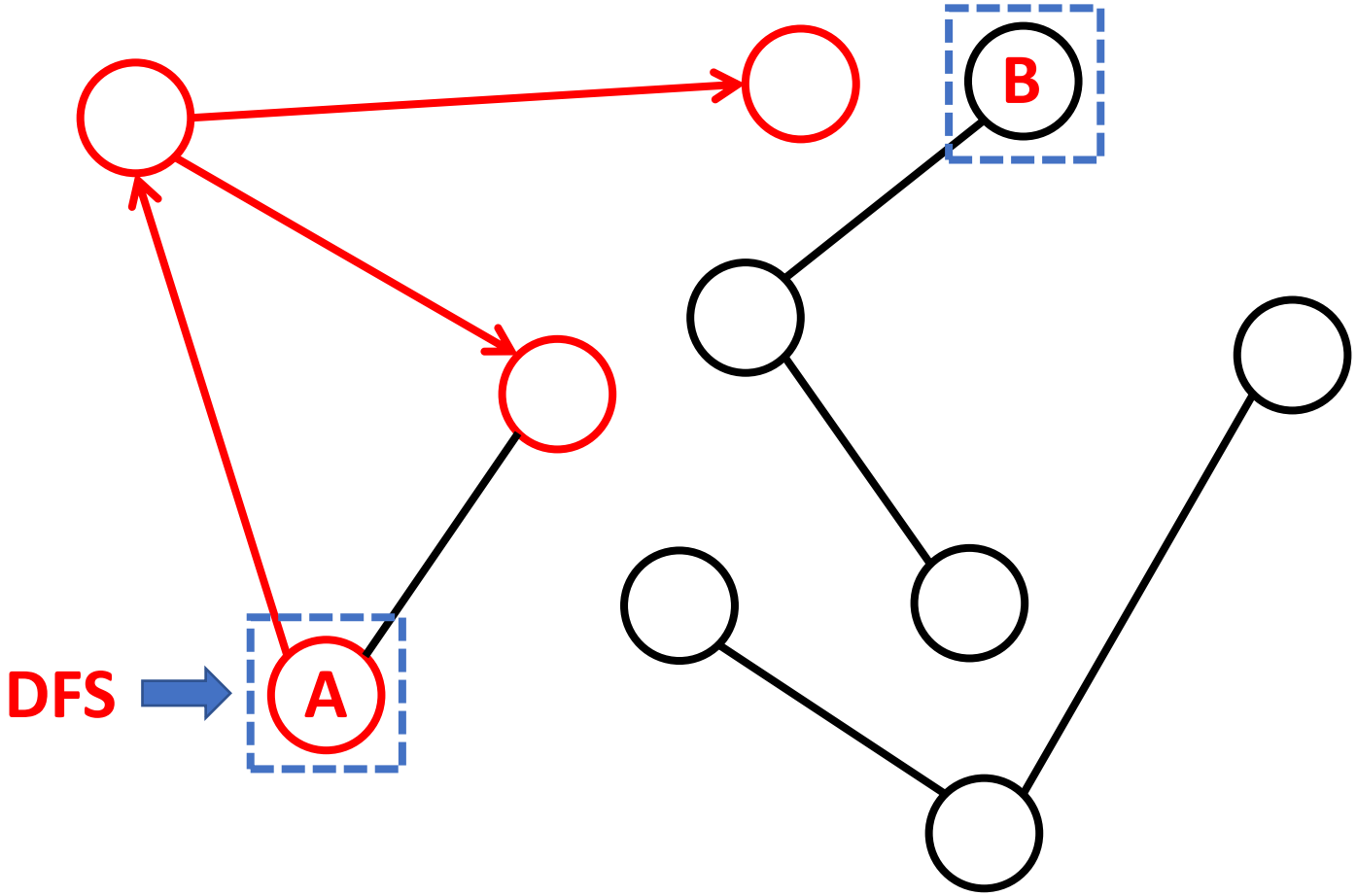
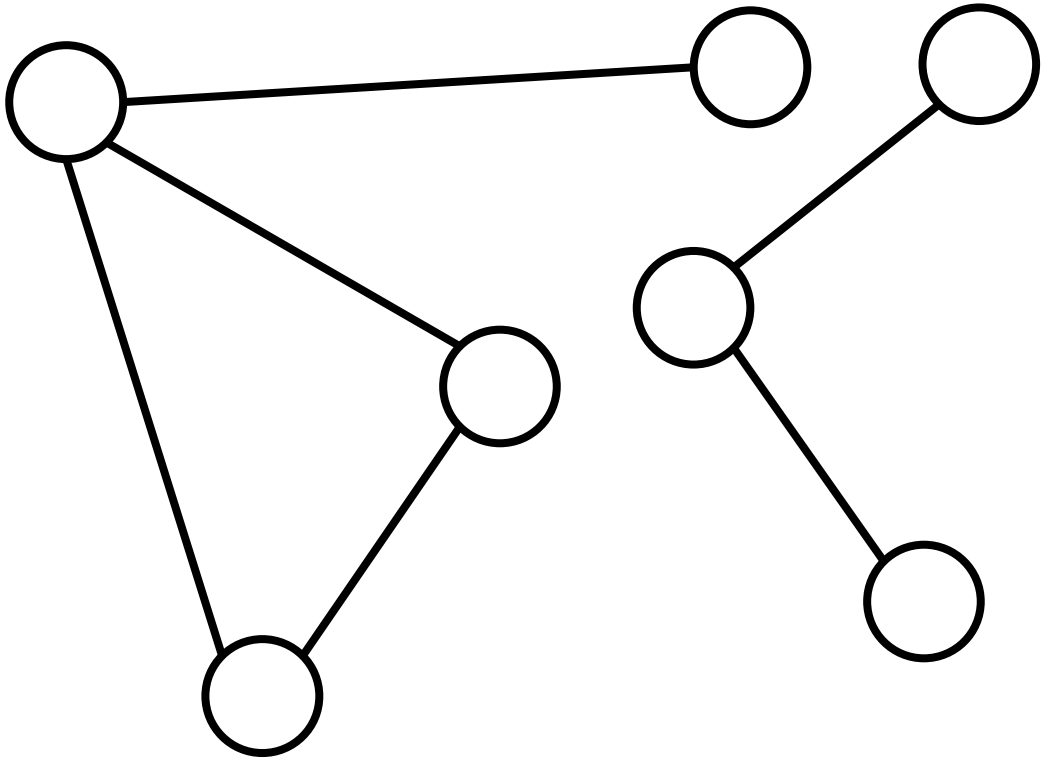


Aplicación – Detectar Ciclos



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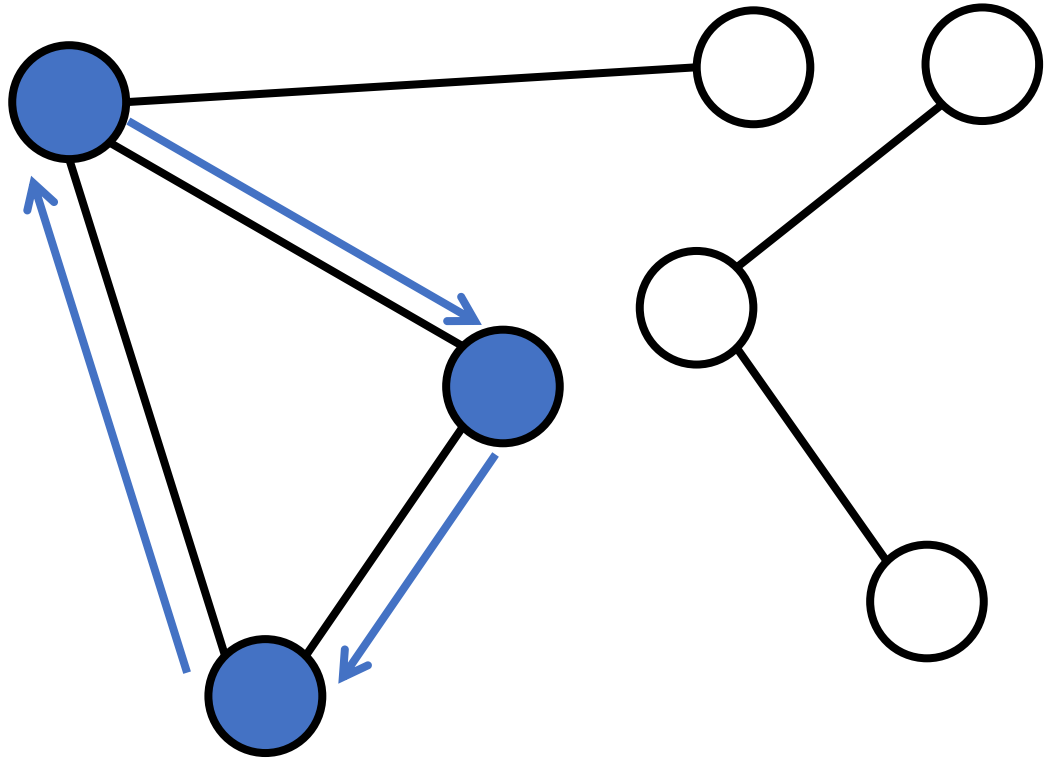

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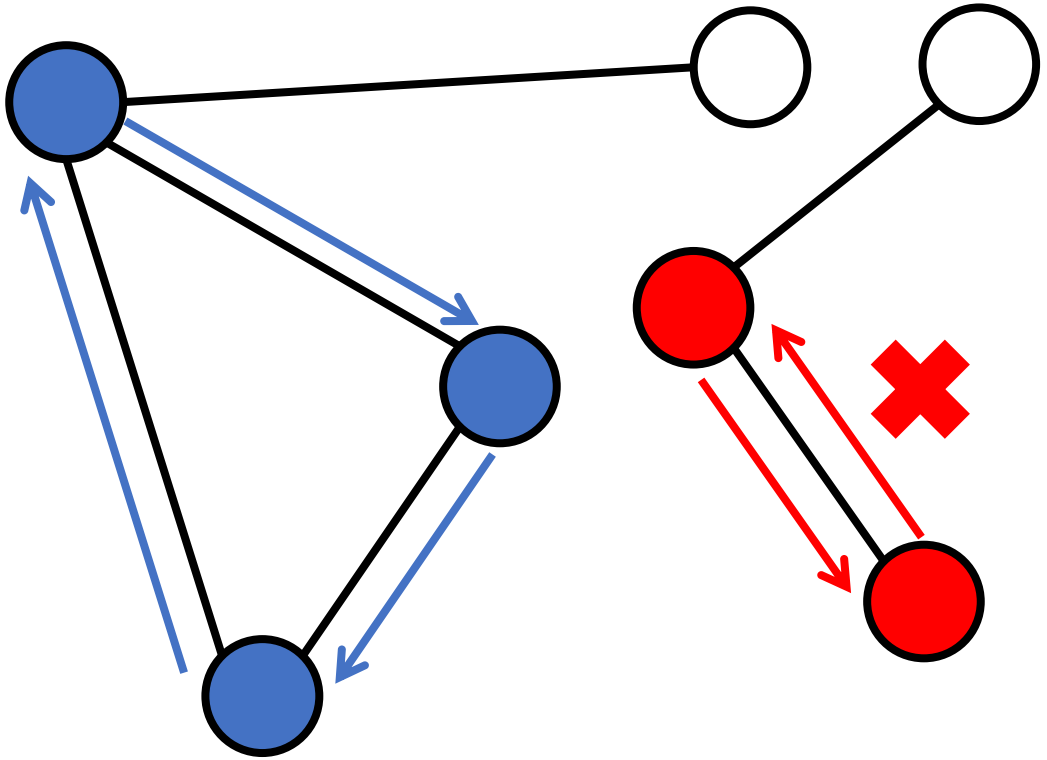

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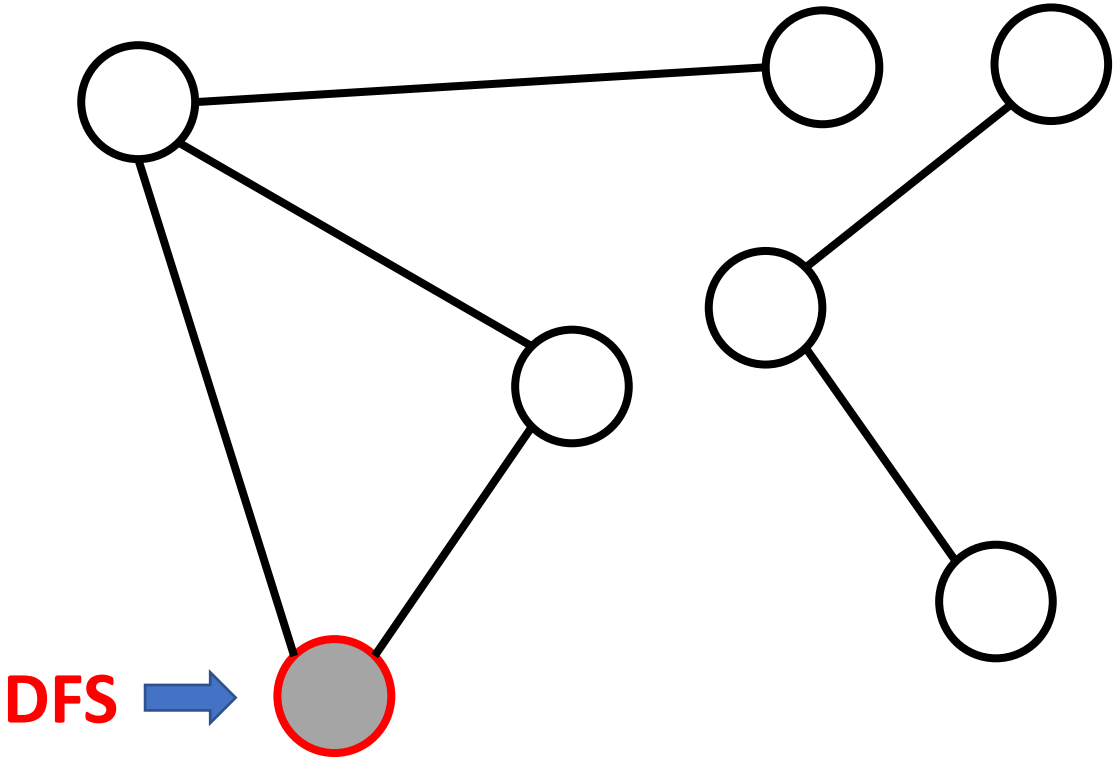

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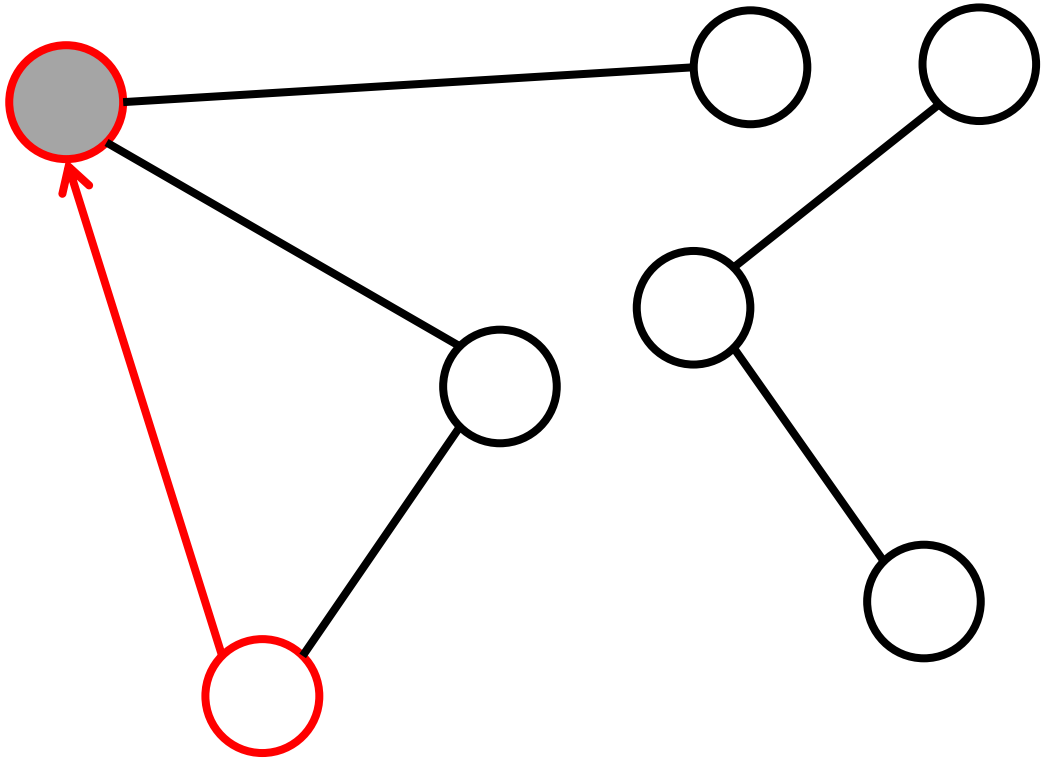

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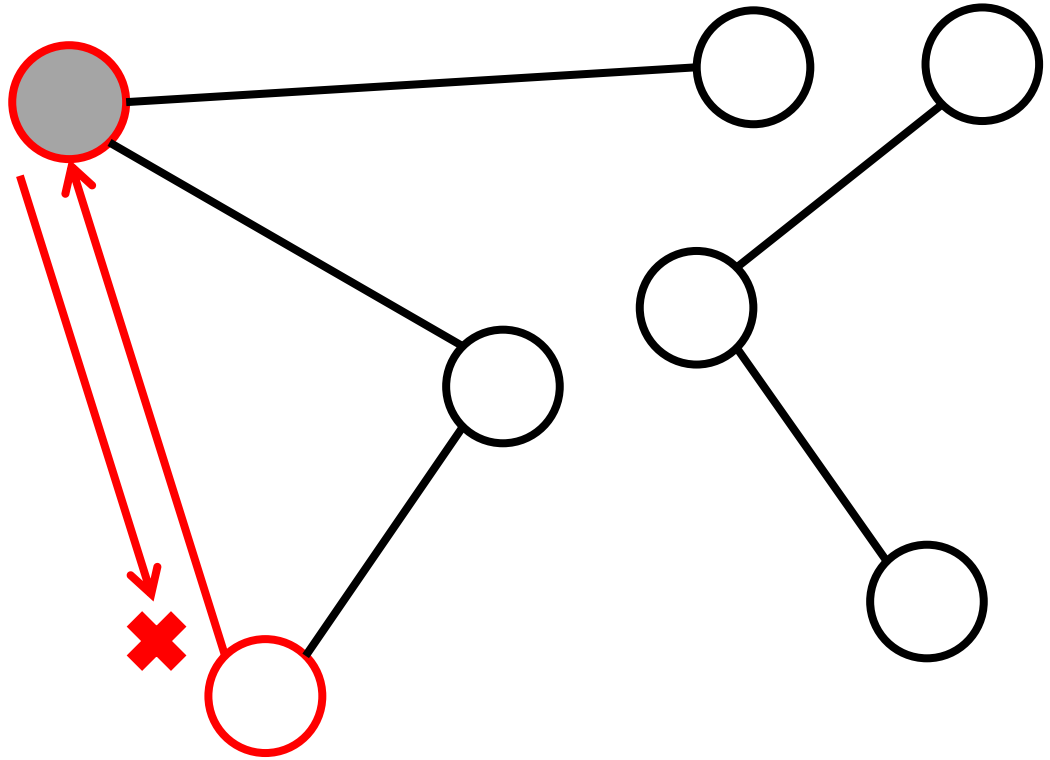

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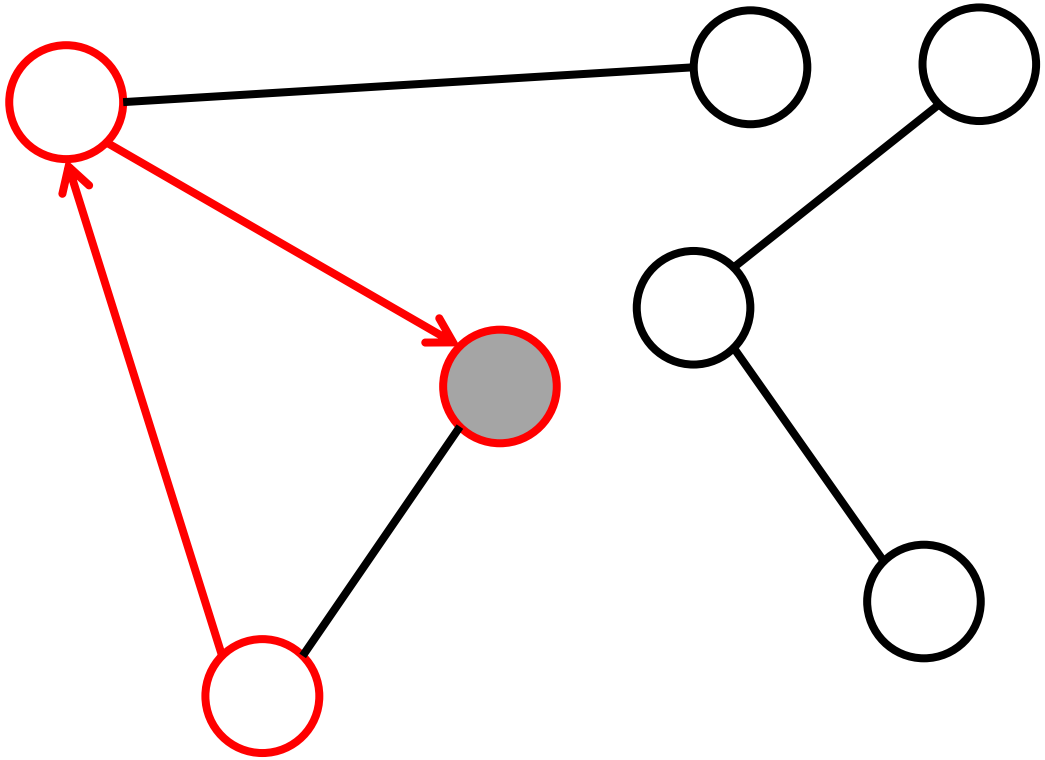

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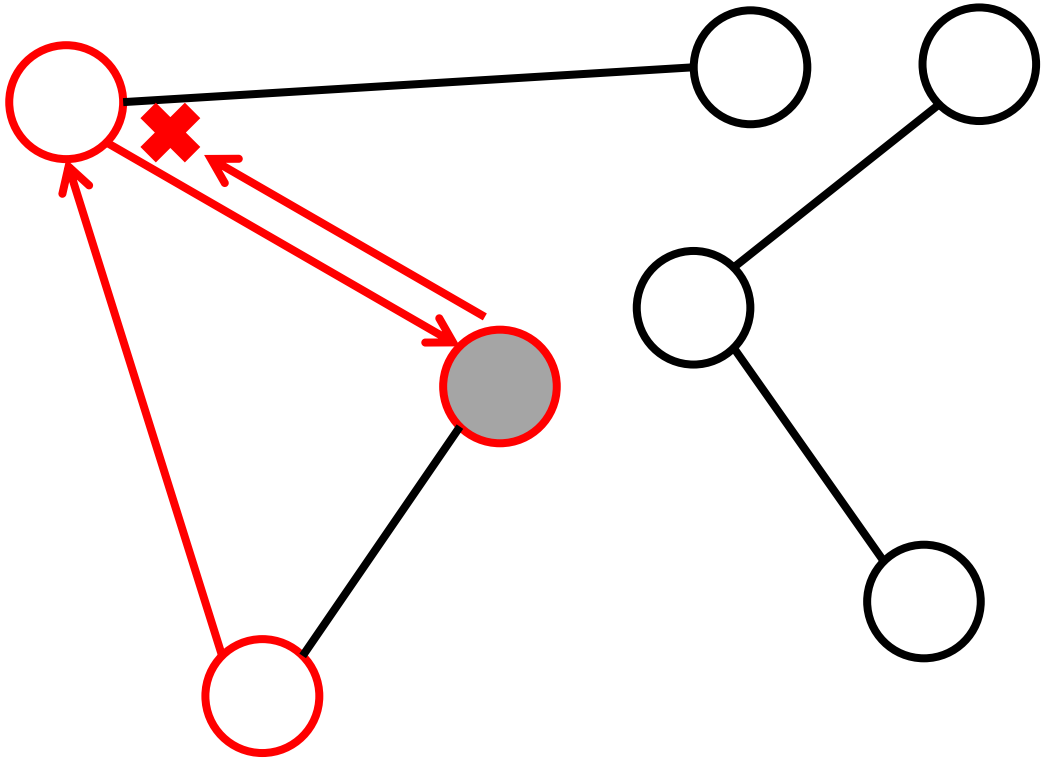

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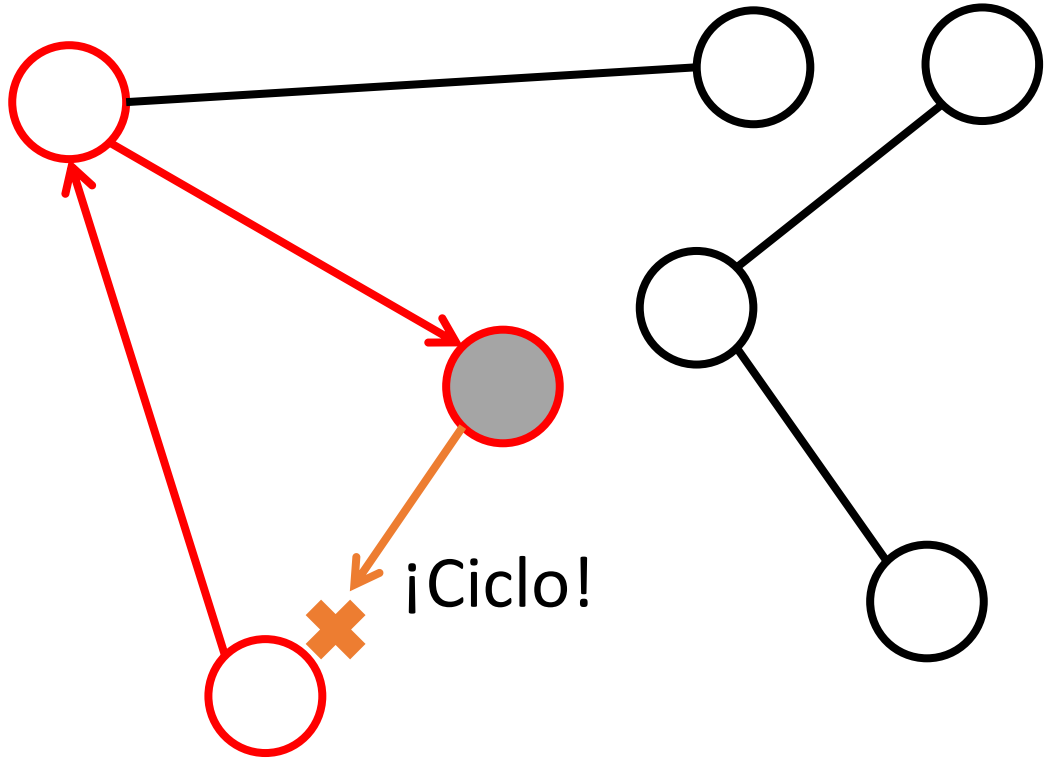

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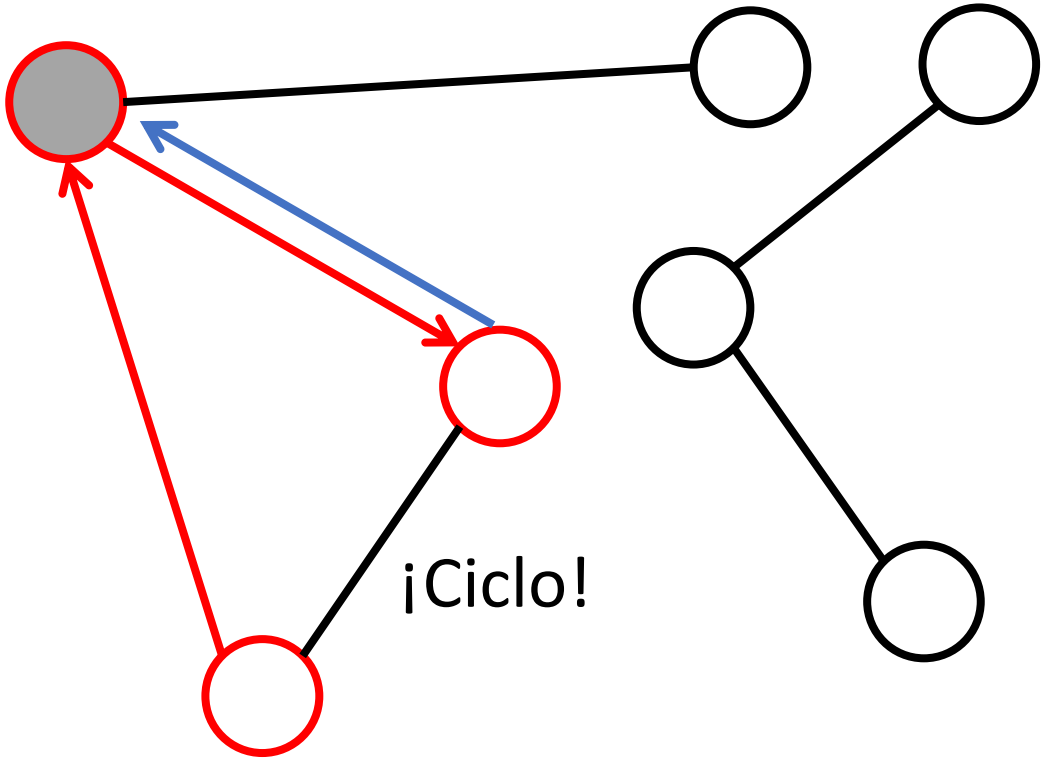
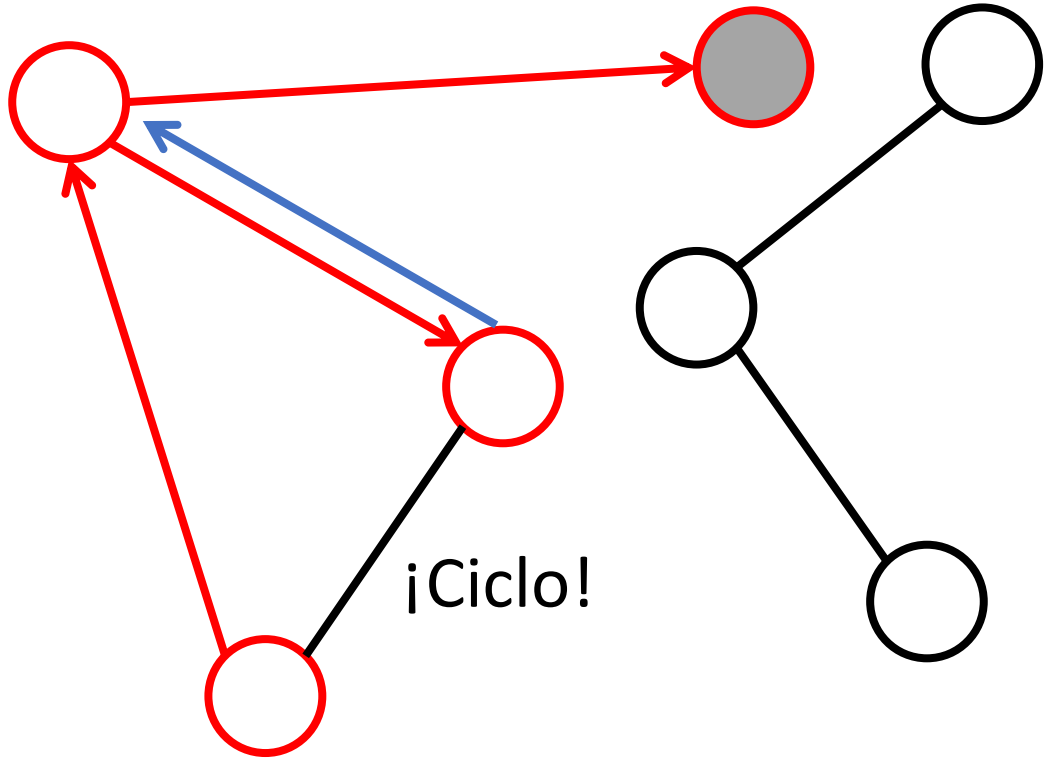
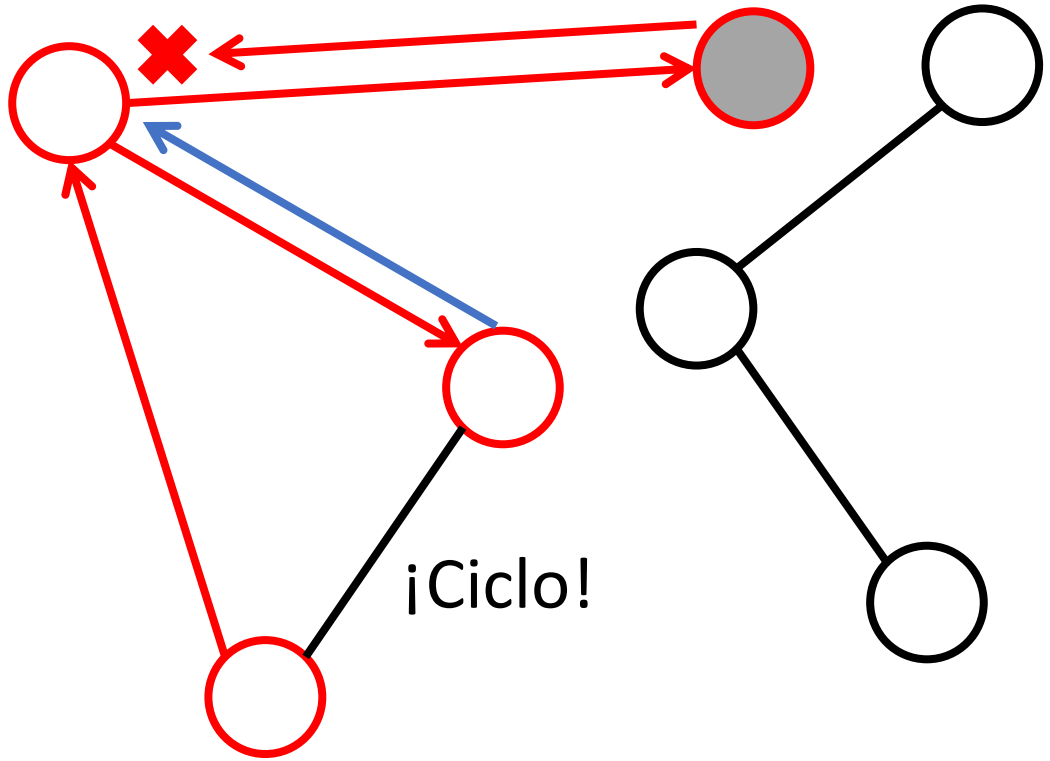


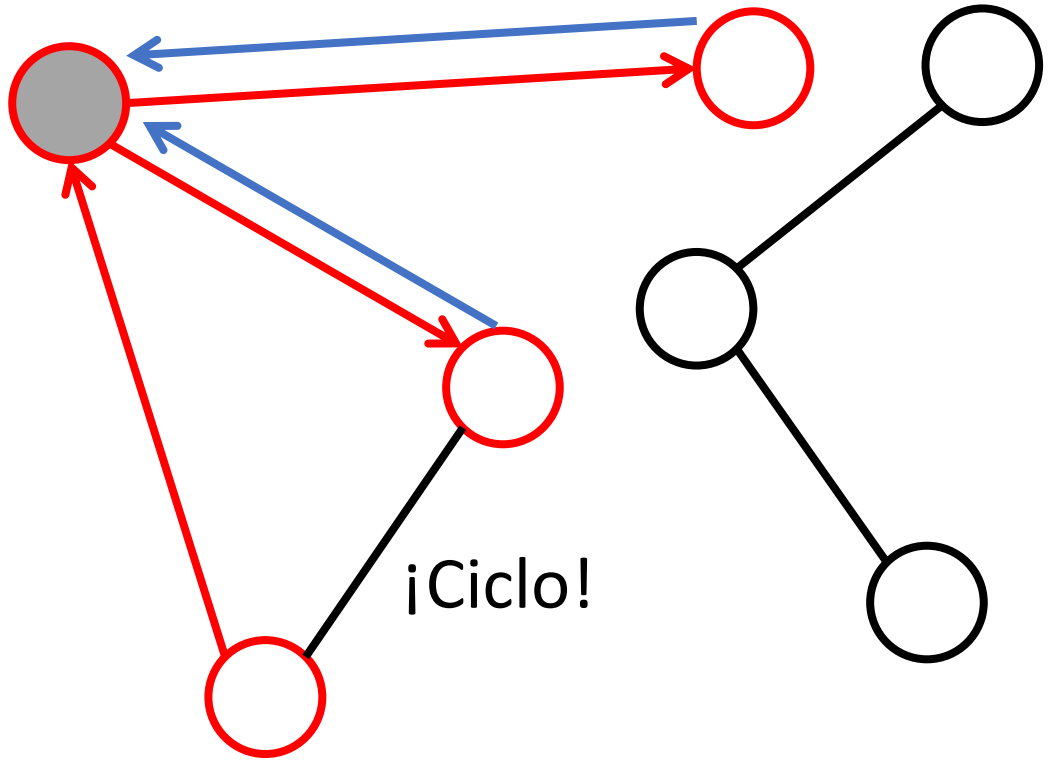
Aplicación – Detectar Ciclos



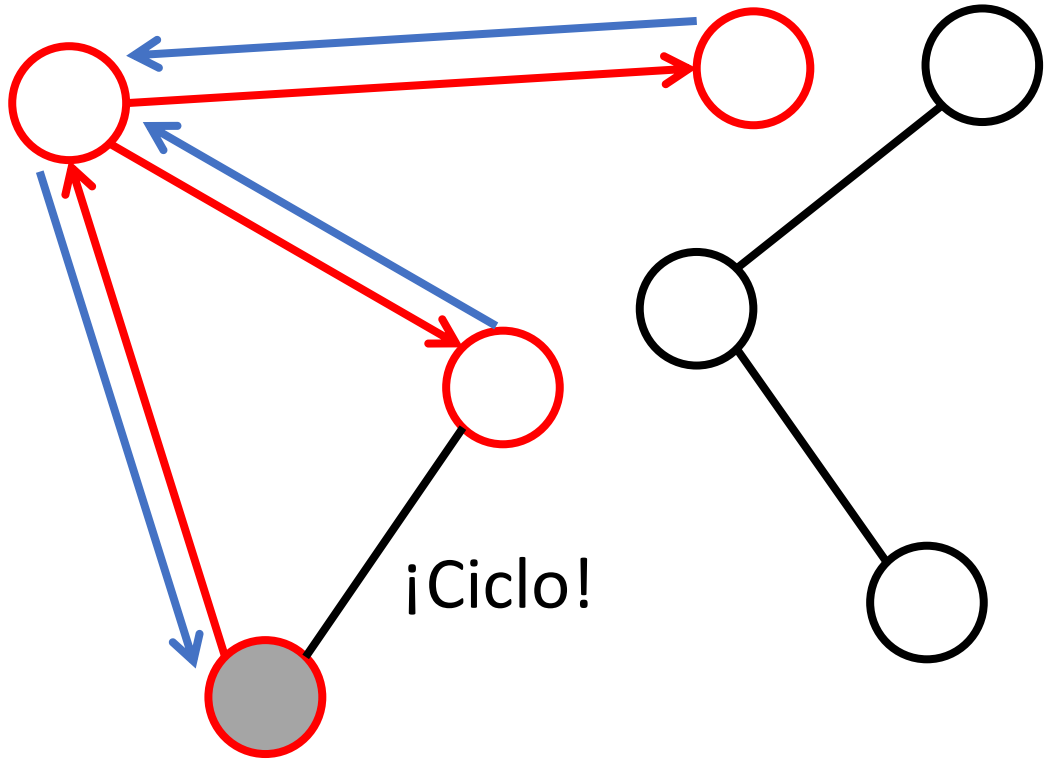
Aplicación – Detectar Ciclos



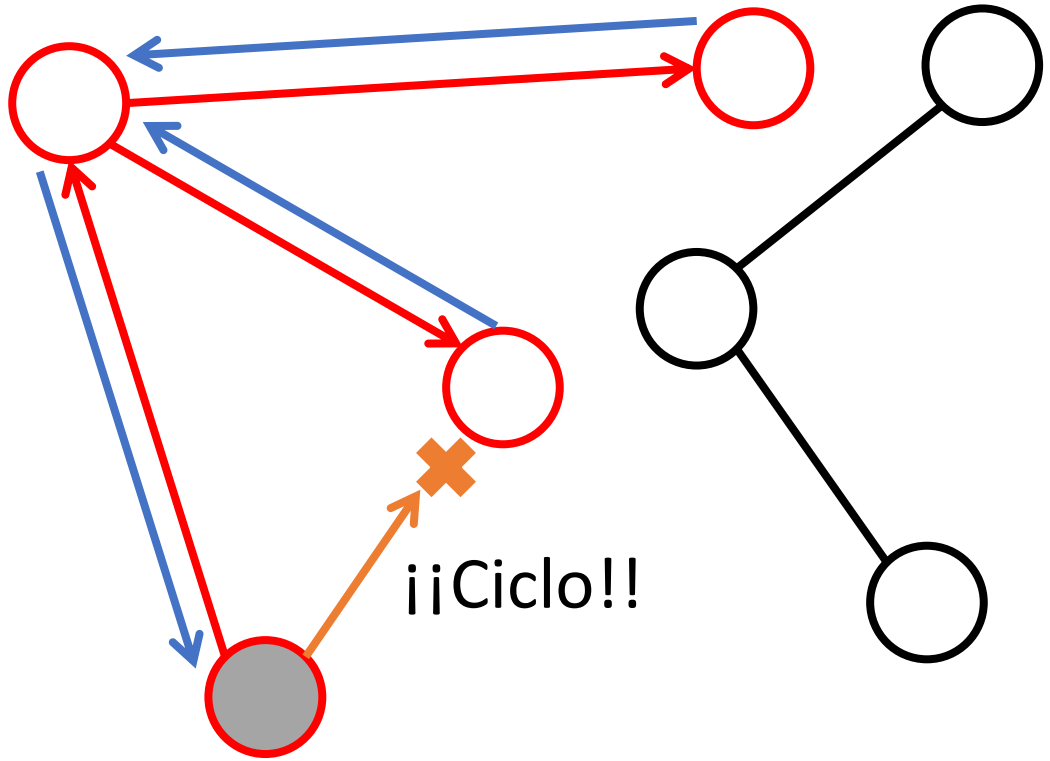
Aplicación – Detectar Ciclos



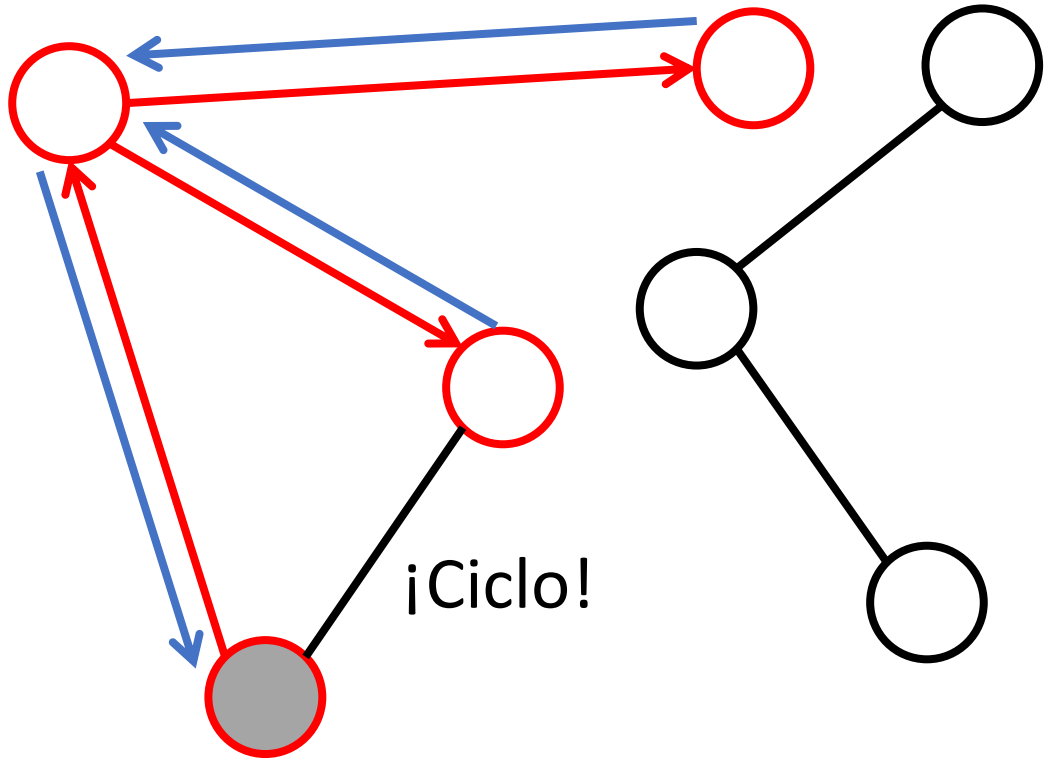
Aplicación – Detectar Ciclos



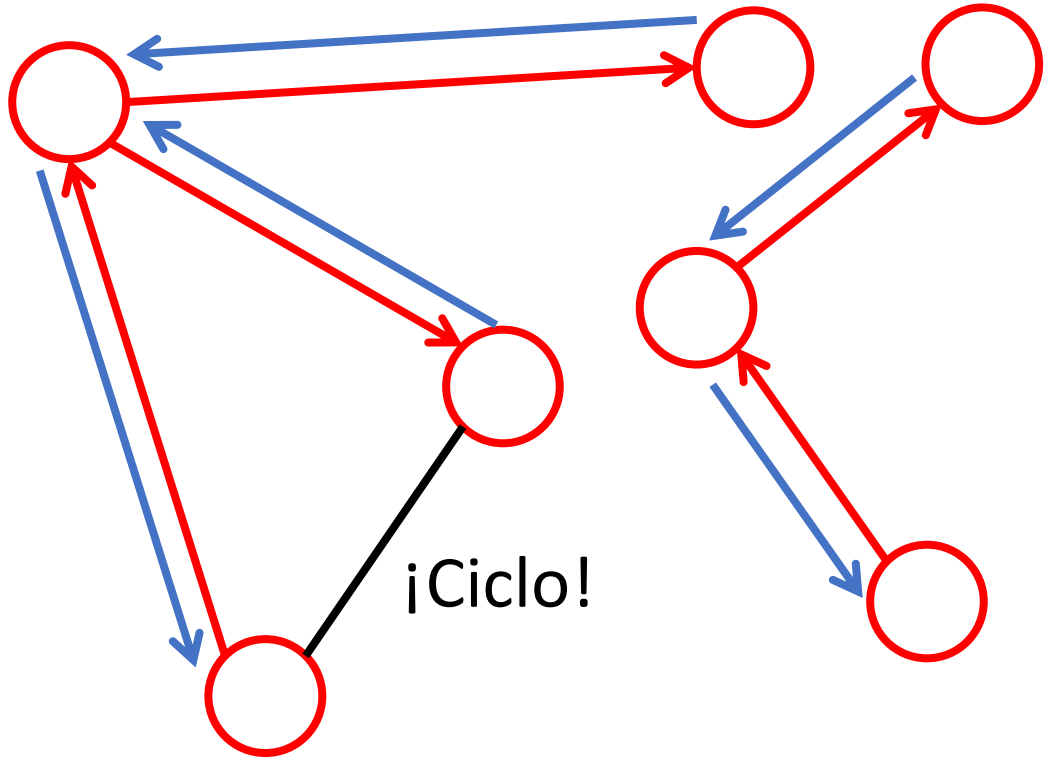
Aplicación – Detectar Ciclos



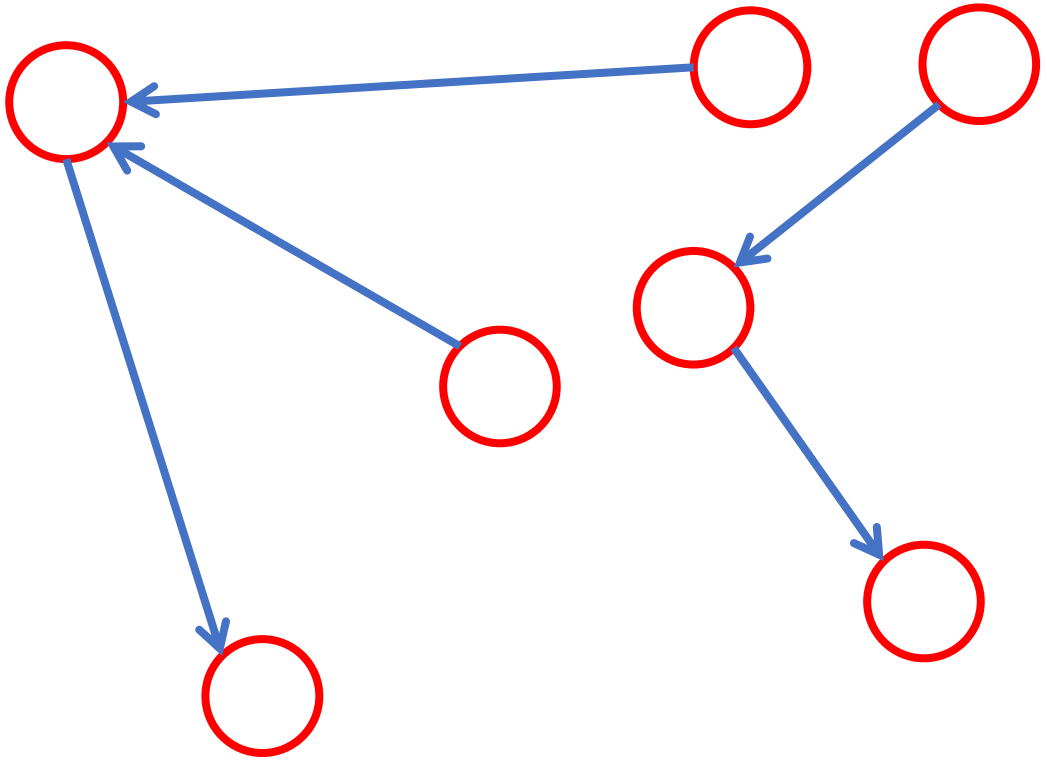
Aplicación – Detectar Ciclos



Aplicación – Detectar Ciclos



Aplicación – Detectar Ciclos



Aplicación – Detectar Ciclos

```
vector<int> g[n];
bool vis[n];

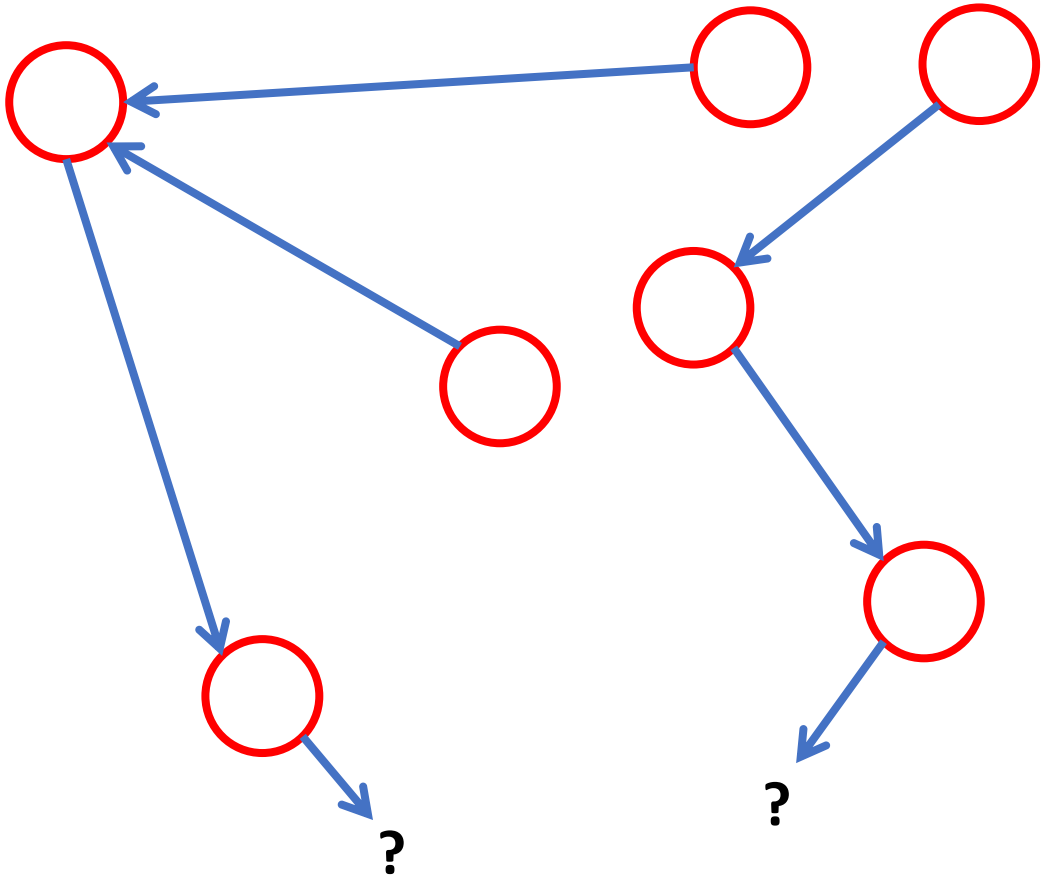

void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v
      // aqui se define el padre
      dfs(v);
    }
  }
}
```

Aplicación – Detectar Ciclos

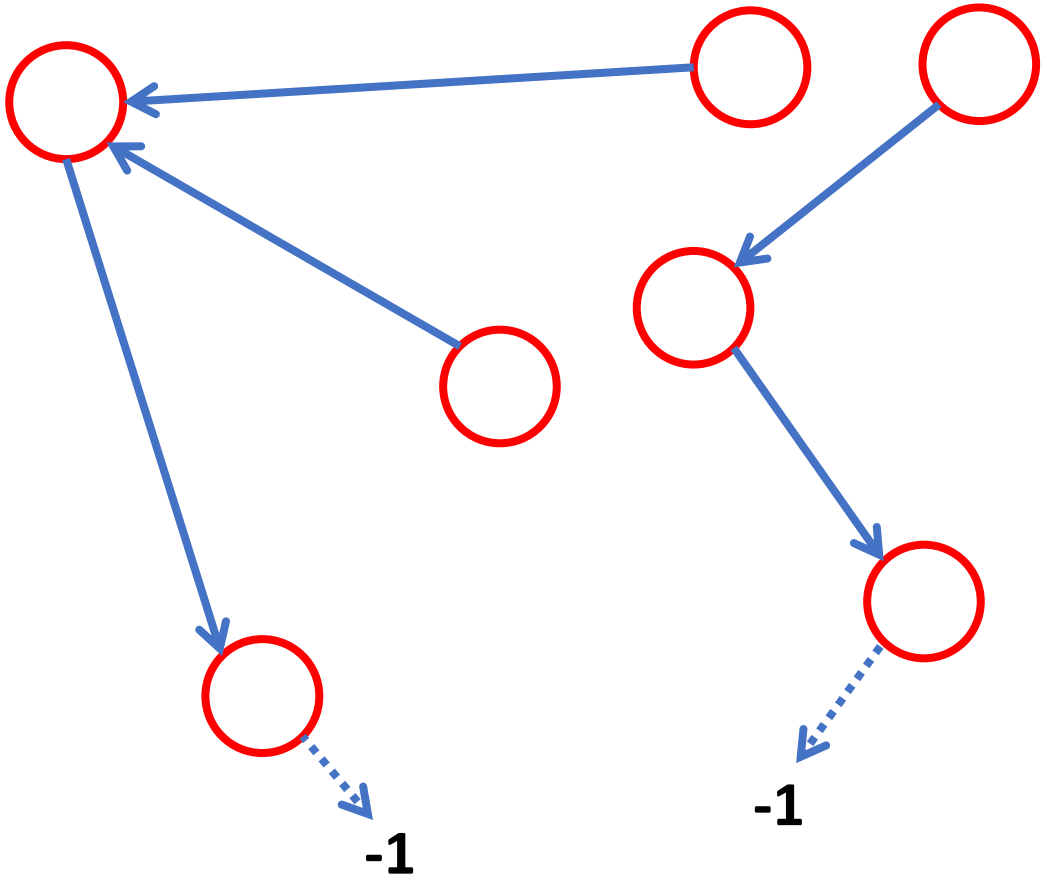
```cpp
vector<int> g[n];
bool vis[n];
int padre[n];

void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v
      padre[v] = u;
      dfs(v);
    }
  }
}
```

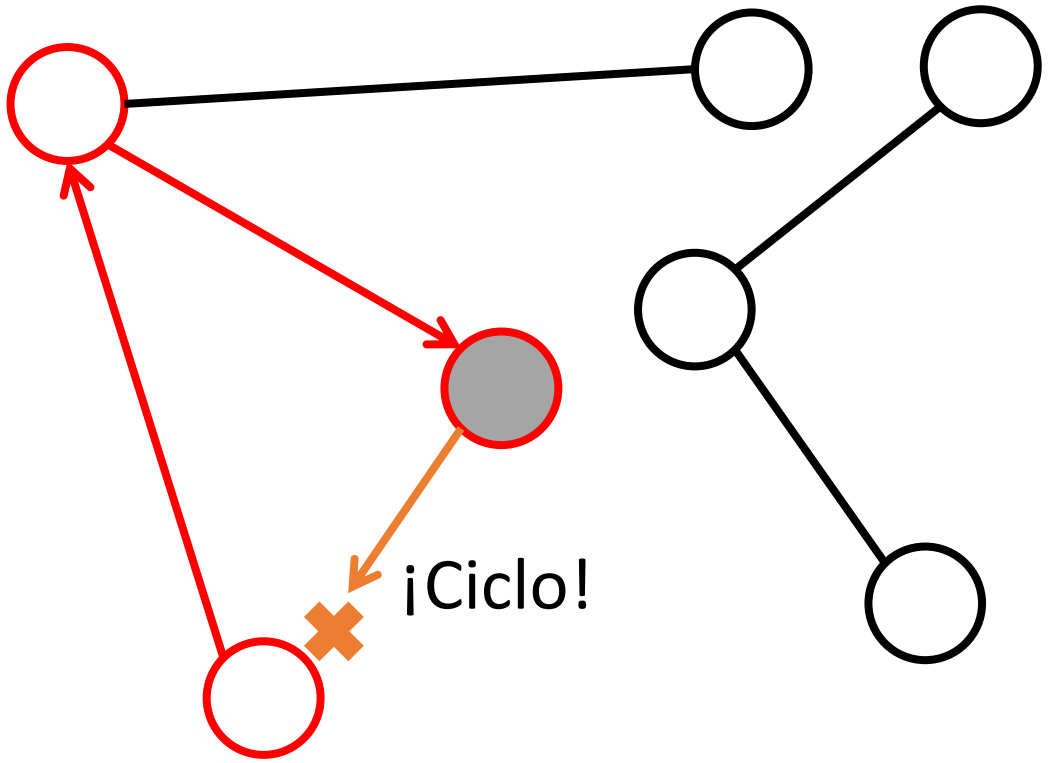
Aplicación – Detectar Ciclos



Aplicación – Detectar Ciclos



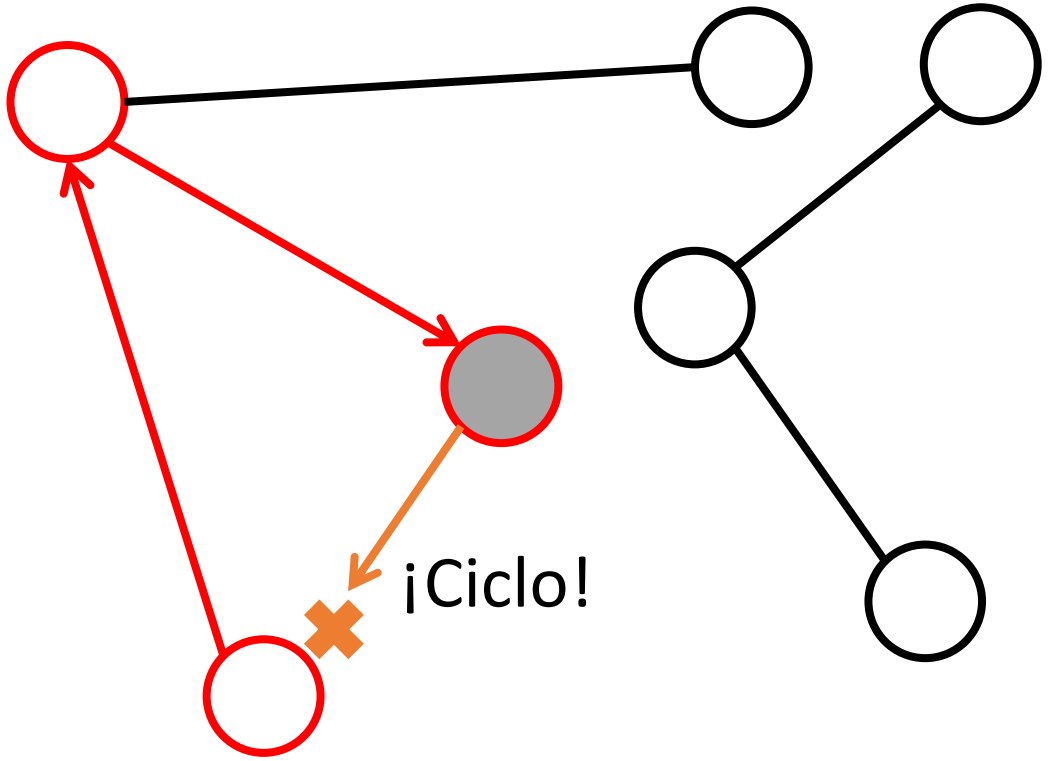
Aplicación – Detectar Ciclos




```
vector<int> g[n];
bool vis[n];
int padre[n];

void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v
      padre[v] = u;
      dfs(v);
    }
  }
}
```

Aplicación – Detectar Ciclos




```cpp
vector<int> g[n];
bool vis[n];
int padre[n];
bool hayCiclo = false;

void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v
      padre[v] = u;
      dfs(v);
    }
    else if (v != padre[u]) {
      hayCiclo = true;
    }
  }
}
```

Aplicación – Orden Topológico

Es un concepto aplicado a grafos dirigidos que no contienen ciclos.

Orden lineal de los vértices del grafo de tal modo que por cada arista dirigida $u \rightarrow v$, u va antes que v en el orden.

Aplicación – Orden Topológico

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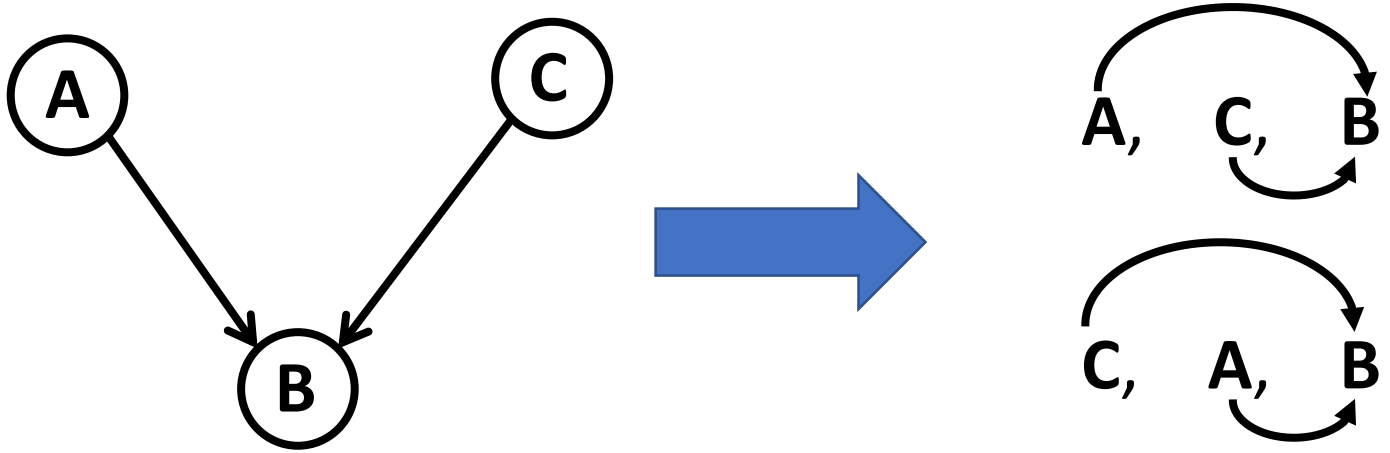
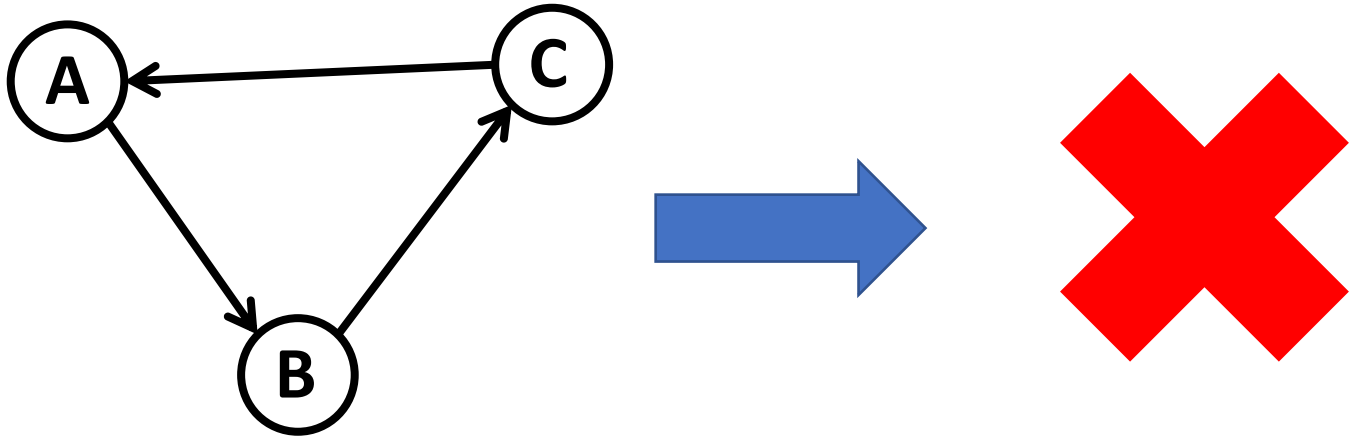


Aplicación – Orden Topológico

Es un concepto aplicado a grafos dirigidos **que no contienen ciclos**.

Orden lineal de los vértices del grafo de tal modo que por cada arista dirigida $u \rightarrow v$, u va antes que v en el orden.



Aplicación – Orden Topológico

Es un concepto aplicado a grafos dirigidos que no contienen ciclos.

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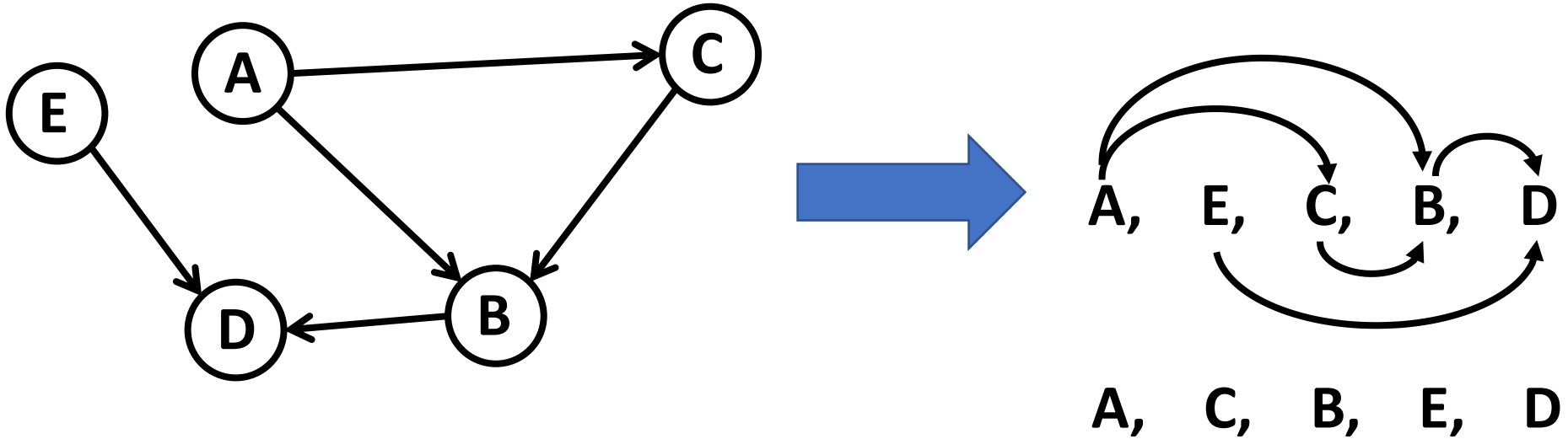
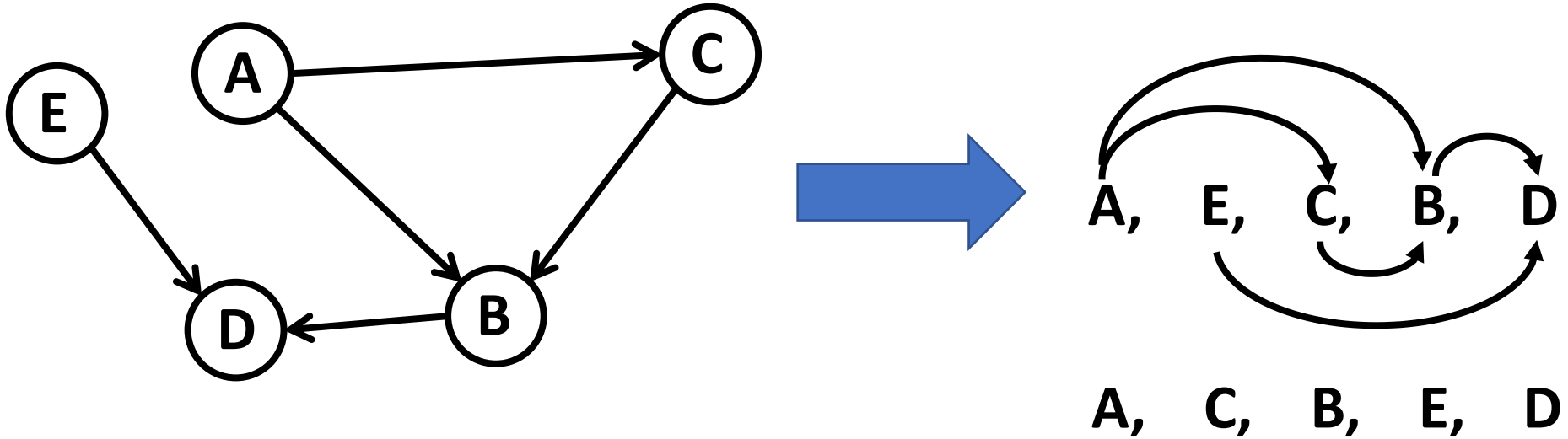


Aplicación – Orden Topológico

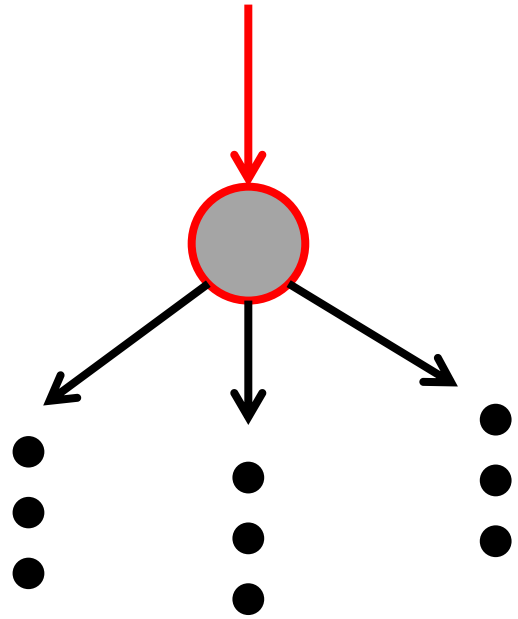
Es un concepto aplicado a grafos dirigidos que no contienen ciclos.

Orden lineal de los vértices del grafo de tal modo que por cada arista dirigida $u \rightarrow v$, u va antes que v en el orden.

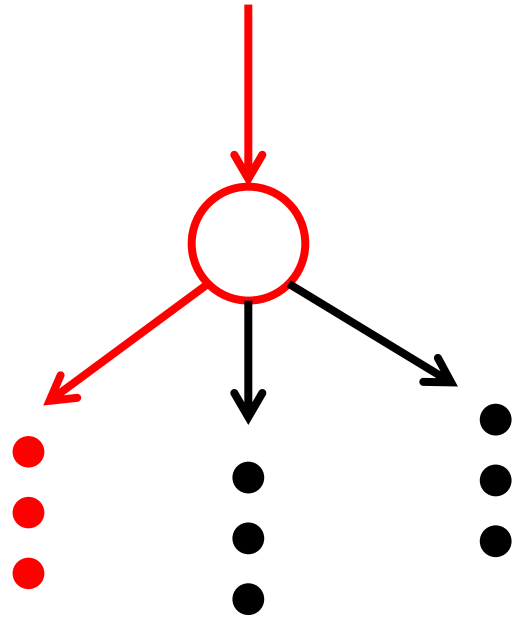
Aprovechamos una propiedad del recorrido DFS para calcular esto.



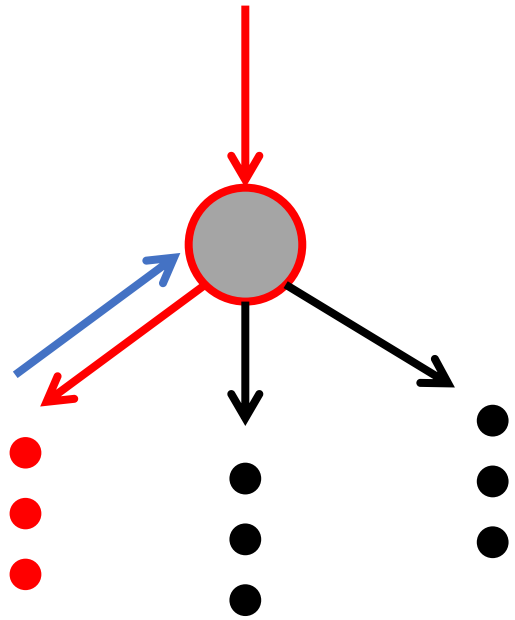
Aplicación – Orden Topológico



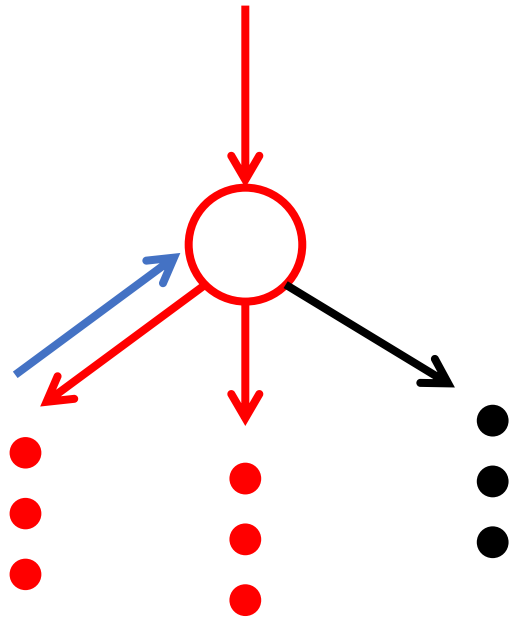
Aplicación – Orden Topológico



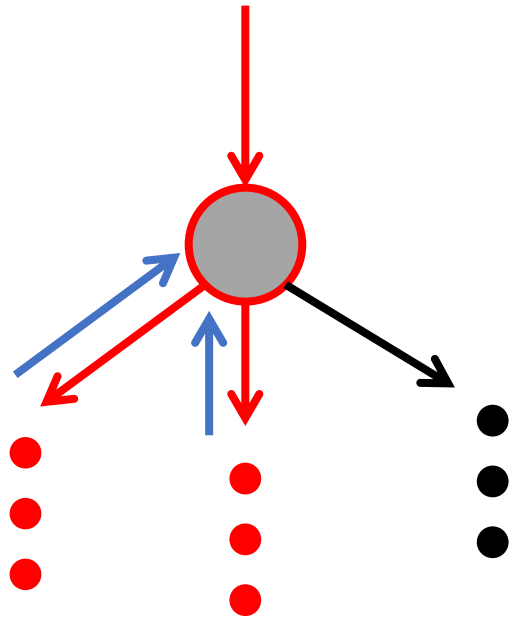
Aplicación – Orden Topológico



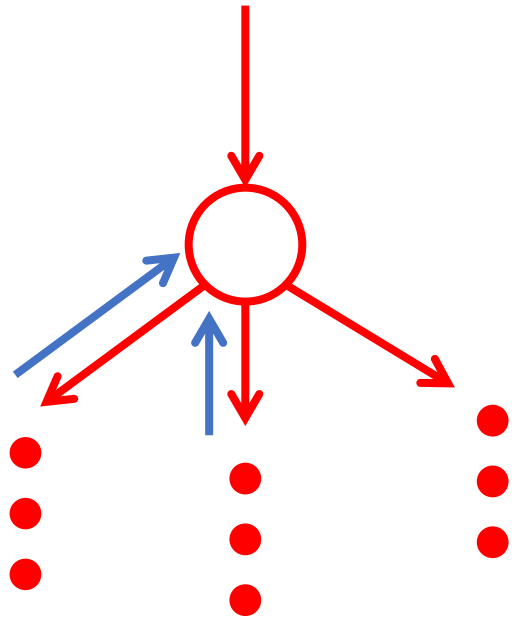
Aplicación – Orden Topológico



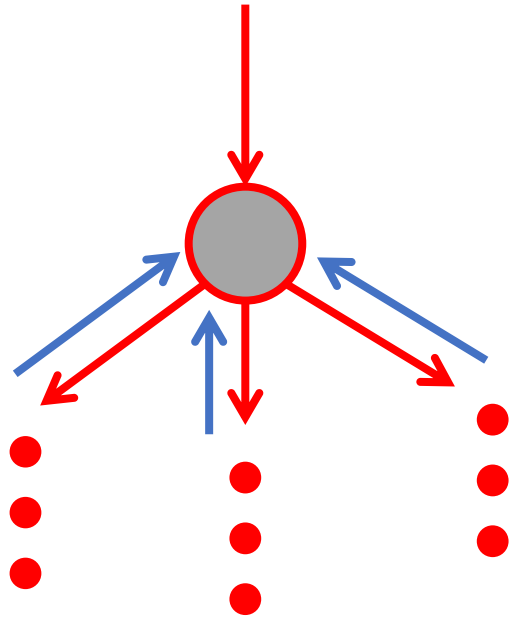
Aplicación – Orden Topológico



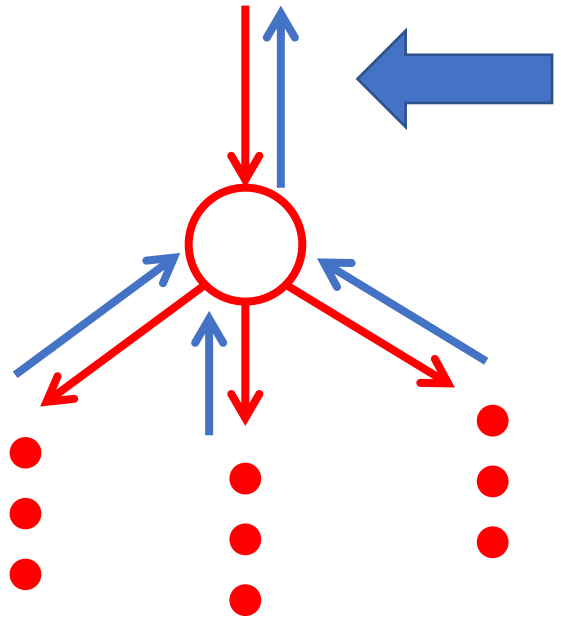
Aplicación – Orden Topológico



Aplicación – Orden Topológico

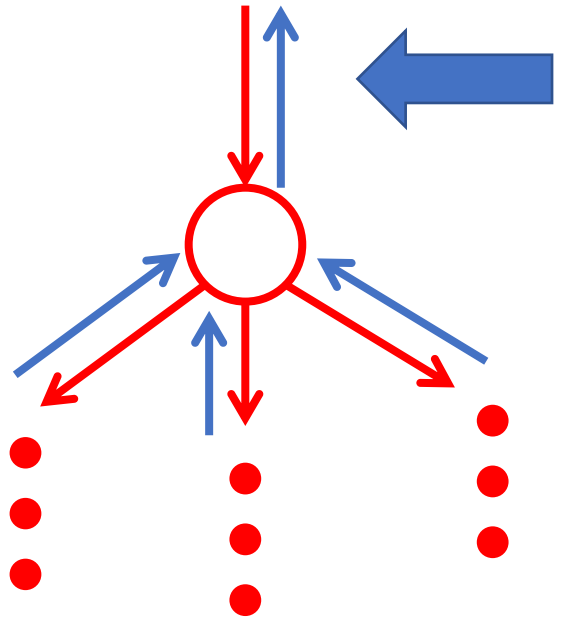


Aplicación – Orden Topológico



Regresar (backtracking) significa que ya se termino de procesar a **todos** sus vecinos.

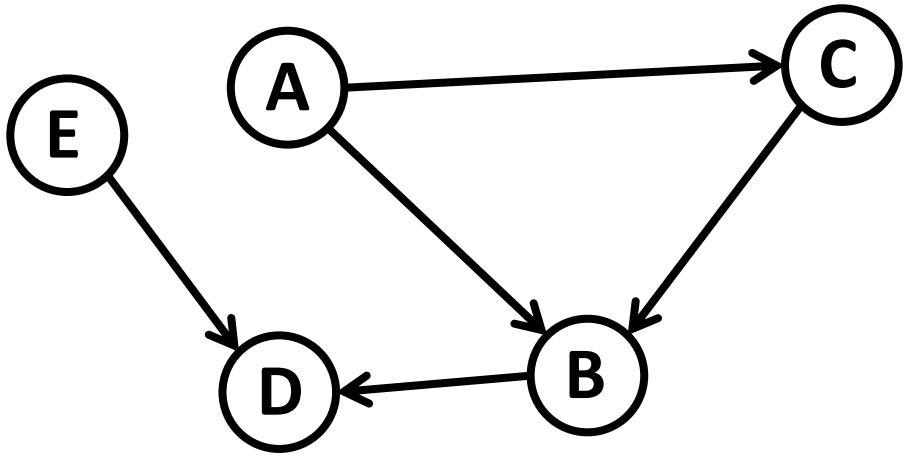
Aplicación – Orden Topológico



Regresar (backtracking) significa que ya se termino de procesar a **todos** sus vecinos.

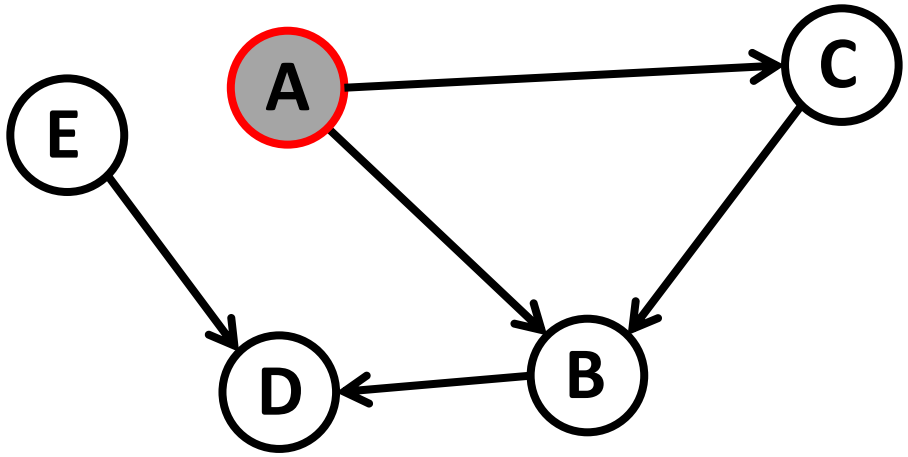
Entonces, podemos poner el nodo **antes** de sus vecinos en el orden topológico.

Aplicación – Orden Topológico



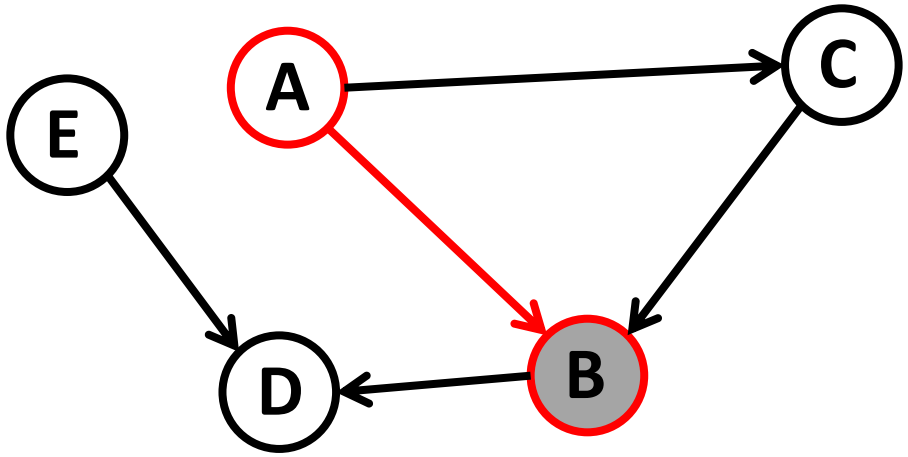
Orden topológico:

Aplicación – Orden Topológico



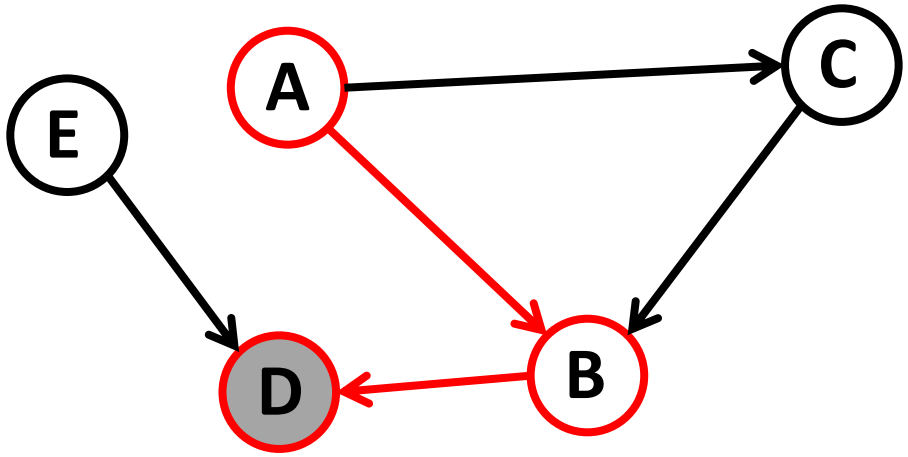
Orden topológico:

Aplicación – Orden Topológico



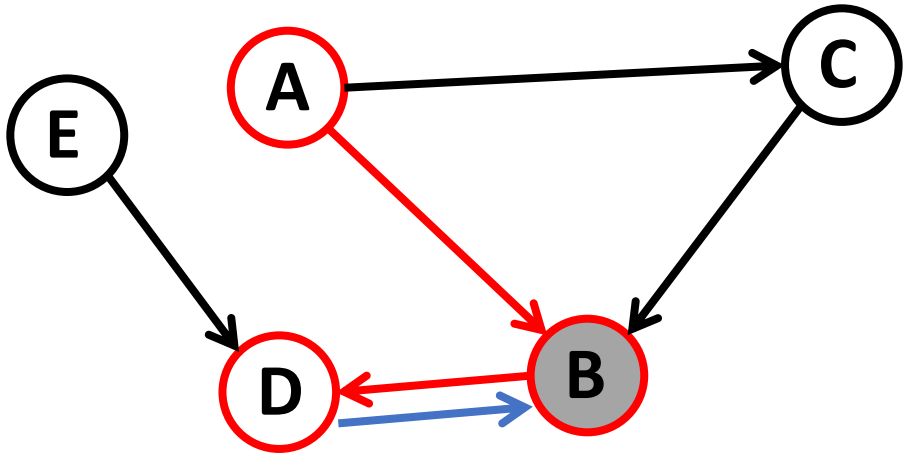
Orden topológico:

Aplicación – Orden Topológico



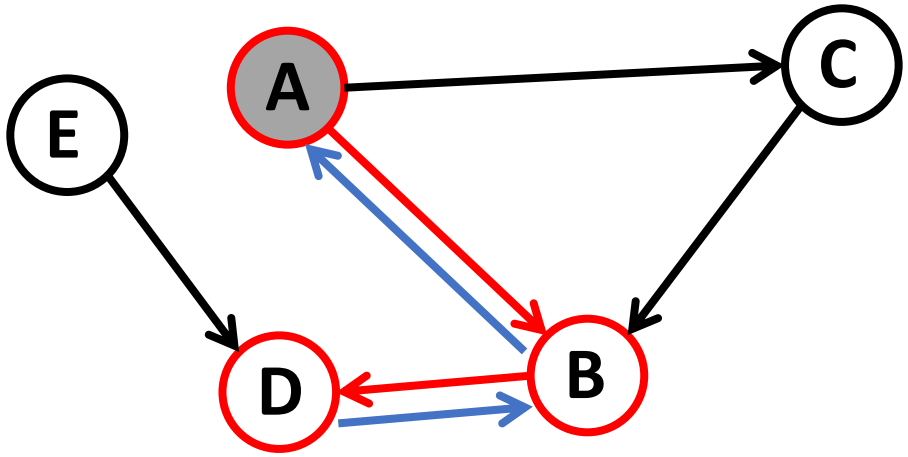
Orden topológico:

Aplicación – Orden Topológico



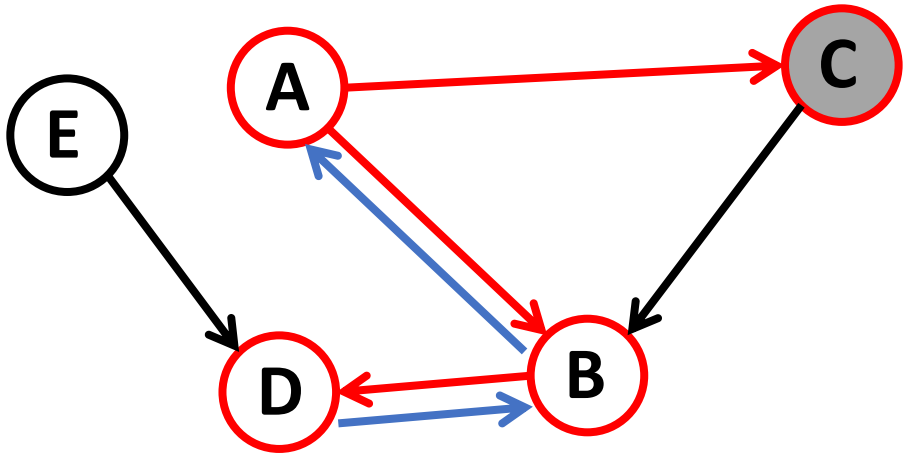
Orden topológico: **D**

Aplicación – Orden Topológico



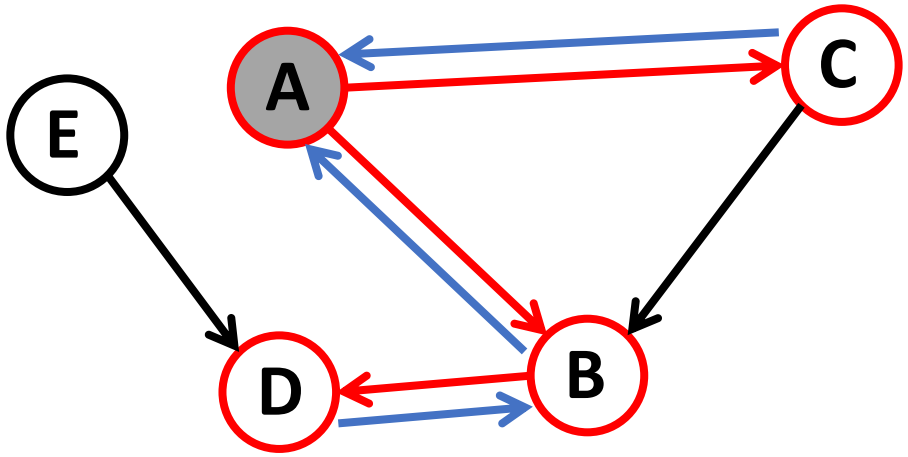
Orden topológico: **B D**

Aplicación – Orden Topológico



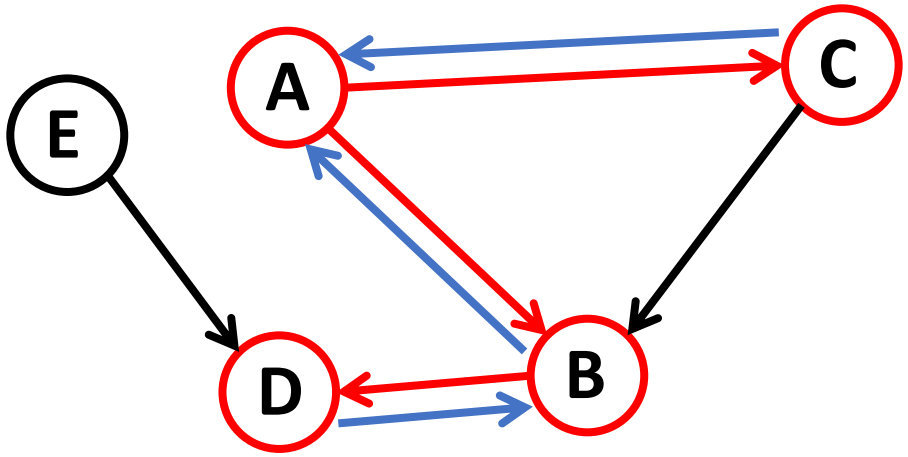
Orden topológico: **B D**

Aplicación – Orden Topológico



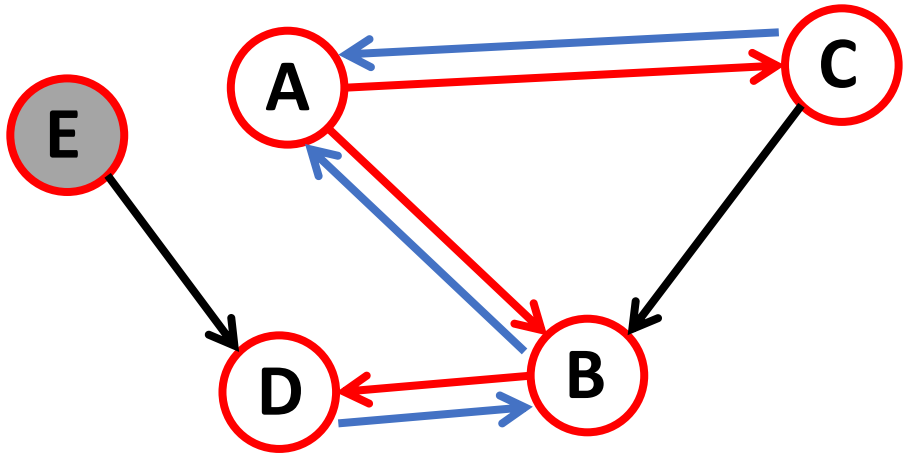
Orden topológico: **C B D**

Aplicación – Orden Topológico



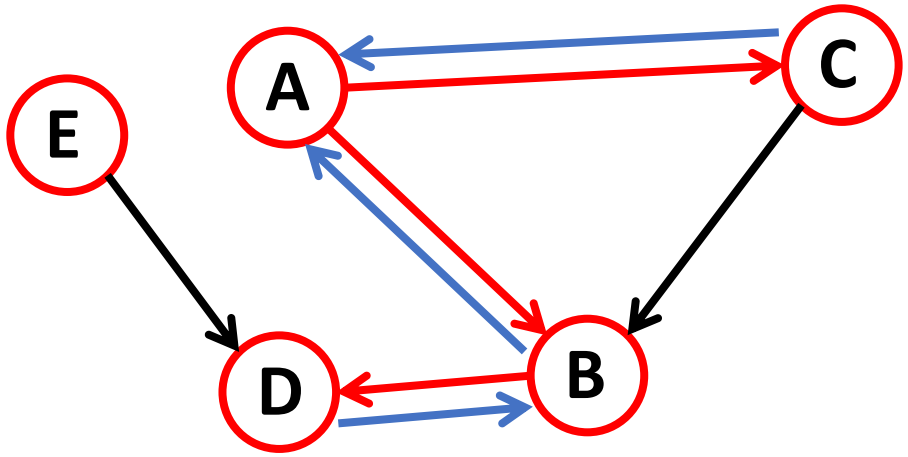
Orden topológico: **A C B D**

Aplicación – Orden Topológico



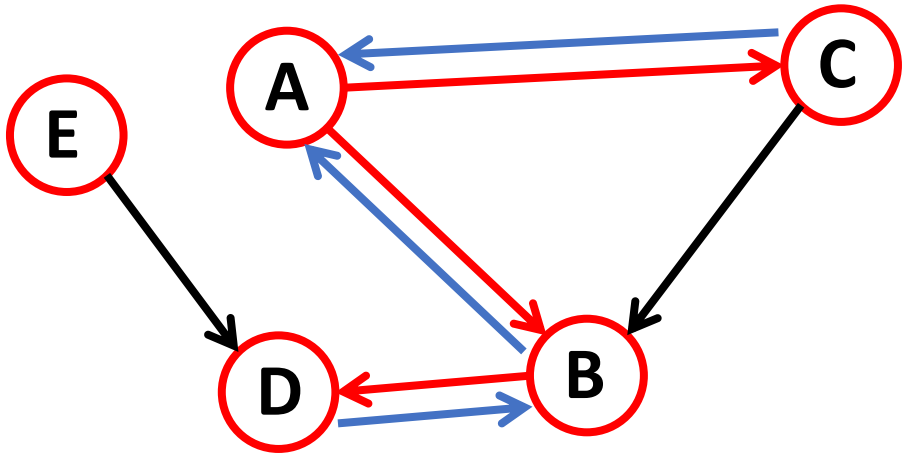
Orden topológico: **A C B D**

Aplicación – Orden Topológico



Orden topológico: **E A C B D**

Aplicación – Orden Topológico



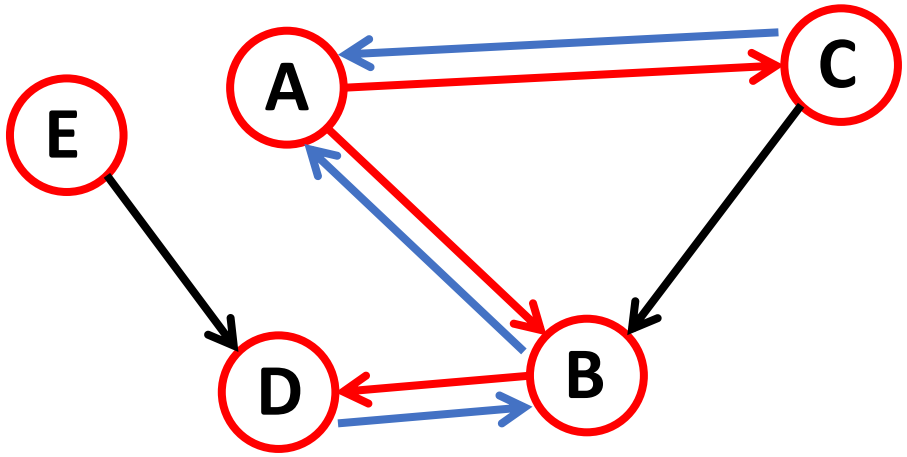

```cpp
vector<int> g[n];
bool vis[n];


void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v (ida)
      dfs(v);
      // v <- u (vuelta)
    }
  }

  // justo antes de salir del nodo (backtracking)
}
```

Orden topológico: **E A C B D**

Aplicación – Orden Topológico




```
vector<int> g[n];
bool vis[n];
vector<int> invTopSort;

void dfs(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) {
      // u -> v (ida)
      dfs(v);
      // v <- u (vuelta)
    }
  }
  invTopSort.push_back(u);
  // justo antes de salir del nodo (backtracking)
}
```

Orden topológico: **E A C B D**

invTopSort : **D B C A E**

El nodo se pone al final de la lista en vez de al inicio, solo hace falta trabajar con la lista invertida.

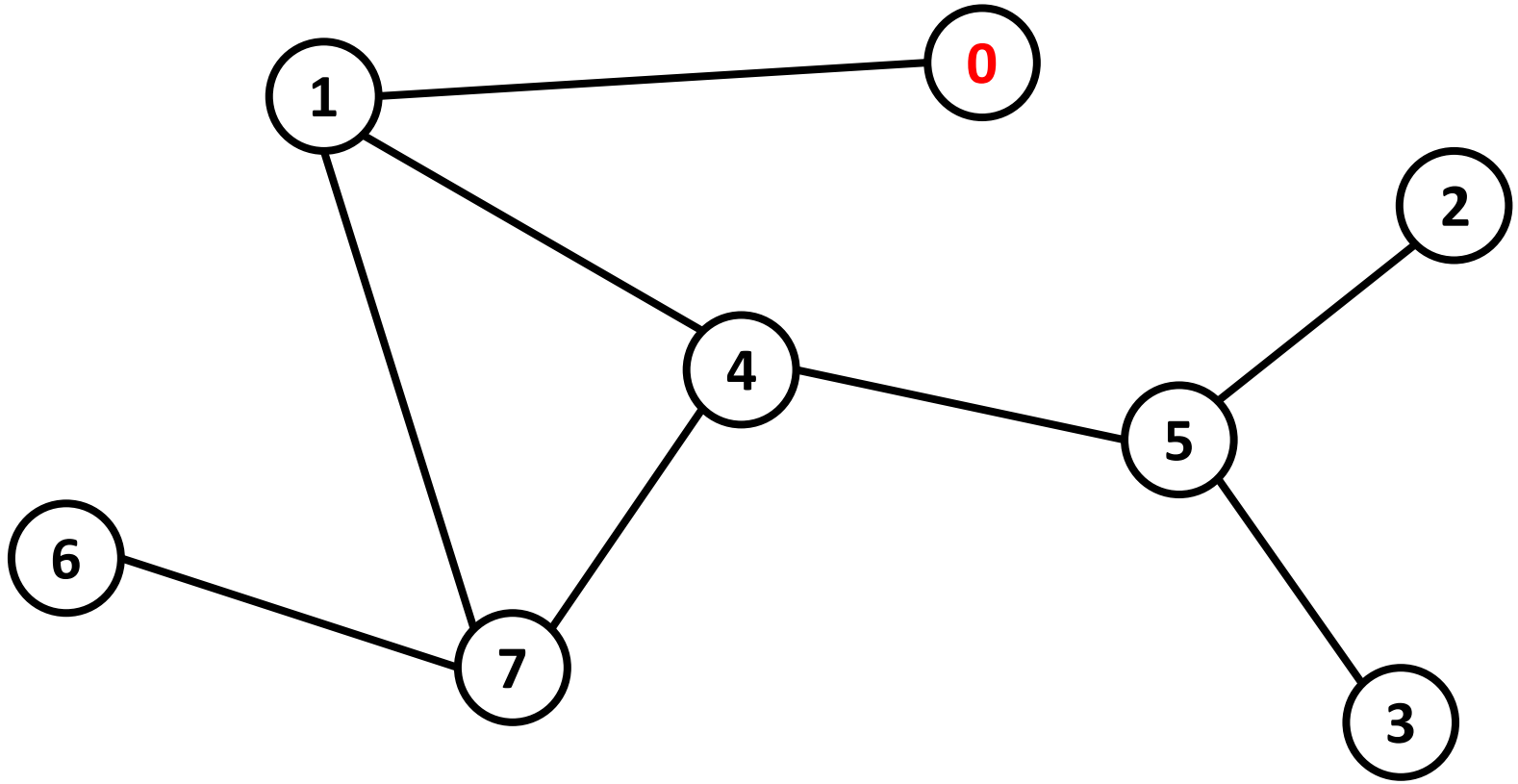
Otros Algoritmos con DFS

- Flood Fill (DFS en matrices)
- Detectar ciclos en grafos dirigidos
- Puentes y Puntos de Articulación
- Componentes Fuertemente Conexos
  - Problema 2-SAT
- Heavy-Light Decomposition de arboles
- Euler Tour en arboles

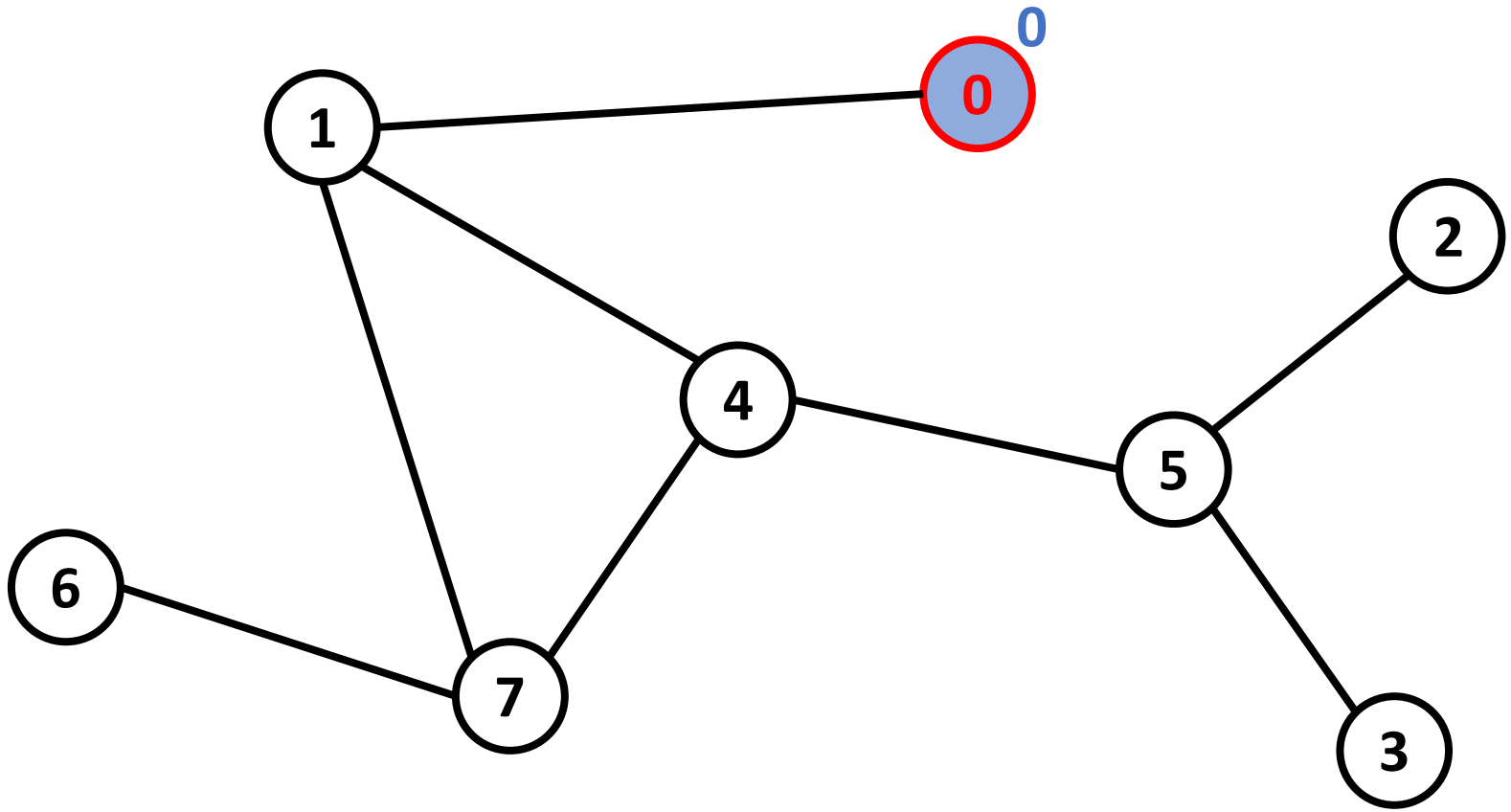
BFS

- Algoritmo de recorrido de grafos
- Empieza en un nodo y visita el resto en orden creciente segun la distancia al nodo inicial
- Es un poco mas dificil de implementar que DFS

BFS

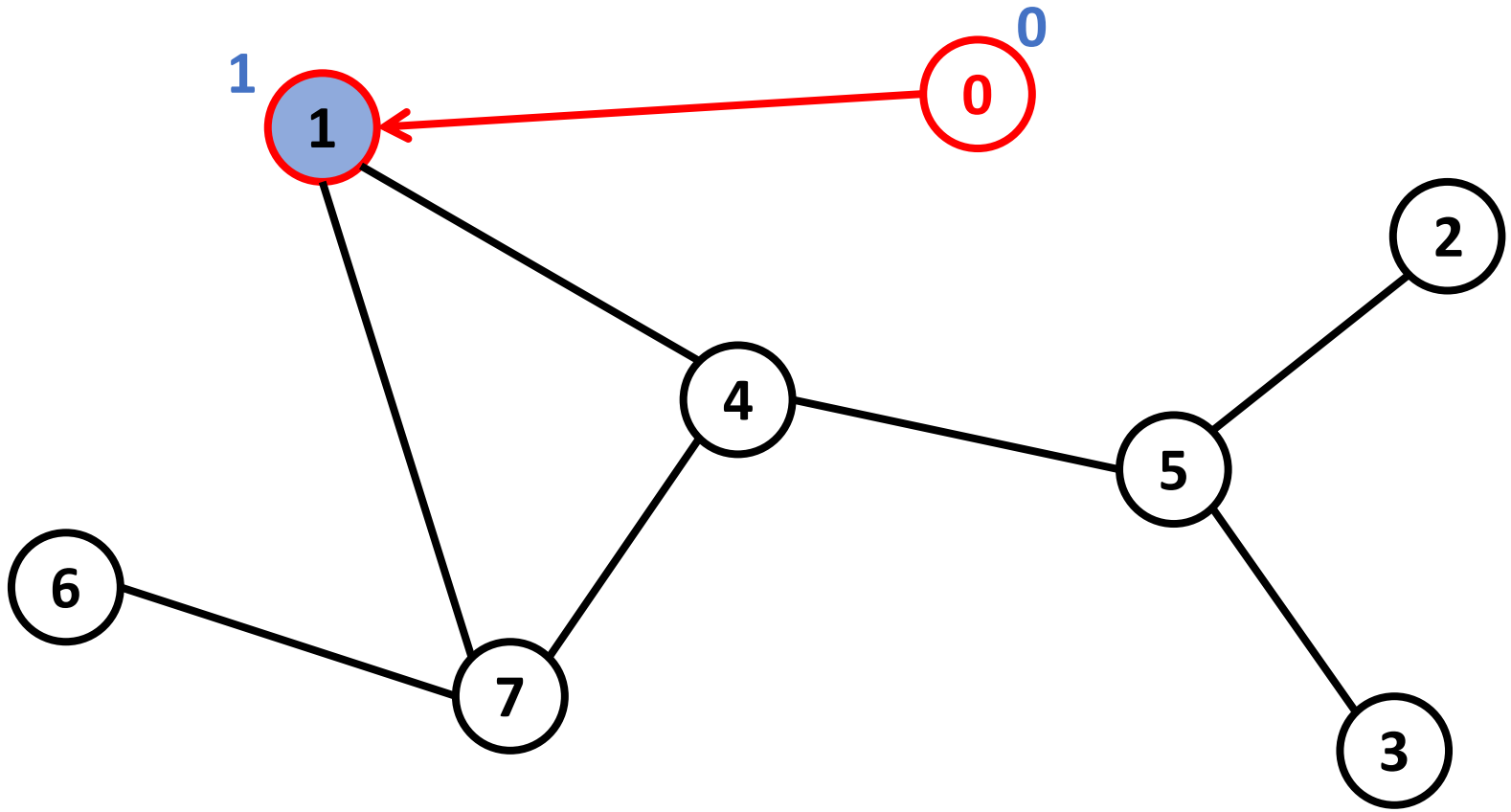


BFS



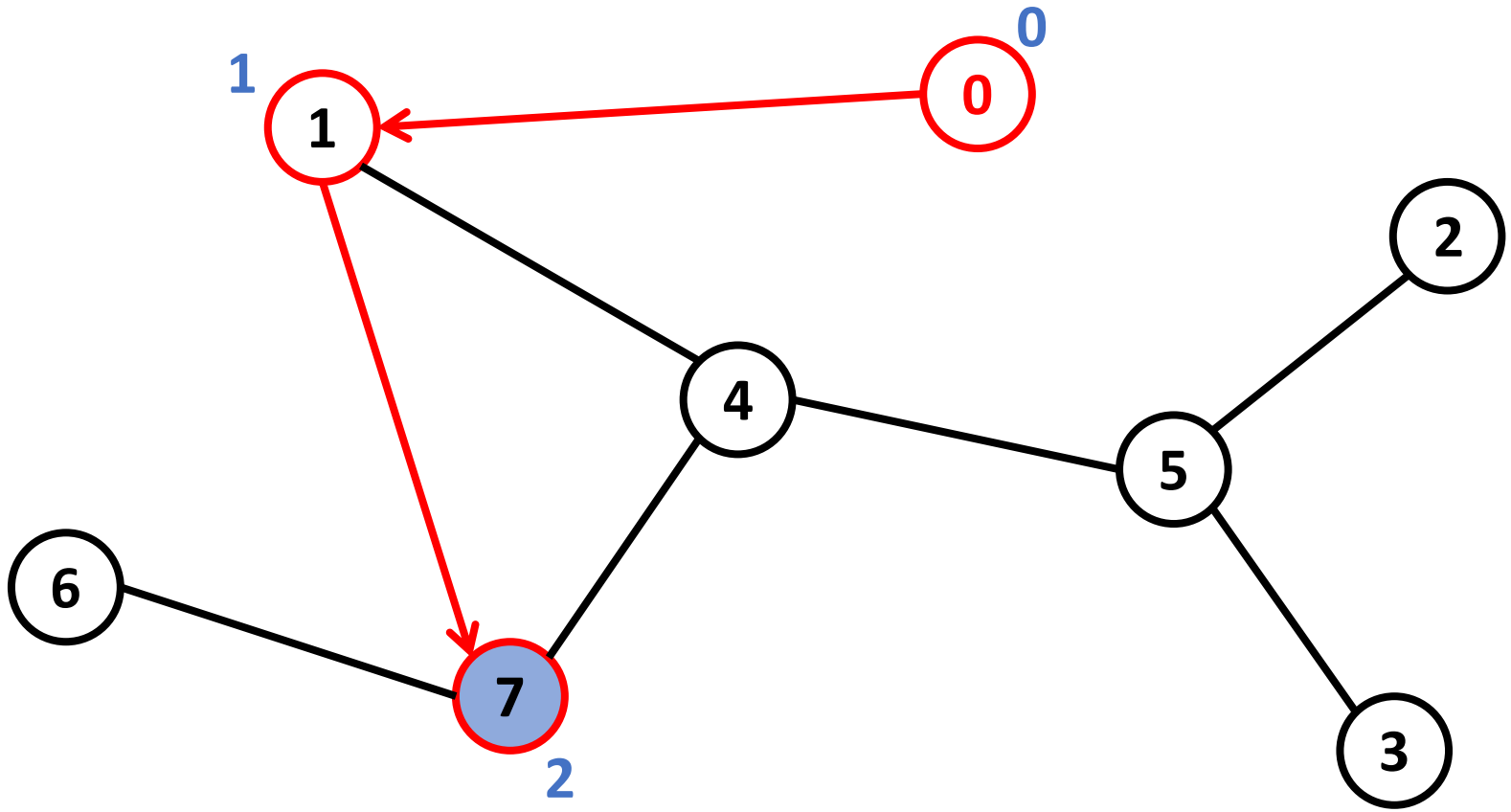
Orden de recorrido: 0

BFS



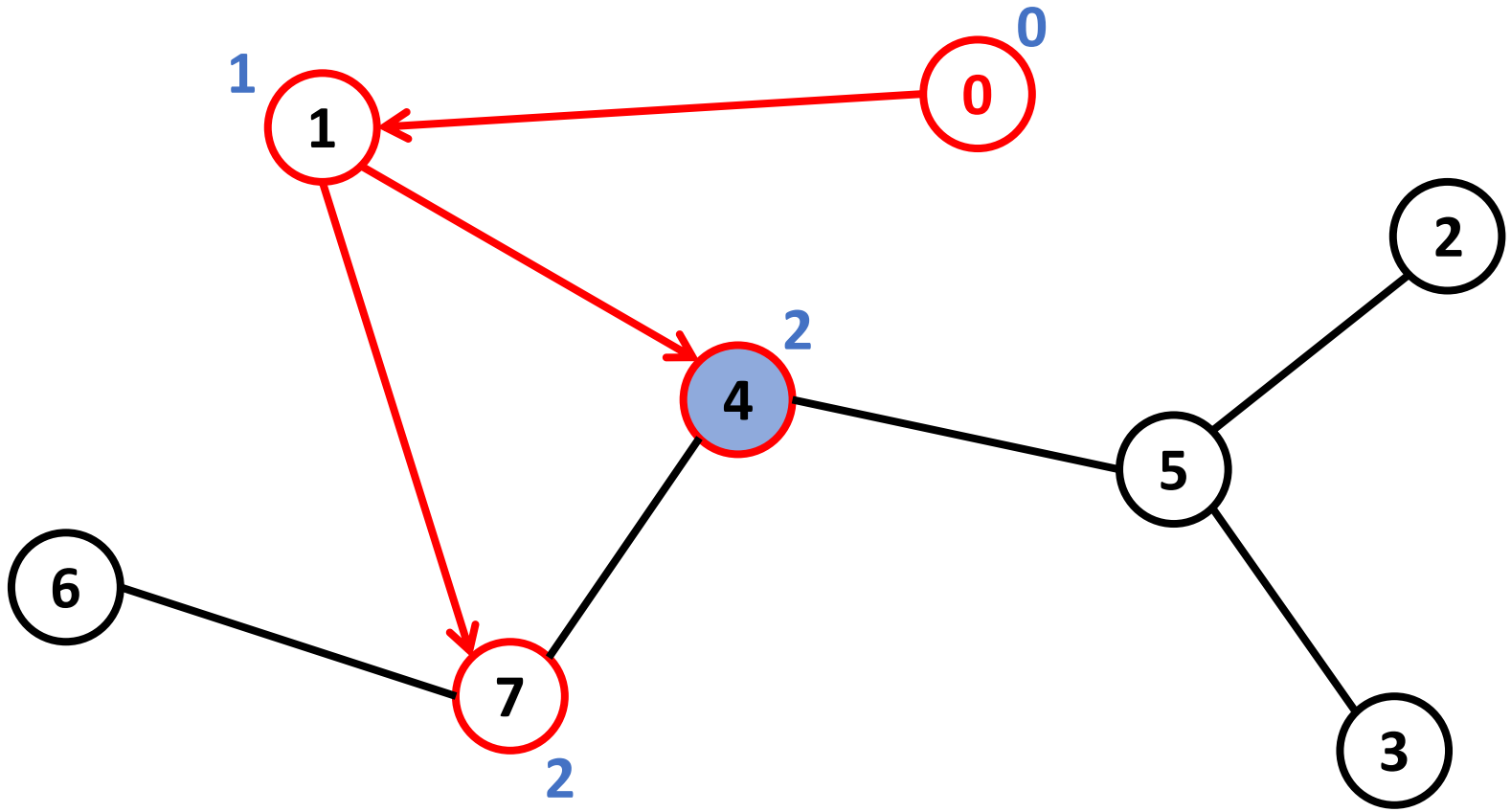
Orden de recorrido: 0, 1

BFS



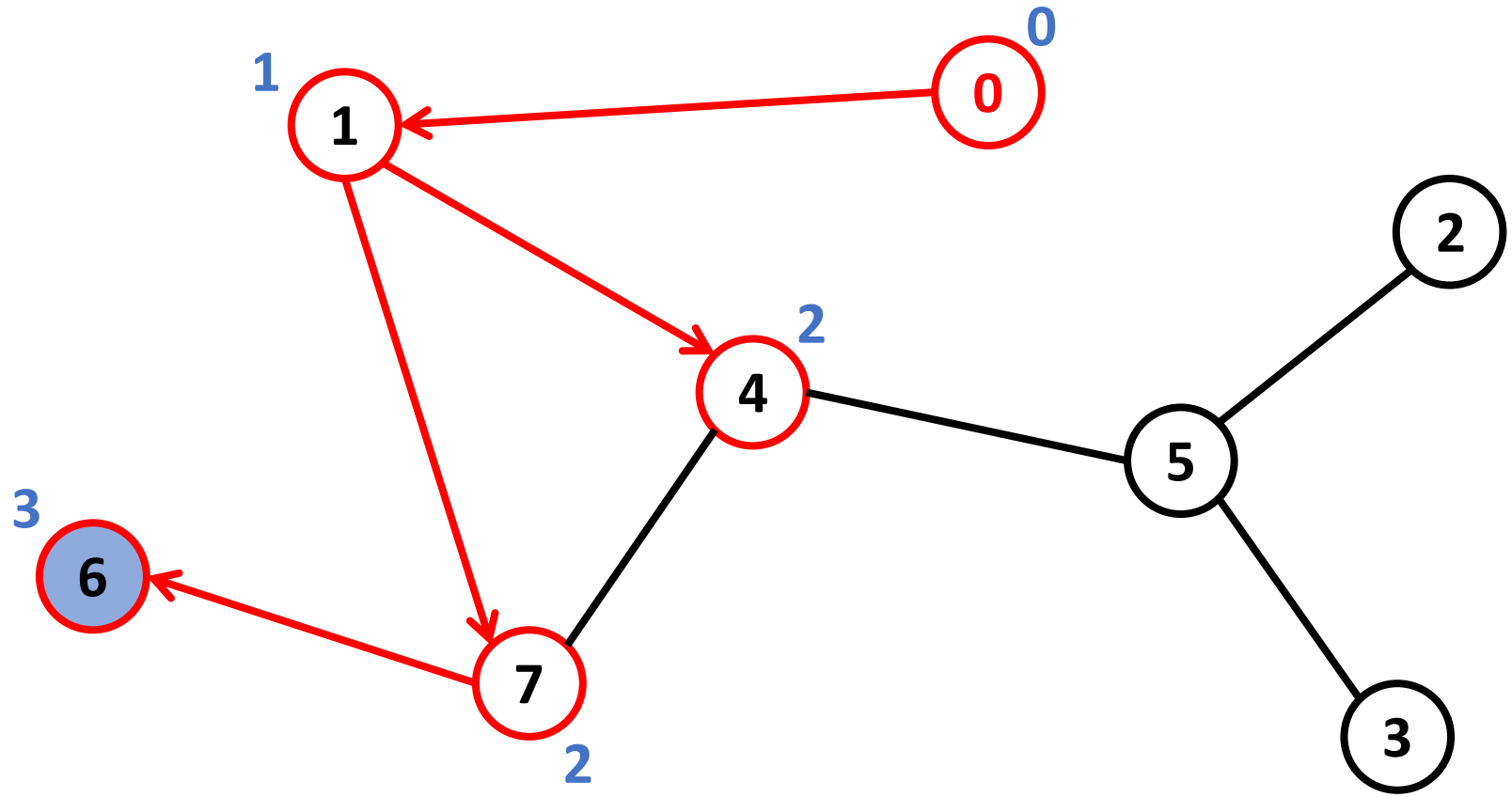
Orden de recorrido: 0, 1, 7

BFS



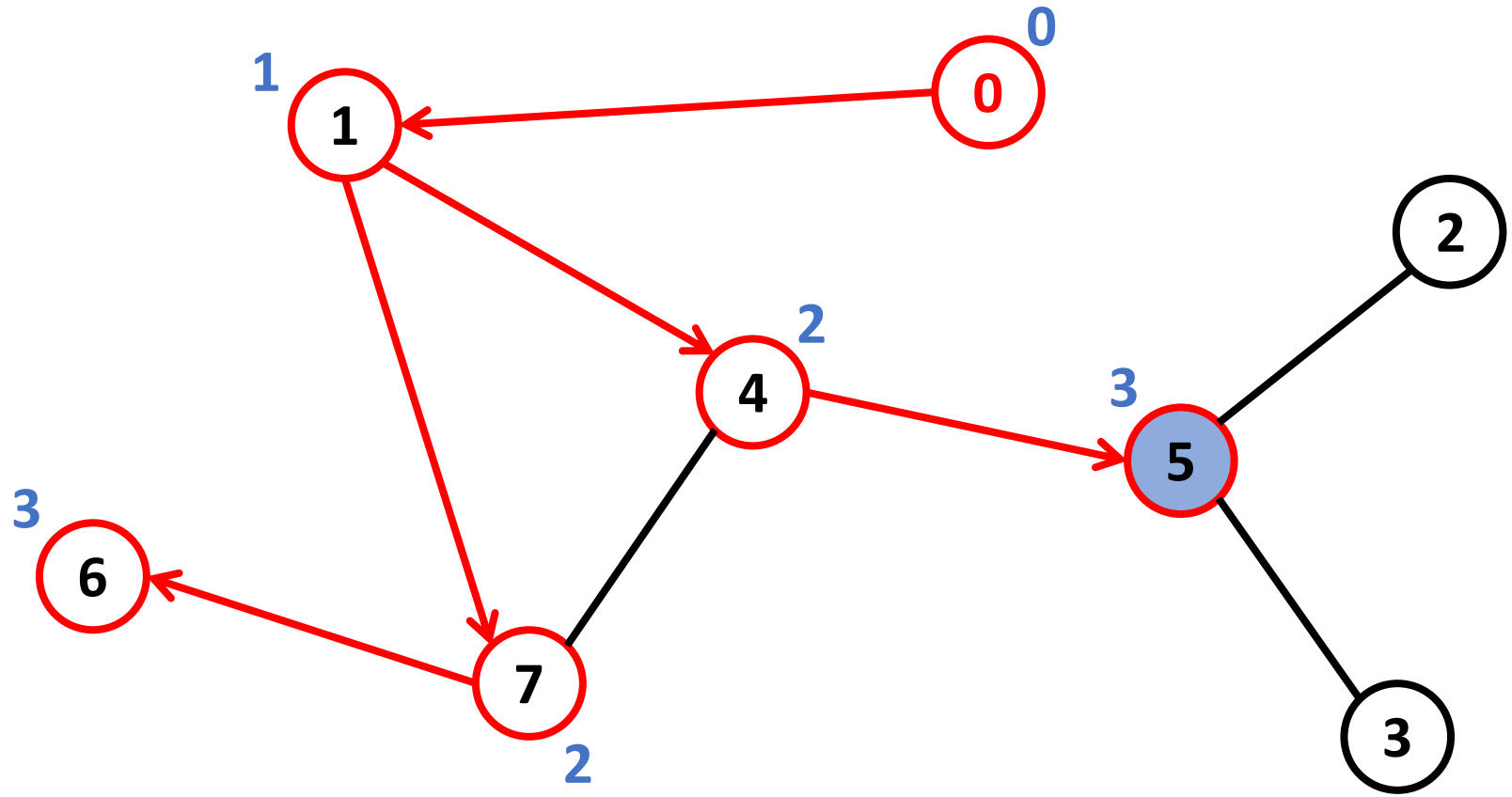
Orden de recorrido: 0, 1, 7, 4

BFS



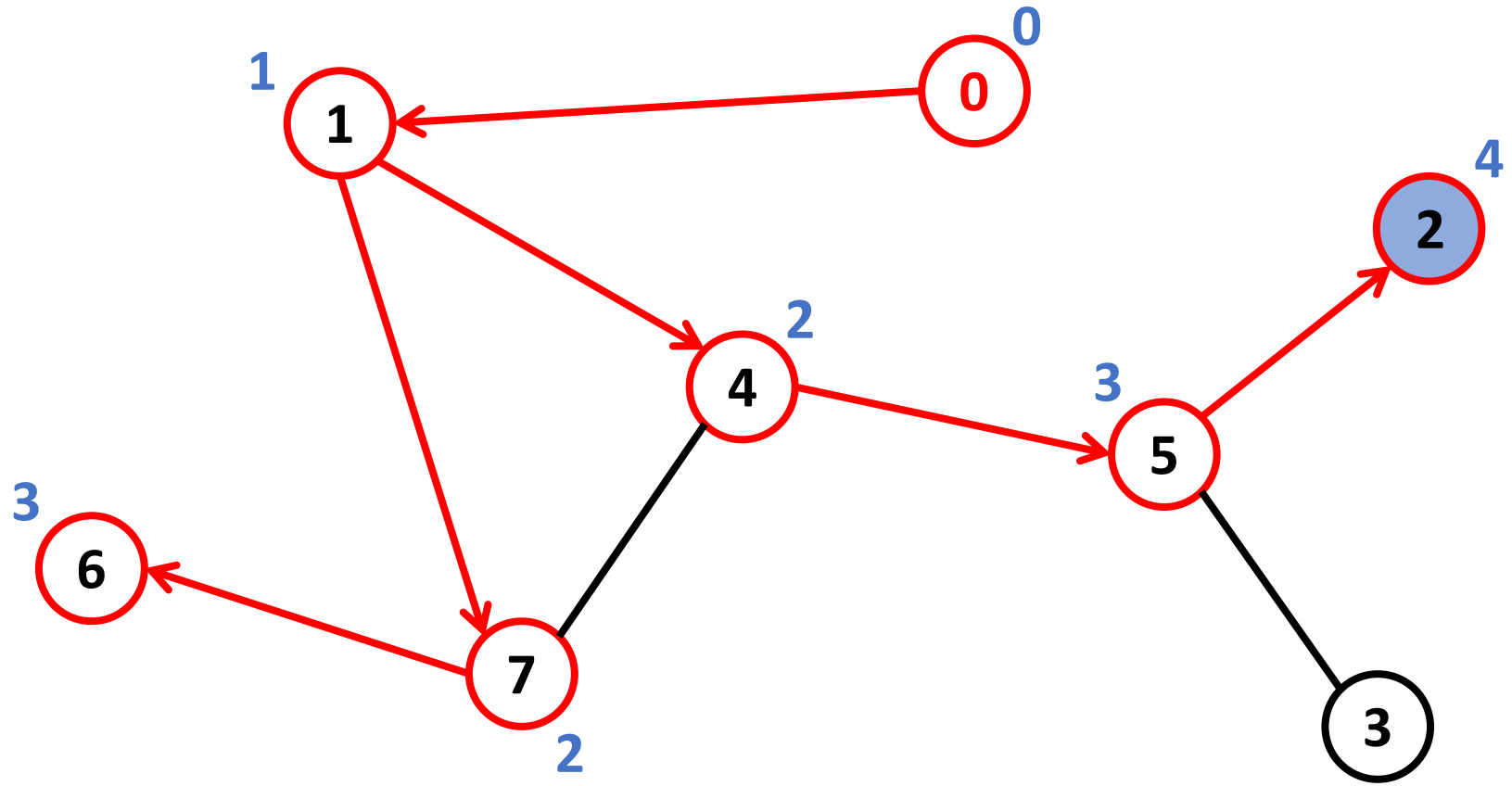
Orden de recorrido: 0, 1, 7, 4, 6

BFS



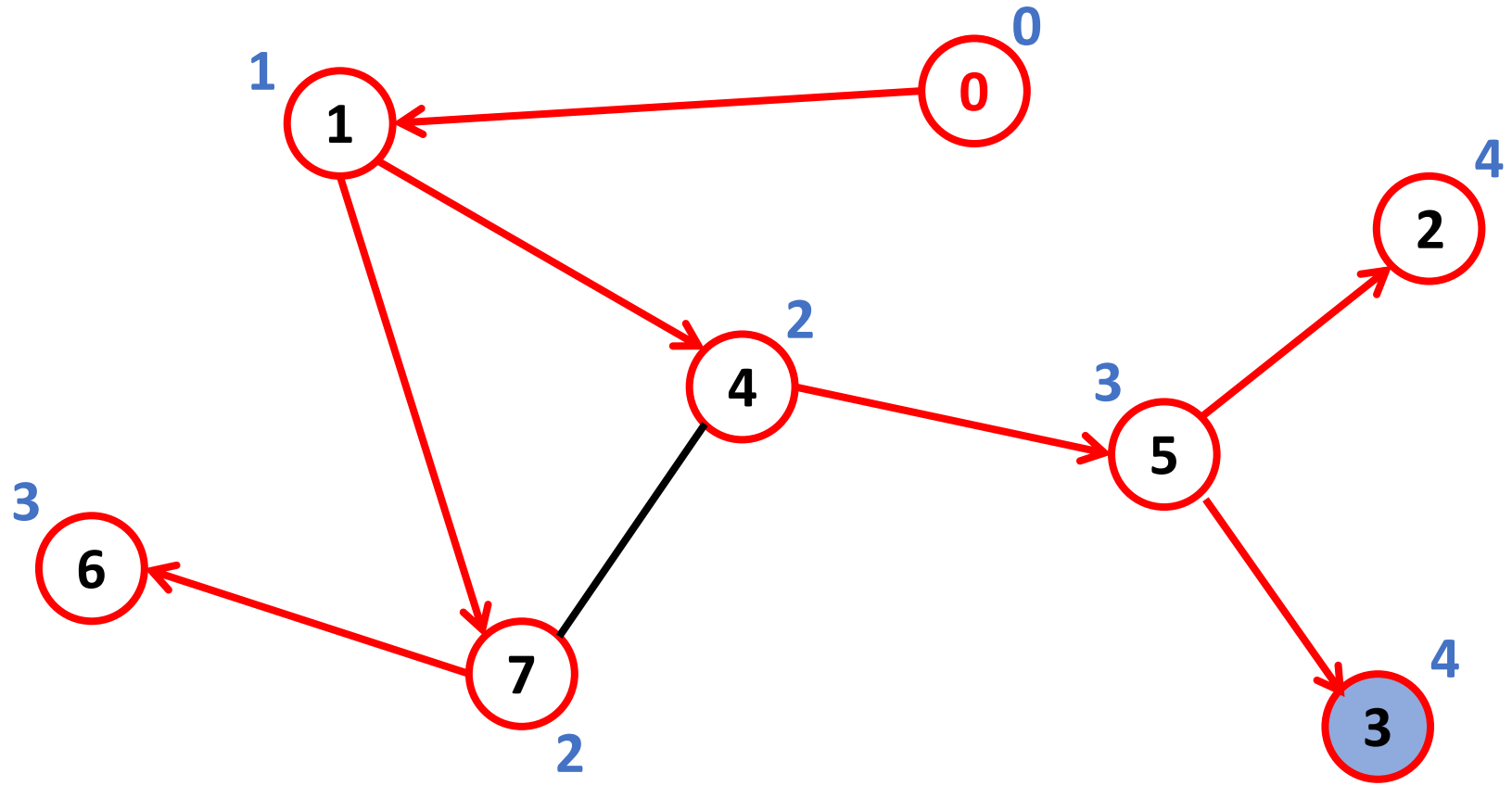
Orden de recorrido: 0, 1, 7, 4, 6, 5

BFS



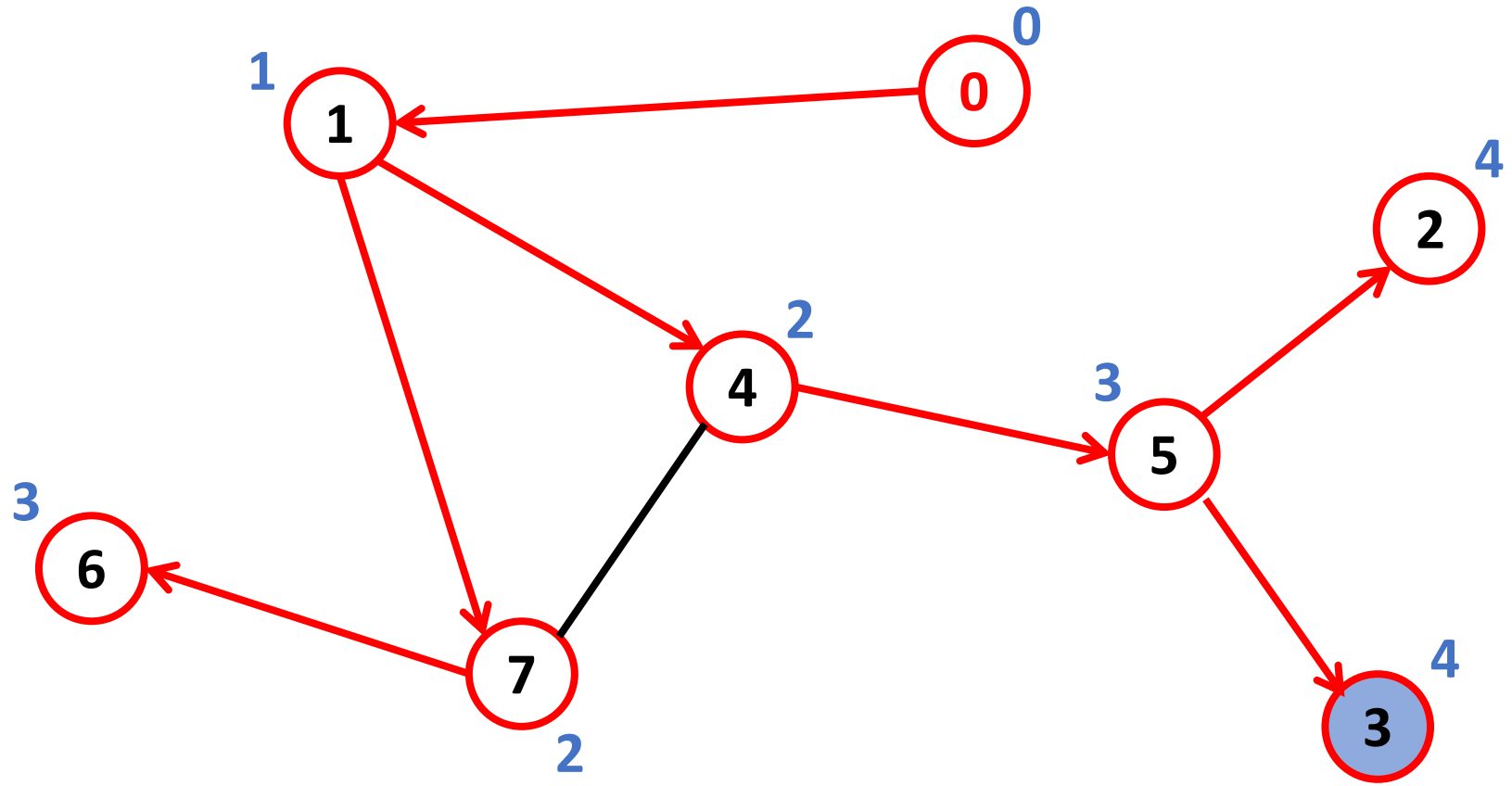
Orden de recorrido: 0, 1, 7, 4, 6, 5, 2

BFS



Orden de recorrido: 0, 1, 7, 4, 6, 5, 2, 3

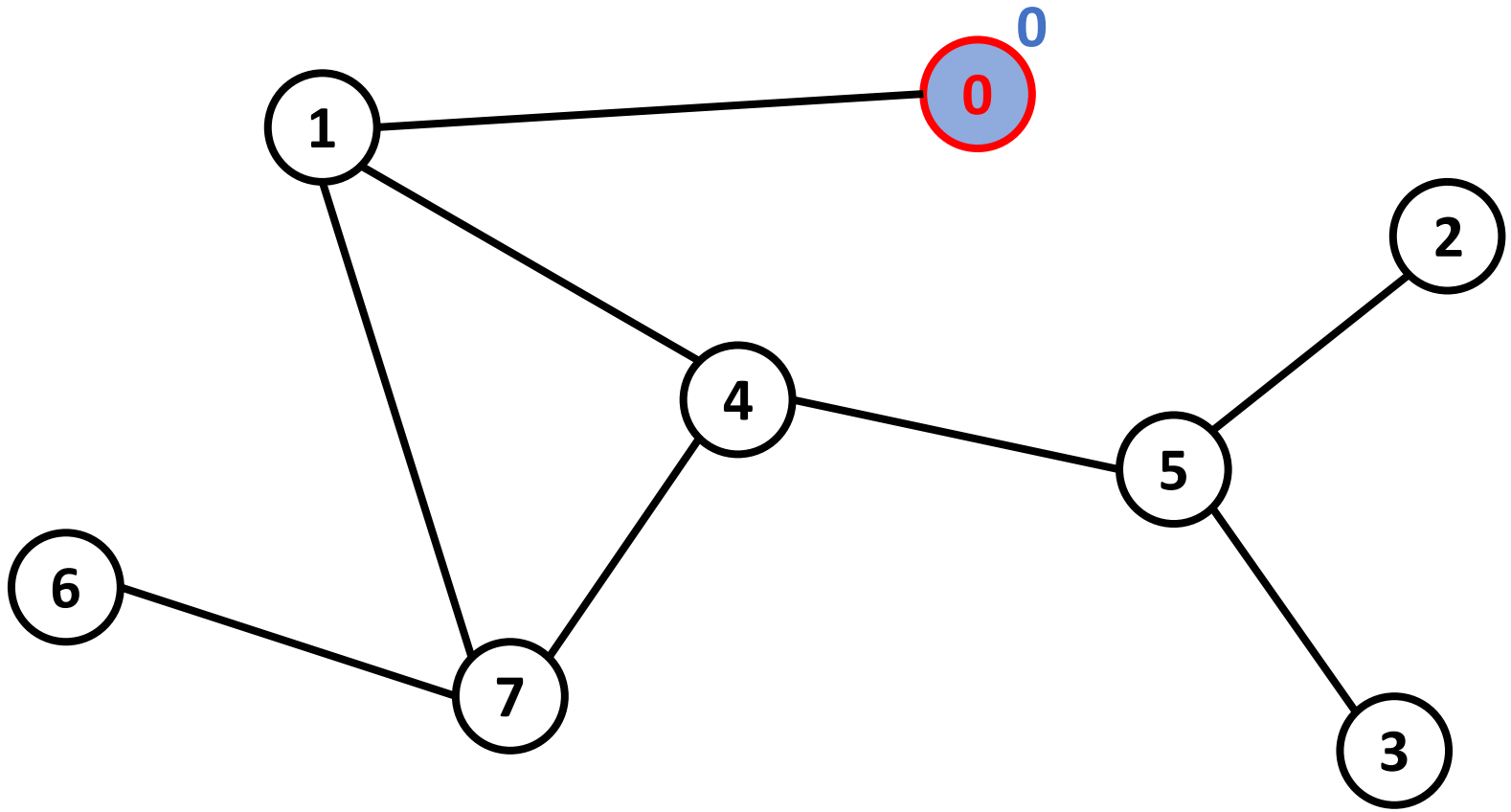
BFS



Orden de recorrido: 0, 1, 7, 4, 6, 5, 2, 3

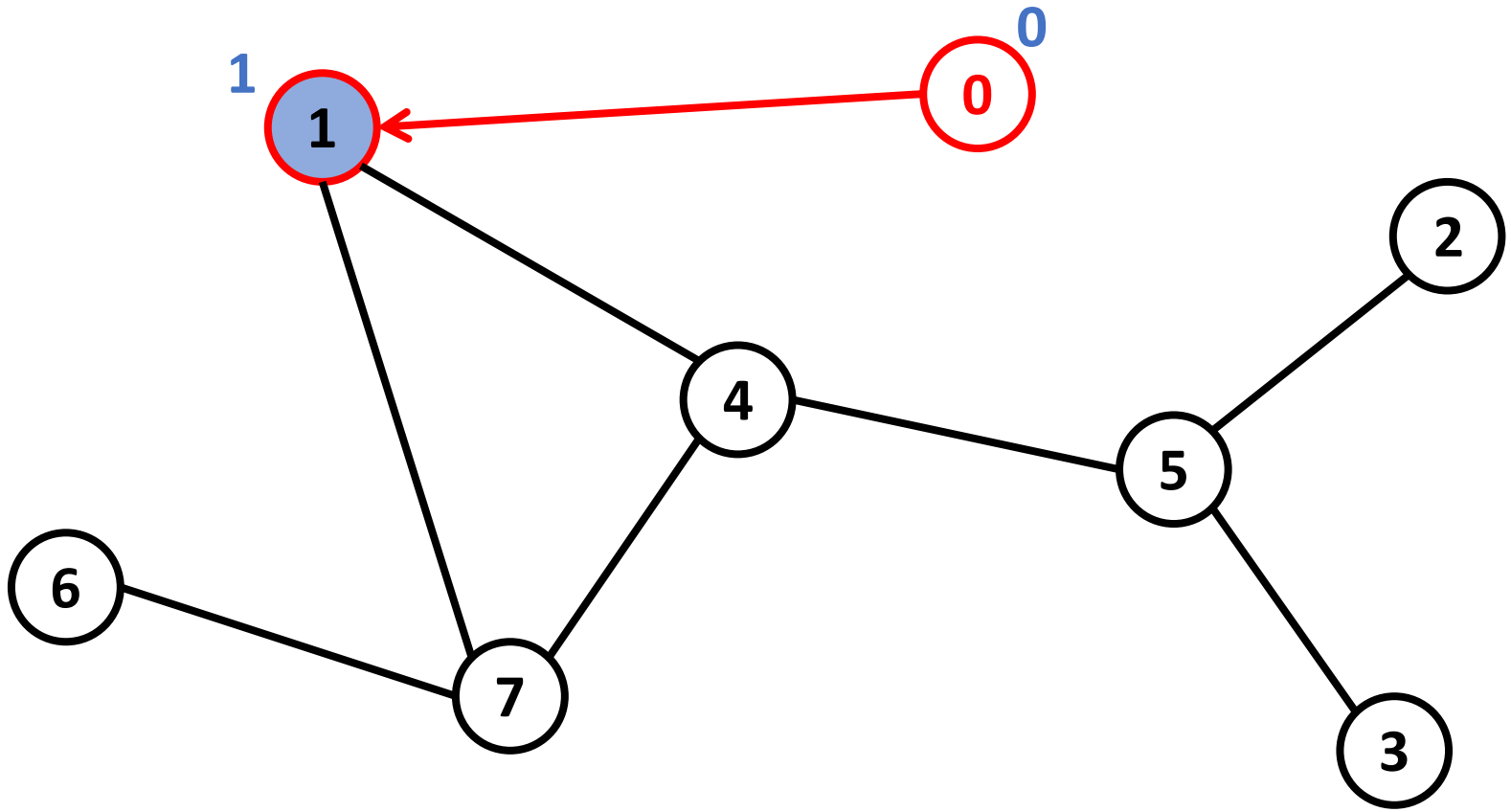
Distancia : 0 1 2 2 3 3 4 4

BFS



Cola: {1}
Nodo: 0

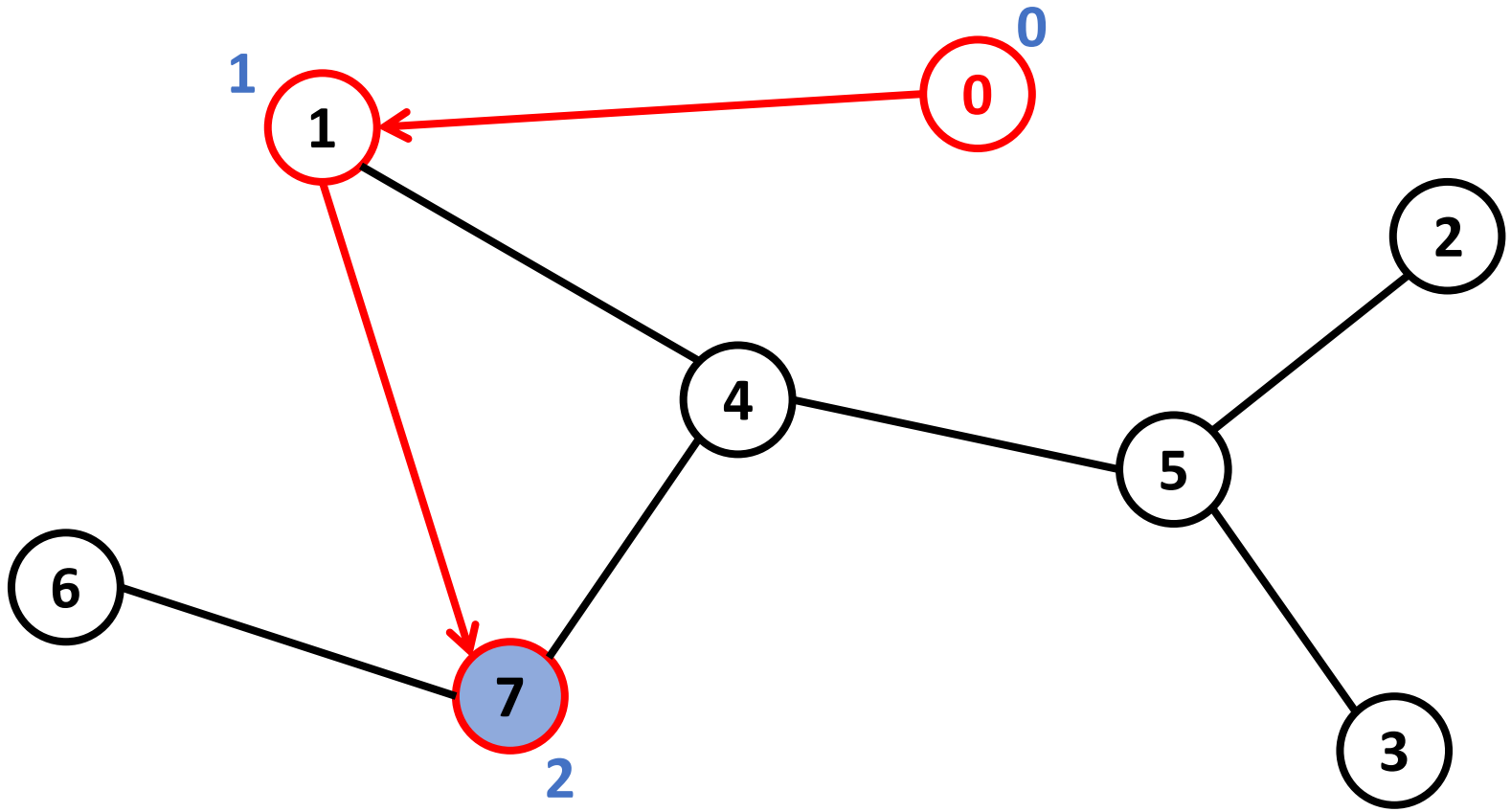
BFS



Cola: {~~1,~~ 7, 4}

Nodo: 1

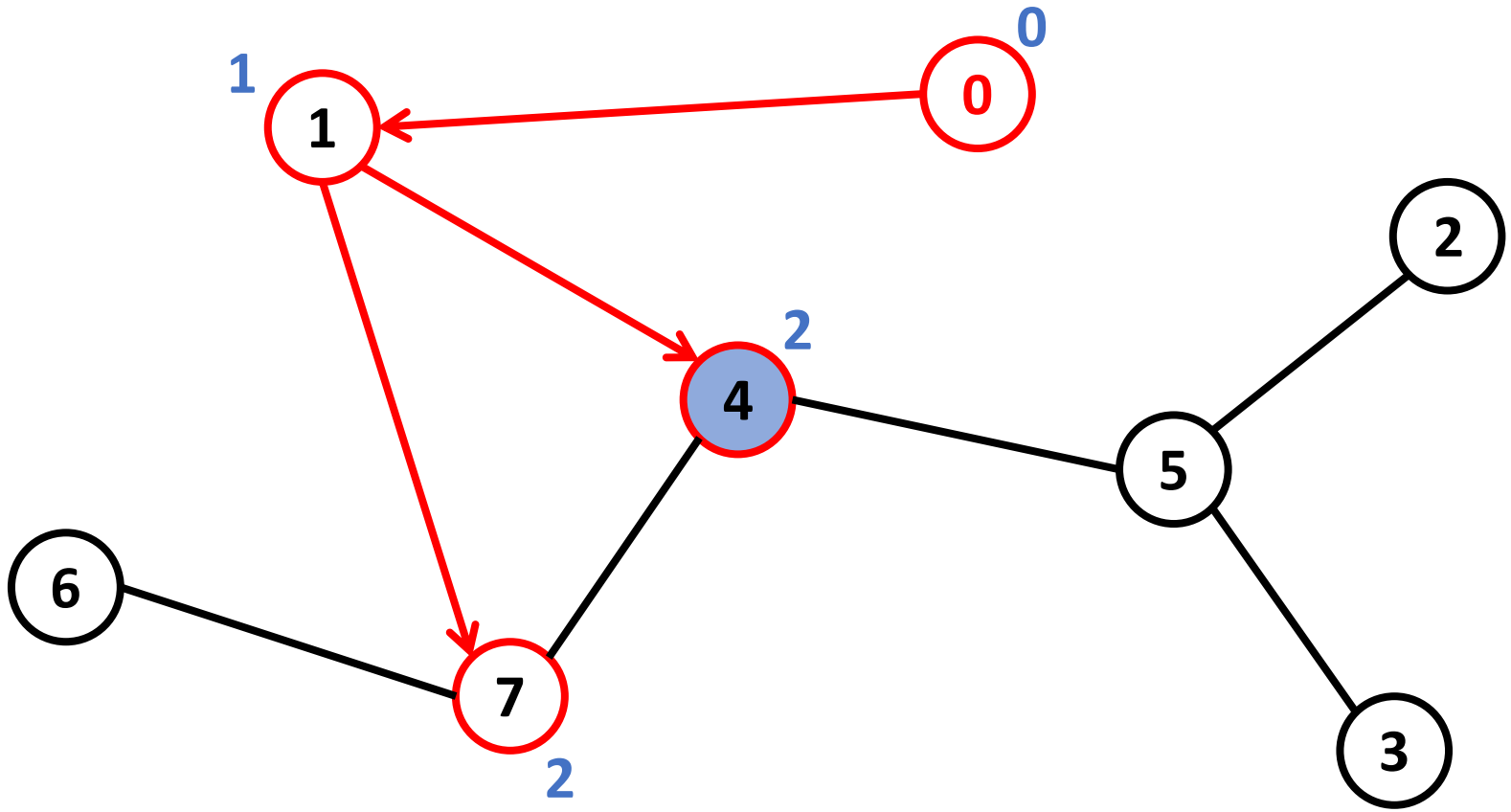
BFS



Cola: {~~1, 7,~~ 4, 6}

Nodo: 7

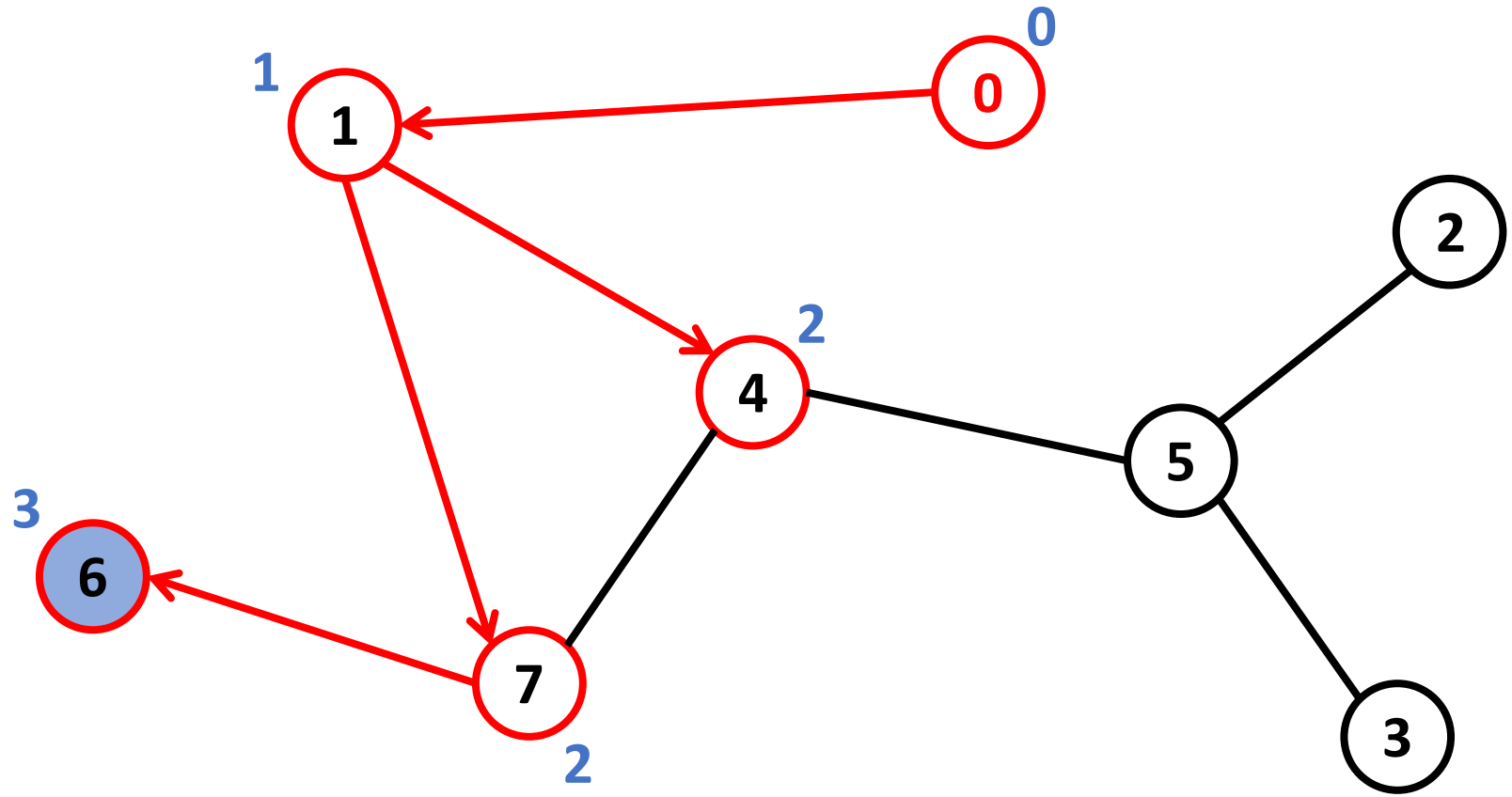
BFS



Cola: {~~1, 7, 4,~~ 6, 5}

Nodo: 4

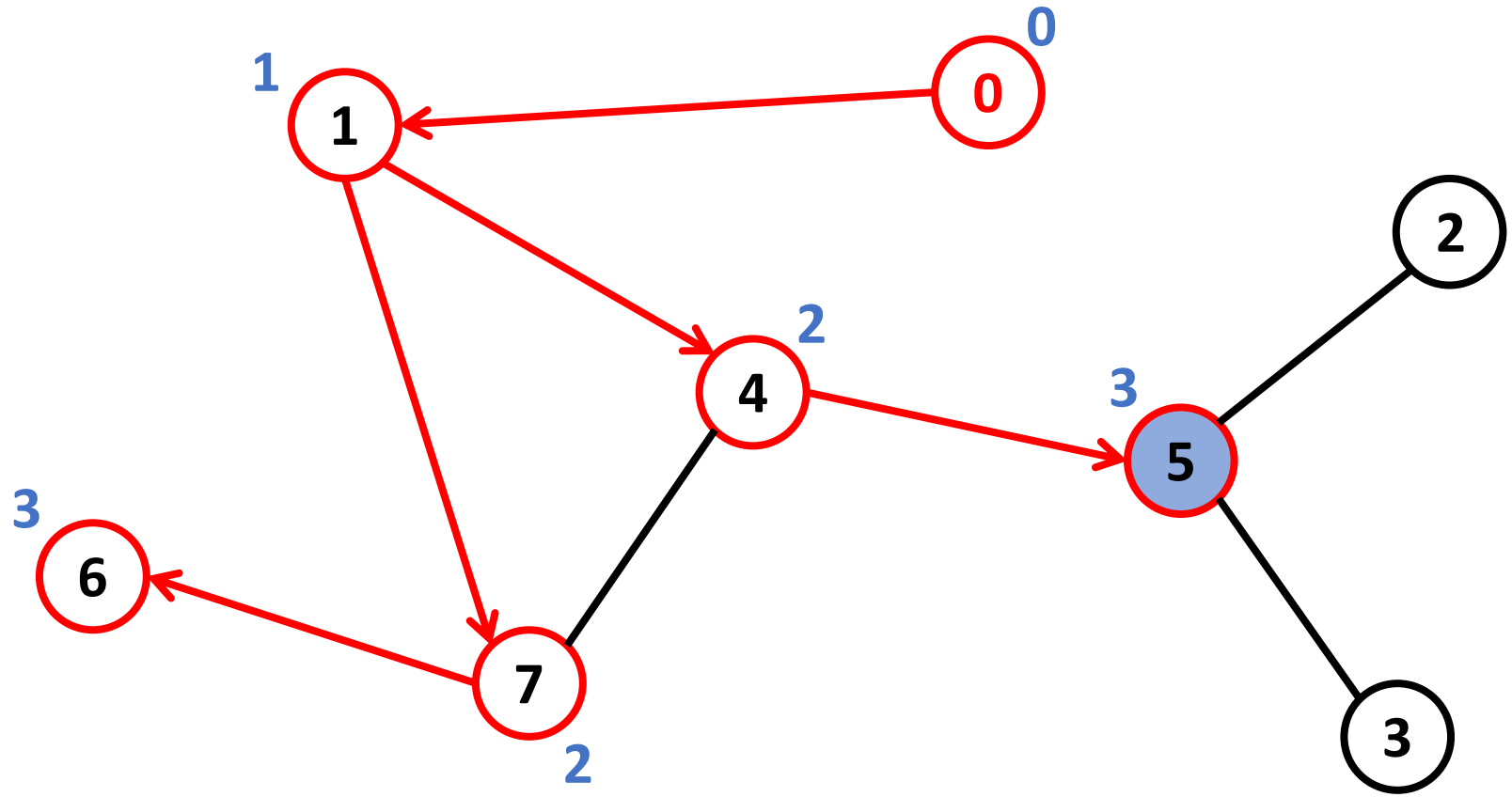
BFS



Cola: {~~1, 7, 4, 6,~~ 5}

Nodo: 6

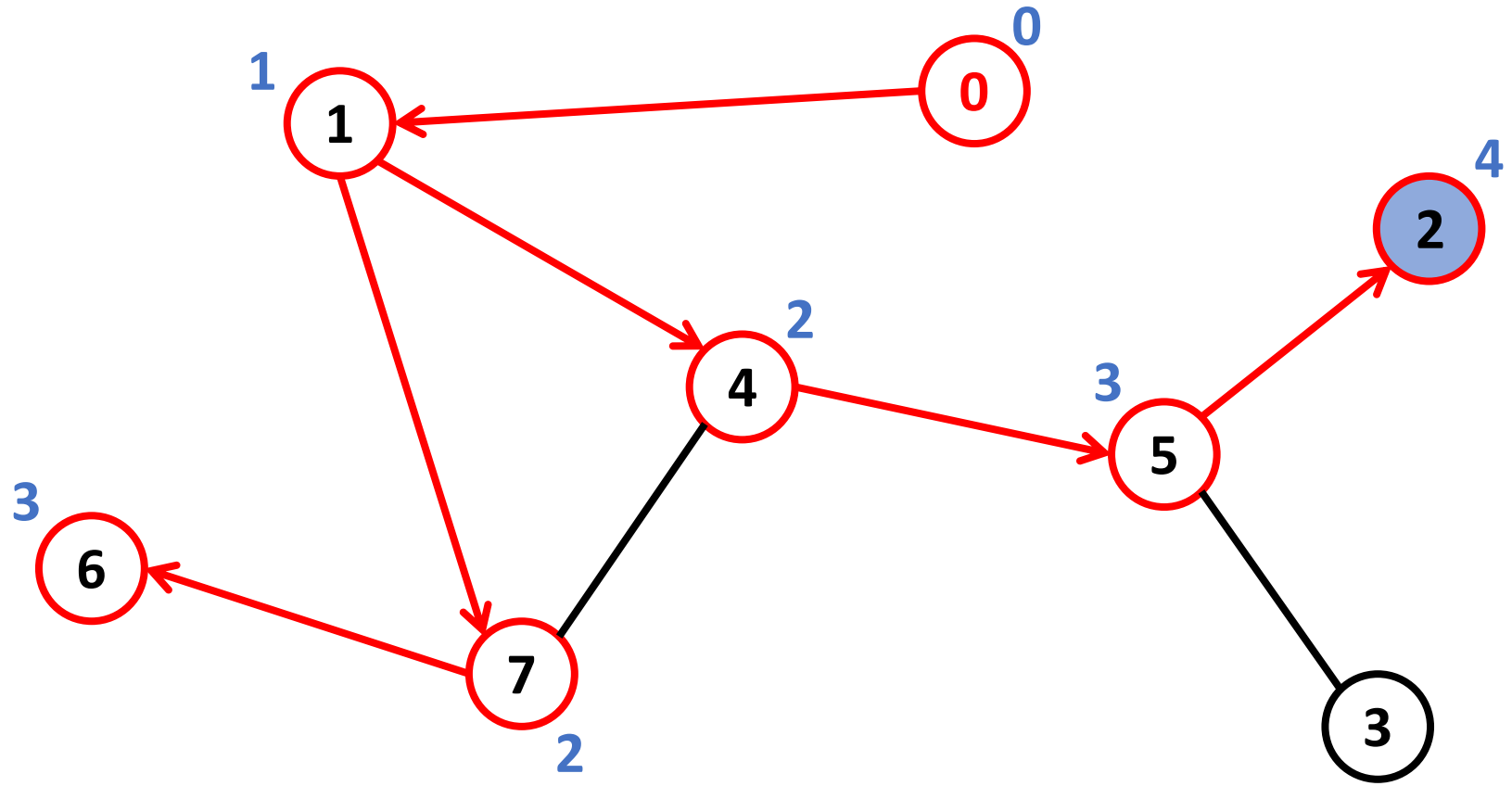
BFS



Cola: {~~1, 7, 4, 6, 5,~~ 2, 3}

Nodo: 5

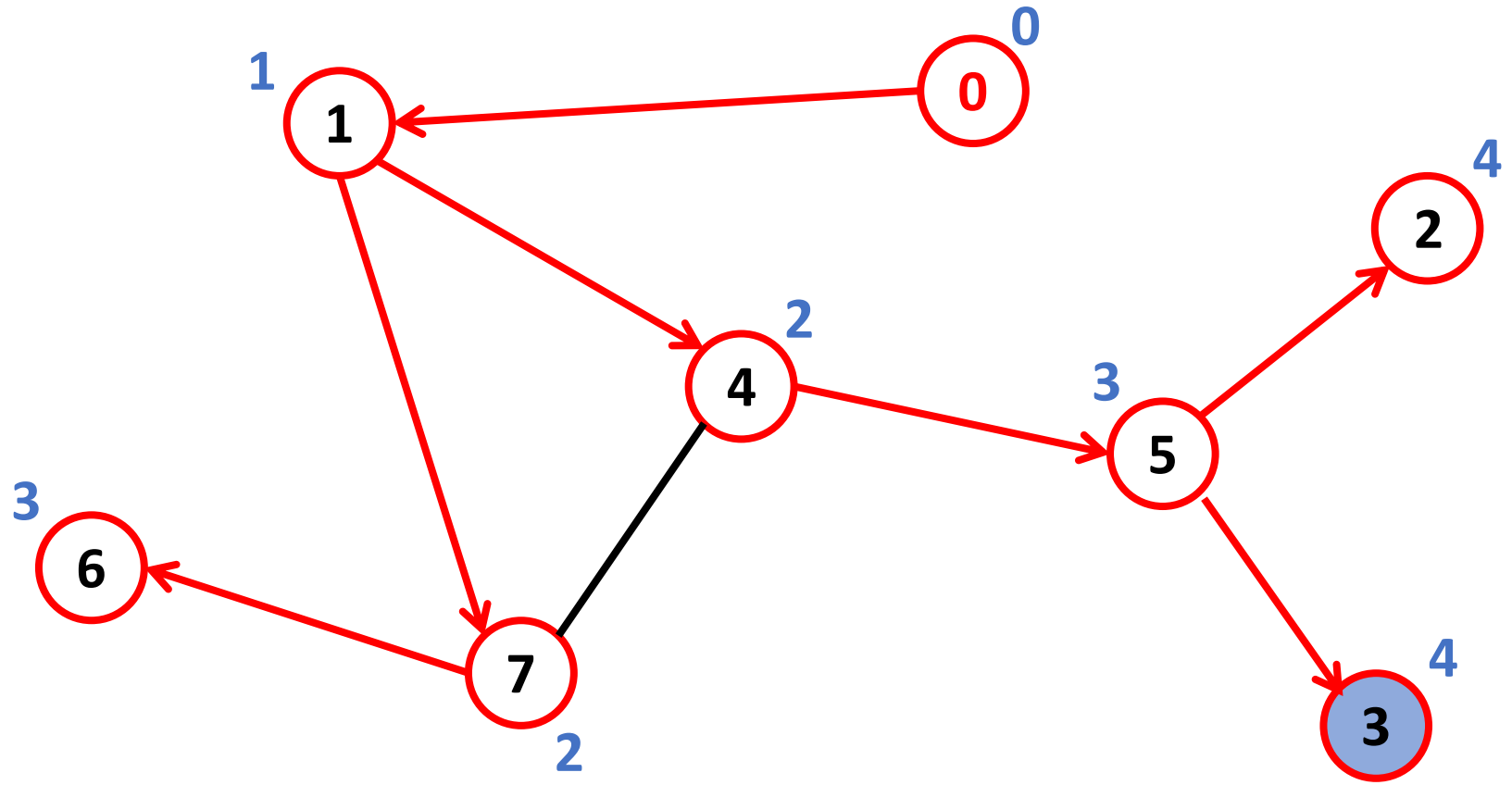
BFS



Cola: {~~1, 7, 4, 6, 5, 2,~~ 3}

Nodo: 2

BFS



Cola: {~~1, 7, 4, 6, 5, 2, 3~~}

Nodo: 3

Código

Necesitamos:

```
vector<int> g[n];

vector<int> nivel(n, -1);

queue<int> q;
```

```
int inicio = 0;
nivel[inicio] = 0;
q.push(inicio);

while (!q.empty()) {
  int u = q.front();
  q.pop();
  for (int v : g[u]) {
    if (nivel[v] == -1) {
      nivel[v] = nivel[u] + 1;
      q.push(v);
    }
  }
}
```