Economic Analysis Macro - Class Notes

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1 Measurement

We focus on the major items of macroeconomic interest: **Economic Activity, Inflation, Unemployment, Interest Rates**

1.1 National Income Accounting

The Three Approaches to National Income Accounting

Product Approach: the dollar amount of output **produced**.

The current **market value** of all **final goods and services newly produced** in the domestic economy, during a specified period of time.

1. **Market Value**

   - Not everything has a market - **imputed values** must be used. Determined based on **production cost**. Example: Fire Department.
   - Most non-market goods and services are not included. Example: Household services.
   - Some market goods and services are not included. Example: Used goods.

2. **Final Goods and Services**

   Goods and services that are **not completely used up** in the production process.

Expenditure Approach: the dollar amount **spent** by purchasers.

The **total spending** on **all final goods and services** produced in the domestic economy during a specified period of time.

$$\boxed{Y = C + I + G + NX} \qquad \text{(\textbf{The National Income Identity})}$$

1. **Consumption**
   Spending by domestic households on final goods and services.

2. **Investment**
   Spending for new capital goods[1] (fixed investment) plus inventory investment.

3. **Gov. Purchases of Goods and Services**
   Government Purchases of goods and services.

   Government Transfer Payments are **not** included in G. (Not payments for goods and services) Example: Medicare, Medicaid, Veterans' benefits and etc.

4. **Net Exports**: Exports - Imports

[1] **Capital goods** are used to produce other goods and services.

Income Approach: the dollar **incomes earned** by production.

The **total income** earned by individuals and businesses in the economy.

The Five Income Measures

- **National Income** = Compensation of Employees + Other Income + Corporate Profits
- **Gross National Product (GNP)** = National Income + Depreciation
- **Gross Domestic Product (GDP)** = GNP + Net Factor Payments
- **Private Disposable Income (PDI)** = GDP + Net Factor Income + Transfer Payments from the Government + Interest Payments on Government Debt - Taxes
- **Net Government Income (NGI)** = Taxes - Government Transfer Payments - Interest Payments on Government Debt

1.2 Inflation

Price Index

Measures the average level of prices for some specified set of goods and services **relative to the prices in a specified base year.**

Three Major Price Indices

- **The Gross Domestic Product (GDP) Deflator:**

$$P = 100 * \frac{\text{nominal GDP}}{\text{real GDP}}$$

- **The Personal Consumption Expenditure (PCE) Deflator:**

$$P = 100 * \frac{\text{nominal PCE}}{\text{real PCE}}$$

- **The Consumer Price Index (CPI):**
  Measures the average prices of a specified basket of goods and services bought by consumers.

Measuring Inflation

Given the price index at time t, denoted as P_t, the inflation rate π_t is expressed as the following

$$\pi_t \approx \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where with a little bit algebra, we derive the **Fisher Equation**

$$i \approx r + \pi$$

i stands for the nominal interest rate, and r is the real interest rate.

Unemployment

2 Aggregate Production and Productivity

When labor and capital are separately increased with the other held constant, the product increases by **diminishing increments**.

2.1 The Cobb-Douglas Production Function

Their assumptions for $Y(K, L)$:

- Output Y is a function of capital K and labor L
- Constant returns to scale:

  If $K - m * K$ and $L - m * L$ then $Y - m * Y$

Their proposed solution:

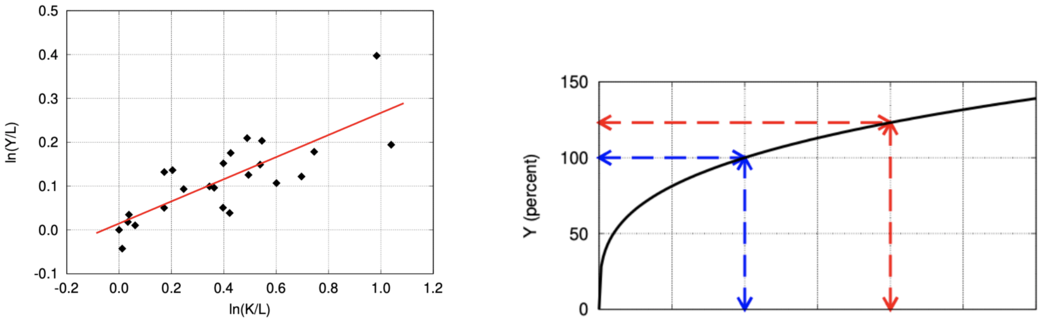
$$\boxed{Y = AK^{\alpha}L^{1-\alpha}} \qquad \textbf{(The Cobb-Douglas Production Function)}$$

Shows how much output can be produced from givenamounts of capital and labor with a given level of total-factor productivity A (also referred to as "technology").

Taking logs of both sides and subtract $\ln(L)$

$$\ln(\frac{Y}{L}) = \ln A + \alpha \ln(\frac{K}{L})$$

Notice that we now have a linear equation that we can analyze



(a) Linear Approx

(b) Diminishing Marginal Return to K

Figure 1

The production function can be drawn as either

- Output as a function of capital (As shown in Figure 1(b))
- Output as a function of labor

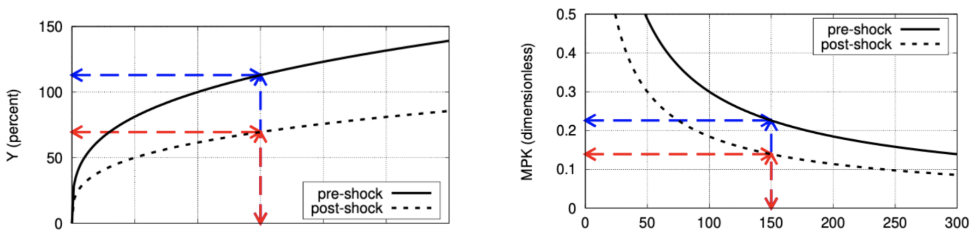
$$\begin{aligned} \text{MPK} &\equiv \frac{\partial Y}{\partial K} \\ &= \alpha A(\frac{L}{K})^{1-\alpha} \end{aligned} \qquad \text{(Marginal Product of Capital)}$$

2.2 Understanding Shocks

Consider the Cobb-Douglas Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

When $AL^{1-\alpha}$ goes down, output goes down across all input K



(a) Reduced Y Across K

(b) Reduced MPC

Figure 2: Shock in $AL^{1-\alpha}$

Moveover, we can see that the slope of the post-shock production curve is smaller than the pre-shock curve, which indicates the decrease of MPK. As shown in Figure 2(b), this also indicates the decrease of **real rental cost of capital**.

Similarily, when AK^α goes down, we will have a lower output across all input of capital.

2.3 Full-Employment Output

Full-employment, or potential, outputis the level of outputwhen the labor market is in the **long-run equilibrium**.

$$Y^P = AK^\alpha L_S^{1-\alpha}$$

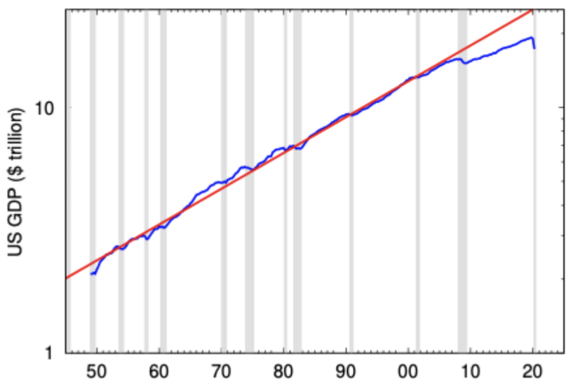
3 The Solow-Swan Model: Long-Run Growth

Over the decades, we discovered that the growth of GDP is roughly **linear** in the **logrithmic domain**.

$$\ln(Y(t)) = c + g_Y t$$

Following this idea, we can get a model for describing the output at time t

$$Y(t) = Y(0)e^{g_Y t}$$



We can see that our log-linear fit is pretty reasonable. The fluctuations about the trend are referred to as **bussiness cycles**.

3.1 Where Does Growth Come From?

Let's begin with the Cobb-Douglas Production Function

$$Y = AK^\alpha L^{1-\alpha}$$

Taking the log of both sides, then take the derivative

$$\boxed{\frac{1}{Y}\frac{dY}{dt} = \frac{1}{A}\frac{dA}{dt} + \alpha\frac{1}{K}\frac{dK}{dt} + (1-\alpha)\frac{1}{L}\frac{dL}{dt}} \qquad \text{(Growth Accounting Formula)}$$

Approximate the result with $\Delta t = 1$, we have

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

Sometimes it's also written as $g_Y = g_A + \alpha g_K + (1-\alpha)g_L$

For example, in the United States where $\alpha \approx 0.3$, we have

$$g_Y = g_A + 0.3g_K + 0.7g_L$$

This tells us how the growth of one or more of the variables in the production function attribute to the overall increase in output.

Research has shown that **productivity growth** is a more important source of variation in growth rates across countries than is growth in capital or labor(a.k.a. factor accumulation).

3.2 The Solow-Swan Model

The Solow-Swan Model was developed in 1950s to determine **capital accumulation**, which affects **economic growth**.

To derive this, we assume that a **constant fraction** s **of output** Y is saved, and the capital **depreciates at a constant rate** δ.

$$\frac{dK}{dt} = sAK^{\alpha}L^{1-\alpha} - \delta K \qquad \text{(Capital Accumulation Equation)}$$

The goal is to solve this differential equation so we get K as a function of t

We make the following assumptions:

- Replace total factor of productivity A with **labor efficiency** E, and assume that $E(t)$ has a **log-linear growth rate**

$$E(t) = A(t)^{1/(1-\alpha)}$$
$$E(t) = E(0)e^{g_E t}$$

- Assume that L also grows in log-linear rate.

$$L(t) = L(0)e^{g_L t}$$

- Instead of examine the aggregate capital K, we look at the the **capital per worker per unit of worker efficiency**, denoted by κ. This is also referred to as **normalized capital**.

$$\kappa(t) = \frac{K(t)}{E(t)L(t)}$$

Plug in our expression for $\kappa(t)$, $L(t)$, and $E(t)$, we have the capital accumulation equation for κ

$$\frac{d\kappa}{dt} = s\kappa^{\alpha} - (g_E + g_L + \delta)\kappa$$

The solution $\kappa(t)$ is important because it is directly related to **per-capita income** by

$$\frac{Y(t)}{L(t)} = \kappa(t)^{\alpha}E(t)$$

Generally we have two solutions of interest

- The steady-state solution κ^*

$$\frac{d\kappa}{dt} = 0$$

- The general solution $\kappa(t)$

3.3 The Solutions

Stead-state Solution

Follows from the capital accumulation equation

$$\frac{d\kappa}{dt} = s\kappa^{\alpha} - (g_E + g_L + \delta)\kappa = 0$$

so

$$s\kappa^{\alpha} = (g_E + g_L + \delta)\kappa$$
$$\kappa^{1-\alpha} = \frac{s}{g_E + g_L + \delta}$$

and

$$\boxed{\kappa^* = (\frac{s}{g_E + g_L + \delta})^{\frac{1}{1-\alpha}}}$$

In the steady state, per-capita income is given by

$$\frac{Y(t)}{L(t)} = \kappa^*(t)^\alpha E(t) = (\frac{s}{g_E + g_L + \delta})^{\frac{1}{1-\alpha}} E(t)$$

and from which follows that

$$g_{Y/L} = g_E$$

In the steady state $\kappa(t) = \kappa^*$ and is not changing, so

$$\kappa(t) = \frac{K(0)e^{g_K t}}{E(0)e^{g_K t} L(0) e^{g_K t}} = \frac{K(0)}{E(0)L(0)} e^{(g_K - g_E - g_L)t} = \kappa^*$$

Therefore $g_{Y/L} = g_E = g_K - g_L$

General Solution

The general solution can be written in 'gap' form

$$\text{gap}(t) = \text{gap}(0)e^{-t/\tau}$$

where

$$\text{gap}(t) = \kappa(t)^{1-\alpha} - \frac{s}{g_E + g_L + \delta}$$

Overtime gap(t) converges to 0, with the **half life** given by

$$\boxed{t_{1/2} = \frac{\ln 2}{(1-\alpha)(g_E + g_L + \delta)}}$$

3.4 Shock Analysis

Three important components

- $\kappa^*(t)^\alpha E(t)$: **The balanced growth**

$$\kappa^* = (\frac{s}{g_E + g_L + \delta})^{\frac{1}{1-\alpha}}$$

- $Y(t)/L(t)$: **Per capita income**

$$\frac{Y(t)}{L(t)} = \kappa(t)^\alpha E(t) = (\frac{K(t)}{E(t)L(t)})^\alpha E(t)$$

- $t_{1/2}$: **Half life** – How fast per capita income converges to the balanced growth after a shock

$$t_{1/2} = \frac{\ln 2}{(1-\alpha)(g_E + g_L + \delta)}$$