

Economic Analysis Macro - Class Notes

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1 Measurement

We focus on the major items of macroeconomic interest: **Economic Activity, Inflation, Unemployment, Interest Rates**

1.1 National Income Accounting

The Three Approaches to National Income Accounting

Product Approach: the dollar amount of output **produced**.

The current **market value** of all **final goods and services newly produced** in the domestic economy, during a specified period of time.

1. Market Value

- Not everything has a market - **imputed values** must be used. Determined based on **production cost**. Example: Fire Department.
- Most non-market goods and services are not included. Example: Household services.
- Some market goods and services are not included. Example: Used goods.

2. Final Goods and Services

Goods and services that are **not completely used up** in the production process.

Expenditure Approach: the dollar amount **spent** by purchasers.

The **total spending** on all **final goods and services** produced in the domestic economy during a specified period of time.

$$\boxed{Y = C + I + G + NX} \quad (\text{The National Income Identity})$$

1. Consumption

Spending by domestic households on final goods and services.

2. Investment

Spending for new capital goods¹ (fixed investment) plus inventory investment.

3. Gov. Purchases of Goods and Services

Government Purchases of goods and services.

Government Transfer Payments are **not** included in G . (Not payments for goods and services) Example: Medicare, Medicaid, Veterans' benefits and etc.

4. Net Exports: Exports - Imports

¹Capital goods are used to produce other goods and services.

Income Approach: the dollar **incomes earned** by production.

The **total income** earned by individuals and businesses in the economy.

The Five Income Measures

- **National Income** = Compensation of Employees + Other Income + Corporate Profits
- **Gross National Product (GNP)** = National Income + Depreciation
- **Gross Domestic Product (GDP)** = GNP + Net Factor Payments
- **Private Disposable Income (PDI)** = GDP + Net Factor Income + Transfer Payments from the Government + Interest Payments on Government Debt - Taxes
- **Net Government Income (NGI)** = Taxes - Government Transfer Payments - Interest Payments on Government Debt

1.2 Inflation

Price Index

Measures the average level of prices for some specified set of goods and services **relative to the prices in a specified base year**.

Three Major Price Indices

- **The Gross Domestic Product (GDP) Deflator:**

$$P = 100 * \frac{\text{nominal GDP}}{\text{real GDP}}$$

- **The Personal Consumption Expenditure (PCE) Deflator:**

$$P = 100 * \frac{\text{nominal PCE}}{\text{real PCE}}$$

- **The Consumer Price Index (CPI):**

Measures the average prices of a specified basket of goods and services bought by consumers.

Measuring Inflation

Given the price index at time t , denoted as P_t , the inflation rate π_t is expressed as the following

$$\pi_t \approx \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where with a little bit algebra, we derive the **Fisher Equation**

$$i \approx r + \pi$$

i stands for the nominal interest rate, and r is the real interest rate.

Unemployment

2 Aggregate Production and Productivity

When labor and capital are separately increased with the other held constant, the product increases by **diminishing increments**.

2.1 The Cobb-Douglas Production Function

Their assumptions for $Y(K, L)$:

- Output Y is a function of capital K and labor L
- Constant returns to scale:

If $K \rightarrow m * K$ and $L \rightarrow m * L$ then $Y \rightarrow m * Y$

Their proposed solution:

$$Y = AK^\alpha L^{1-\alpha} \quad (\text{The Cobb-Douglas Production Function})$$

Shows how much output can be produced from given amounts of capital and labor with a given level of total-factor productivity A (also referred to as "technology").

Taking logs of both sides and subtract $\ln(L)$

$$\ln\left(\frac{Y}{L}\right) = \ln A + \alpha \ln\left(\frac{K}{L}\right)$$

Notice that we now have a linear equation that we can analyze

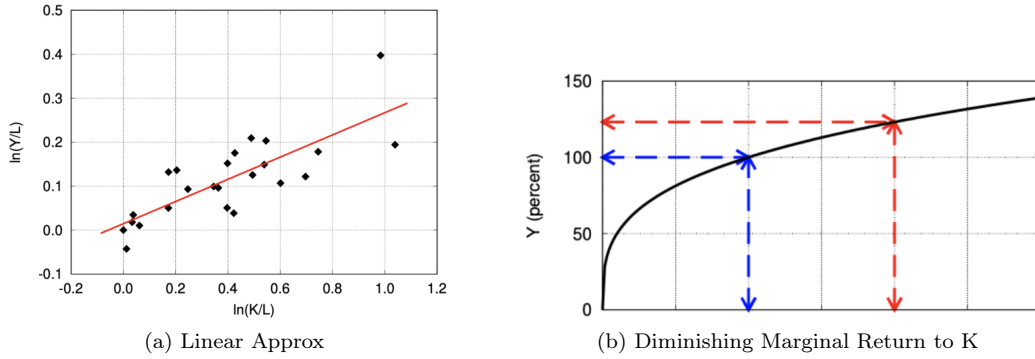


Figure 1

The production function can be drawn as either

- Output as a function of capital (As shown in Figure 1(b))
- Output as a function of labor

$$\begin{aligned} \text{MPK} &\equiv \frac{\partial Y}{\partial K} \\ &= \alpha A \left(\frac{L}{K}\right)^{1-\alpha} \end{aligned} \quad (\text{Marginal Product of Capital})$$

2.2 Understanding Shocks

Consider the Cobb-Douglas Production Function

$$Y = AK^\alpha L^{1-\alpha}$$

When $AL^{1-\alpha}$ goes down, output goes down across all input K

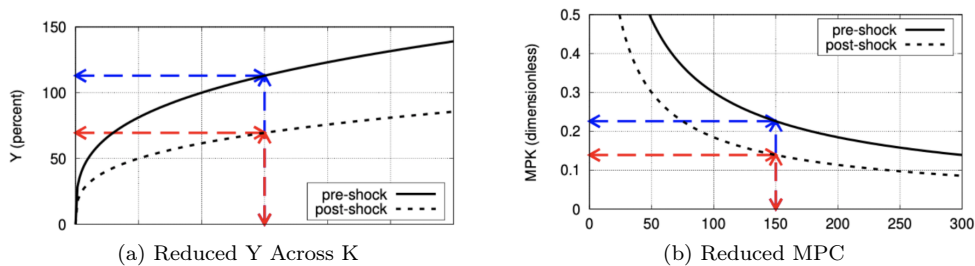


Figure 2: Shock in $AL^{1-\alpha}$

Moreover, we can see that the slope of the post-shock production curve is smaller than the pre-shock curve, which indicates the decrease of MPK . As shown in Figure 2(b), this also indicates the decrease of **real rental cost of capital**.

Similarly, when AK^α goes down, we will have a lower output across all input of capital.

2.3 Full-Employment Output

Full-employment, or potential, output is the level of output when the labor market is in the **long-run equilibrium**.

$$Y^P = AK^\alpha L_S^{1-\alpha}$$

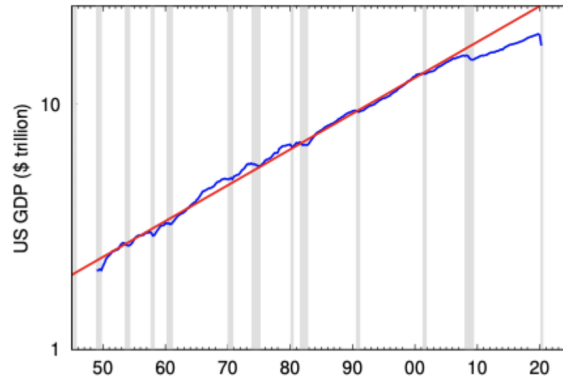
3 The Solow-Swan Model: Long-Run Growth

Over the decades, we discovered that the growth of GDP is roughly **linear** in the **logarithmic domain**.

$$\ln(Y(t)) = c + g_Y t$$

Following this idea, we can get a model for describing the output at time t

$$Y(t) = Y(0)e^{g_Y t}$$



We can see that our log-linear fit is pretty reasonable. The fluctuations about the trend are referred to as **business cycles**.

3.1 Where Does Growth Come From?

Let's begin with the Cobb-Douglas Production Function

$$Y = AK^\alpha L^{1-\alpha}$$

Taking the log of both sides, then take the derivative

$$\boxed{\frac{1}{Y} \frac{dY}{dt} = \frac{1}{A} \frac{dA}{dt} + \alpha \frac{1}{K} \frac{dK}{dt} + (1-\alpha) \frac{1}{L} \frac{dL}{dt}} \quad (\text{Growth Accounting Formula})$$

Approximate the result with $\Delta t = 1$, we have

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta L}{L}$$

Sometimes it's also written as $g_Y = g_A + \alpha g_K + (1-\alpha)g_L$

For example, in the United States where $\alpha \approx 0.3$, we have

$$g_Y = g_A + 0.3g_K + 0.7g_L$$

This tells us how the growth of one or more of the variables in the production function attribute to the overall increase in output.

Research has shown that **productivity growth** is a more important source of variation in growth rates across countries than is growth in capital or labor (a.k.a. factor accumulation).

3.2 The Solow-Swan Model

The Solow-Swan Model was developed in 1950s to determine **capital accumulation**, which affects **economic growth**.

To derive this, we assume that a **constant fraction s of output Y** is saved, and the capital **depreciates at a constant rate δ** .

$$\frac{dK}{dt} = sAK^\alpha L^{1-\alpha} - \delta K \quad (\text{Capital Accumulation Equation})$$

The goal is to solve this differential equation so we get K as a function of t

We make the following assumptions:

- Replace total factor of productivity A with **labor efficiency E** , and assume that $E(t)$ has a **log-linear growth rate**

$$\begin{aligned} E(t) &= A(t)^{1/(1-\alpha)} \\ E(t) &= E(0)e^{g_E t} \end{aligned}$$

- Assume that L also grows in log-linear rate.

$$L(t) = L(0)e^{g_L t}$$

- Instead of examine the aggregate capital K , we look at the **capital per worker per unit of worker efficiency**, denoted by κ . This is also referred to as **normalized capital**.

$$\kappa(t) = \frac{K(t)}{E(t)L(t)}$$

Plug in our expression for $\kappa(t)$, $L(t)$, and $E(t)$, we have the capital accumulation equation for κ

$$\frac{d\kappa}{dt} = s\kappa^\alpha - (g_E + g_L + \delta)\kappa$$

The solution $\kappa(t)$ is important because it is directly related to **per-capita income** by

$$\frac{Y(t)}{L(t)} = \kappa(t)^\alpha E(t)$$

Generally we have two solutions of interest

- The steady-state solution κ^*

$$\frac{d\kappa}{dt} = 0$$

- The general solution $\kappa(t)$

3.3 The Solutions

Stead-state Solution

Follows from the capital accumulation equation

$$\frac{d\kappa}{dt} = s\kappa^\alpha - (g_E + g_L + \delta)\kappa = 0$$

so

$$\begin{aligned} s\kappa^\alpha &= (g_E + g_L + \delta)\kappa \\ \kappa^{1-\alpha} &= \frac{s}{g_E + g_L + \delta} \end{aligned}$$

and

$$\boxed{\kappa^* = \left(\frac{s}{g_E + g_L + \delta} \right)^{\frac{1}{1-\alpha}}}$$

In the steady state, per-capita income is given by

$$\frac{Y(t)}{L(t)} = \kappa^*(t)^\alpha E(t) = \left(\frac{s}{g_E + g_L + \delta}\right)^{\frac{1}{1-\alpha}} E(t)$$

and from which follows that

$$g_{Y/L} = g_E$$

In the steady state $\kappa(t) = \kappa^*$ and is not changing, so

$$\kappa(t) = \frac{K(0)e^{g_K t}}{E(0)e^{g_K t}L(0)e^{g_K t}} = \frac{K(0)}{E(0)L(0)} e^{(g_K - g_E - g_L)t} = \kappa^*$$

Therefore $g_{Y/L} = g_E = g_K - g_L$

General Solution

The general solution can be written in 'gap' form

$$\text{gap}(t) = \text{gap}(0)e^{-t/\tau}$$

where

$$\text{gap}(t) = \kappa(t)^{1-\alpha} - \frac{s}{g_E + g_L + \delta}$$

Overtime $\text{gap}(t)$ converges to 0, with the **half life** given by

$$t_{1/2} = \frac{\ln 2}{(1-\alpha)(g_E + g_L + \delta)}$$

3.4 Shock Analysis

Four important components

- $\kappa^*(t)^\alpha E(t)$: **The balanced growth**

$$\kappa^* = \left(\frac{s}{g_E + g_L + \delta}\right)^{\frac{1}{1-\alpha}}$$

- $Y(t)/L(t)$: **Per capita income**

$$\frac{Y(t)}{L(t)} = \kappa(t)^\alpha E(t) = \left(\frac{K(t)}{E(t)L(t)}\right)^\alpha E(t)$$

- $t_{1/2}$: **Half life** – How fast per capita income converges to the balanced growth after a shock

$$t_{1/2} = \frac{\ln 2}{(1-\alpha)(g_E + g_L + \delta)}$$

- $g_{Y/L}$: **Long term growth rate**

$$g_{Y/L} = g_E$$

3.5 Endogenous Growth Theory - the Romer Model

If we are looking for the factors that help the economy grows faster, we soon realize that factors such as the saving rate, labor-force growth rate can rise and fall forever, but efficiency can always improve.

In endogenous growth theory, we look at **technology as a production input**. It differs from capital and labor in two important characteristics:

- **Non-rivalry:** More than one person can use the factor at any given time.
- **Non-excludability:** One person cannot prevent others from using the factor.

Endogenous growth theory is an attempt to explain how and why technology can increase endogenously and, thereby sustained increases in income-per-worker. It is often referred to as the **Romer Model**.

The Romer Model

In the Romer Model, **labor** is allocated to the production of

- **Goods and services** L_P
- **New technology** L_E

The total labor supply $L = L_P + L_E$ is assumed to be fixed. Let γ denotes the ratio of total labor participating in R&D.

$$L_E = \gamma L$$

The production function for technology

$$\frac{dE}{dt} = \chi E L_E$$

where χ indicates how productive labor is in producing ideas.

If we take the technology production function and divide both side by E

$$\frac{1}{E} \frac{dE}{dt} \equiv g_E = \chi L_E$$

Substituting L_E , we get the **Romer Model**

$$g_E = \chi \gamma L$$

4 Business Cycles

5 IS Curve

From the production function we can derive the economy's **full-employment output level** (how much the economy could produce). This is the basis for the economy's **aggregate supply**.

Now the focus shifts to how much is the demand in the economy, a.k.a the **aggregate demand**.

Aggregate demand and aggregate supply are crucial for explaining **short-run fluctuations** in economic activity.

5.1 Planned Expenditures

There are two types of expenditure

- **Actual Expenditures** Y : Total amount of spending on domestically produced goods and services that households, businesses, the government, and foreigners actually make. **This is equivalent to the total output actually produced in the economy.**
- **Planned Expenditures** Y^{pl} : Total amount of spending on domestically produced goods and services that households, businesses, the government, and foreigners want to make. **This is equivalent to the aggregate demand.**

The economy is in **goods market equilibrium** when actual expenditures equal planned expenditures

$$Y = Y^{pl}$$

Similarly to the national income identity, Y^{pl} can be expressed as

$$Y^{pl} = C + I^{pl} + G + NX$$

The Consumption Function

Consumption expenditures are represented by the **consumption function**

$$C = \overline{C} + \text{mpc} \cdot Y^d - \zeta_c r_L$$

where

$\overline{C} \equiv$ Autonomous Consumption Expenditure

$Y^d \equiv$ Disposable Income = $Y - T$

$\text{mpc} \equiv$ Marginal Propensity to Consume

$\zeta_c \equiv$ Sensitivity of C to r_L

$r_L \equiv$ Real Lending Interest Rate