Spontaneous particle aggregation with delay

immediate

1 The original spontaneous aggregation model

The individual-based stochastic model introduced in reference [1] under the name 'direct aggregation model' consists of a group of $N \geq 2$ biological agents (cells or animals), characterized by their positions $\mathbf{x}_i(t) \in \mathbb{R}^d$, with spatial dimension $d \in \{1,2,3\}$ and $i \in [N]$, where we have denoted the set of indices by $[N] := \{1, 2, \ldots, N\}$. Every individual senses the average density of its close neighbours, given by

$$\vartheta_i(t) = \frac{1}{N-1} \sum_{j \neq i} W(\mathbf{x}_i(t) - \mathbf{x}_j(t)) \,, \qquad \text{for} \quad i \in [N] \,, \tag{1.1}$$

where $W(\mathbf{x}) = w(|\mathbf{x}|)$ with the weight function $w : \mathbb{R}^+ \to \mathbb{R}^+$ assumed to be bounded, nonnegative, nonincreasing and integrable on $\mathbb{R}^d$. Without loss of generality we impose the normalization

$$\int_{\mathbb{R}^d} W(\mathbf{x}) \, \mathrm{d}\mathbf{x} = 1 \,. \tag{1.2}$$

A generic example of w is the (properly normalized) characteristic function of the interval $[0, R]$, corresponding to the sampling radius $R > 0$. The average density ϑ_i is then simply the fraction of individuals located within the distance R from the i-th individual. The individual positions are subject to a random walk with modulated amplitude, described by the system of coupled SDEs

$$\mathrm{d}\mathbf{x}_i(t) \; = \; G(\vartheta_i) \, \mathrm{d}\mathbf{B}_i^t \,, \qquad \text{for} \quad i \in [N] \,, \tag{1.3}$$

where $\mathbf{B}_i^t$ are independent d-dimensional Brownian motions. The *response function* $G : \mathbb{R}^+ \to \mathbb{R}^+$ is assumed to be globally bounded, nonnegative and decreasing. The monotonicity of G is implied by the modeling assumption that the individuals respond to higher perceived population densities in their vicinity by reducing the amplitude of their random walk.

2 Spontaneous aggregation model with delay

From the modeling point of view it makes sense to consider two types of delay:

- **Transmission-type delay**, where we assume that there is non-negligible time for the transmission of information from agent x_j to agent x_i. I.e., agent i at time t makes its decision based on the information about agent j that was actual at time $t - \tau$, where $\tau > 0$ is the transmission delay. For simplicity we shall assume that $\tau > 0$ is a global constant, taking the same value for all pairs of agents i, j. This means that formula (1.1) is replaced by

$$\vartheta_i(t) = \frac{1}{N-1} \sum_{j \neq i} W(\mathbf{x}_i(t) - \mathbf{x}_j(t-\tau)) \,, \qquad \text{for} \quad i \in [N] \,. \tag{2.1}$$

- **Reaction-type delay**, where we assume that the information transmission is instantaneous (or, strictly speaking, the transmission delay is negligible), but the agents need a non-negligible time to carry out their actions. This means that agent i at time t reacts to the information $x_i - x_j$ that was actual at time $t - \tau$. In this case we replace formula (1.1) by

$$\vartheta_i(t) = \frac{1}{N-1} \sum_{j \neq i} W(\mathbf{x}_i(t-\tau) - \mathbf{x}_j(t-\tau)), \qquad \text{for} \quad i \in [N]. \tag{2.2}$$

References

[1] M. Burger, J. Haskovec, and M.-T. Wolfram. Individual-based and mean-field modelling of direct aggregation. *Physica D - Nonlinear Phenomena*, 260:145–158, 2012.