№8.1.5

$y'' + y = -x \qquad x \in [0;1] \qquad h = \frac{1}{2}$

$y(x) = C_1 Y_1(x) + C_2 Y_2(x) + Y(x)$

$Y_1'' + Y_1 = 0, \quad Y_1(0) = 0, \quad Y_1'(0) = 1, \quad x \in [0,1]$

$Y_2'' + Y_2 = 0, \quad Y_2(0) = 1, \quad Y_2'(0) = 0, \quad x \in [0,1]$

$Y'' + Y = -x, \quad Y(0) = 0, \quad Y'(0) = 0, \quad x \in [0,1]$

Сведем к ур-ниям первого порядка

$$\begin{cases} Z_1 = Y_1' \\ Z_1' = -Y_1 \\ x \in [0,1] \\ Y_1(0) = 0 \quad Z_1(0) = 1 \end{cases} \qquad \begin{cases} Z_2 = Y_2' \\ Z_2' = -Y_2 \\ x \in [0,1] \\ Y_2(0) = 1 \quad Z_2(0) = 1 \end{cases} \qquad \begin{cases} Z = Y' \\ Z' = -Y - x \\ x \in [0,1] \\ Y(0) = 0 \quad Z(0) = 0 \end{cases}$$

метод Эйлера

$(Y_1)_0 = 0 \; (Z_1)_0 = 1$

$(Y_1)_1 = (Y_1)_0 + (Z_1)_0 h = 0 + 1 \cdot \frac{1}{2} = \frac{1}{2}$

$(Z_1)_1 = (Z_1)_0 + (-(Y_1)_0) h = 1 + 0 \cdot h = 1$

$(Y_1)_2 = (Y_1)_1 + (Z_1)_1 h = \frac{1}{2} + 1 \cdot \frac{1}{2} = 1$

$(Z_1)_2 = (Z_1)_1 + (-(Y_1)_1) h = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\overline{Y}_1 = (0 \quad \frac{1}{2} \quad 1)$

$(Y_2)_0 = 1 \quad (Z_2)_0 = 0$

$(Y_2)_1 = (Y_2)_0 + (Z_2)_0 h = 1 + 0 \cdot h = 1$

$(Z_2)_1 = (Z_2)_0 + (-(Y_2)_0)h = 0 - 1 \cdot \frac{1}{2} = -\frac{1}{2}$

$(Y_2)_2 = (Y_2)_1 + (Z_2)_1 \cdot h = 1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\overline{Y}_2 = (1 \quad 1 \quad \frac{3}{4})$

$(Y)_0 = 0 \qquad Z_0 = 0$

$(Y)_1 = Y_0 + (Z)_0 h = 0$

$(Z)_1 = (Z)_0 + (-(Y)_0 - 0)h = 0$

$(Y)_2 = (Y)_1 + (Z)_1 \cdot h = 0$

$\overline{Y} = (0;0;0)$

$\overline{y} = C_1 (0 \quad \frac{1}{2} \quad 1)^T + C_2 (1 \quad 1 \quad \frac{3}{4})^T + (0 \quad 0 \quad 0)^T$

№ 8.2.5

$(x^2+1)y''(x) + xy'(x) - x^2 y(x) = 2x$ $\qquad y(0)=0$

$2y(1) + y'(1) = 0$

$x \in [0;1]$

$O(h^2)$

$$y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i+1}}{h^2}$$

$$y'(x_i) = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y'(x) = \frac{y_{i-2} - 4_{i-1} + 3y_i}{2h}$$

$$\begin{cases} y_0 = 0 \\ (x_1^2+1)\cdot\frac{y_2 - 2y_1 + \cancel{y_0}^{0}}{h^2} + x_1\frac{y_2 - \cancel{y_0}^{0}}{2h} - x_1^2 y_1 = 2x_1 \\ (x_2^2+1)\ \frac{y_3 - 2y_2 + y_1}{h^2} + x_2\frac{y_3 - y_1}{2h} - x_2^2 y_2 = 2x_2 \\ 2y_3 + \frac{y_1 - 4y_2 + 3y_3}{2h} = 0 \end{cases}$$

$h = \frac{1}{3}$ $\quad x_1 = \frac{1}{3}$ $\quad x_2 = \frac{2}{3}$ $\quad x_3 = 1$

$$\begin{cases} \frac{10}{9}\cdot\frac{1}{9}\cdot(y_2 - 2y_1) + \frac{1}{\cancel{3}}\cdot\frac{\cancel{9}}{\cancel{6}_2}y_2 - \frac{1}{9}y_1 = \frac{2}{3} \quad \Big|\cdot 81 \\ \frac{13}{9}\cdot\frac{1}{9}\,(y_3 - 2y_2 + y_1) + \frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}(y_3 - y_1) - \frac{4}{9}y_2 = \frac{4}{3} \quad \Big|\cdot 81 \\ 4y_3 + 3y_1 - 12y_2 + 9y_3 = 0 \end{cases}$$

$$\begin{cases} 10y_2 - 20y_1 + \frac{81}{2}y_2 - 9y_1 = 54 \\ 13y_3 - 26y_2 + 13y_1 + 81y_3 - 81y_1 - 36y_2 = 108 \\ 4y_3 + 3y_1 - 12y_2 + 9y_3 = 0 \end{cases}$$

решаем СЛАУ (wolfram)

$y_1 = \frac{-1982}{1835}$

$y_2 = \frac{824}{1835}$ $\qquad$ Ответ $\bar{y} = \begin{pmatrix} 0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix}$

$y_3 = \frac{1218}{1835}$

$y_0 = 0$