№ 5.18.

$f(x)$	1	5	7	11	16
x	-1	1	2	3	5

$f''(2.00)$ - ?

$$f''(x) = \sum_{i=1}^{5} c_i f(x_i)$$

$$0 = 1''|_{x=2} = c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = x''|_{x=2} = -c_1 + c_2 + 2c_3 + 3c_4 + 5c_5$$

$$2 = (x^2)''|_{x=2} = c_1 + c_2 + 4c_3 + 9c_4 + 25c_5$$

$$12 = (x^3)''|_{x=2} = -c_1 + c_2 + 8c_3 + 27c_4 + 125c_5$$

$$48 = (x^4)''|_{x=2} = c_1 + c_2 + 16c_3 + 81c_4 + 625c_5$$

Решаем слау

$$c_1 = -\frac{1}{72} \quad c_2 = \frac{9}{8} \quad c_3 = \frac{-20}{9} \quad c_4 = \frac{9}{8} \quad c_5 = \frac{-1}{72}$$

$$f''(2) = -\frac{1}{72} \cdot 1 + \frac{9}{8} \cdot 5 - \frac{20}{9} \cdot 7 + \frac{9}{8} \cdot 11 - \frac{1}{72} \cdot 16 = \frac{53}{24}$$

№ 5.1.24

$$f'(x) \approx \frac{-3f(x-2h) + 10f(x-h) - 13f(x) + 6f(x+h)}{2h}$$

$$f'(x) = \frac{1}{2h}\Big(-3\Big(f - 2hf' + \frac{(2h)^2}{2!}f'' - \frac{(2h)^3}{3!}f''' + \frac{(2h)^4}{4!}f^{IV} +$$

$$+10\Big(f - hf' + \frac{h^2}{2!}f'' - \frac{h^3}{3!}f''' + \frac{h^4}{4!}f^{IV} -$$

$$-13(f) +$$

$$+6\Big(f + hf' + \frac{h^2}{2!}f'' + \frac{h^3}{3!}f''' + \frac{h^4}{4!}f^{IV} \dots \Big) =$$

$$= \frac{1}{2h}\Big(\underbrace{(-3+10-13+6)}_{=0}f + \underbrace{(6-10+6)}_{=2}hf' + (-12+10+6)\frac{h^2f''}{2!} +$$

$$+O(h^3)\Big) = \frac{1}{2h}\left(2hf' + 2h^2f'' + O(h^3)\right) = f' + \underbrace{hf''}_{=O(h)} + O(h^2) =$$

$$= f' + O(h)$$

№5.1.9

$y_{n+3} - 12y_{n+2} + 12y_{n+1} + 80y_n = 0$

$\tilde{y}_n = \mu^n$

$\mu^{n+3} - 12\mu^{n+2} + 12\mu^{n+1} + 80\mu^n = 0 \quad | : \mu^n$

$\mu^3 - 12\mu^2 + 12\mu + 80 = 0$

$\mu_1 = -2$

$\mu^2 - 14\mu + 40 = 0$

$\mu = 7 \pm \sqrt{49-40} \qquad \mu_2 = 4 \quad \mu_3 = 10$

$y_n = c_1(-2)^n + c_2(4)^n + c_3(10)^n$

№6.1.34

$\overline{1, N-1}$

$y_{n+1} - y_{n-1} = 2h\cos(hn) \qquad n = \overline{1, N-1}$

$y_0 = 0$

$y_1 = h$

$h = const \neq 0$

$y_{n+1} - y_{n-1} = $ ~~...~~

$\tilde{y}_n = \mu^n$

$\mu^{n+1} - \mu^{n-1} = 0 \quad | : \mu^{n-1}$

$\mu^2 - 1 = 0$

$\mu^2 = 1 \qquad \mu_1 = 1 \qquad \mu_2 = -1$

$\tilde{y}_n = c_1 \cdot 1^n + c_2 \cdot (-1)^n = c_1 + c_2(-1)^n$

[signature]

$$y_{n+1} - y_{n-1} = (2h\cos hn + 0 \cdot \sin hn)\, 1^n$$

$$y_n = (A\sin hn + B\cos hn)\, 2h$$

$$A\sin(hn+h) + B\cos(hn+h) - A\sin(hn-h) - B\cos(hn-h)$$

$$= A \cdot 2\sin(h) \cdot \cos hn - 2B\sin(hn)\sin(h) =$$

$$= 2A\sin(h)\cos(hn) - 2B\sin(hn)\sin(h) = \cos hn$$

$$B = 0 \qquad A = \frac{1}{2\sin(h)} \qquad Y_n = \frac{\cancel{2}h\sin(hn)}{\cancel{2}\sin(h)}$$

$$y_0 = 0 \qquad 0 = C_1 + C_2 \qquad C_1 = -C_2$$

$$y_1 = h \qquad h = C_1 - C_2 + \frac{h\sin(h^2)}{\sin(h)}$$

$$h = 2C_1 + \frac{h\sin(h^2)}{\sin(h)}$$

$$C_1 = h\left(1 - \frac{\sin(h^2)}{\sin(h)}\right) \qquad C_2 = -h\left(1 - \frac{\sin(h^2)}{\sin(h)}\right)$$

$$y_n = \ldots$$