№6.2.16

a) $\frac{y_{l-2} - 2y_{l-1} + y_l}{h^2} - y_l = 0 \qquad l = \overline{2, L}$

б) $y'' - y = 0 \quad y(0) = 1 \quad y'_x(0) = -1 \qquad y_0 = 1 \quad y_1 = 1 - h$

$$\psi^{(h)} = L_{(h)}[y]_h - f^{(h)} = \begin{cases} [y]_0 - 0 \\ [y]_1 + h - 1 \\ \frac{[y]_{l-2} - 2[y]_{l-1} + [y]_l}{h^2} - [y]_l \end{cases}$$

$$[y]_{l+k} = [y(x+kh)]_l =$$

$$= \left[y(x) + kh y'(x) + \frac{(kh)^2}{2!} y''(x) + \ldots + \frac{(kh)^n}{n!} y^{n}(x) + O(h^{n+1}) \right]_l =$$

$$= [y]_l + kh[y']_l + \frac{(kh)^2}{2!}[y'']_l + \ldots \frac{(kh)^n}{n!}[y^n]_l + O(h^{n+1})$$

$K = 0, \pm 1, \pm 2, \ldots$

$$[y]_1 = [y]_0 + h[y']_0 + \frac{h^2}{2}[y'']_0 + O(h^3)$$

$$\psi^h = \begin{cases} [y]_0 - 0 = 1 \\ [y]_0 + h[y']_0 + O(h^2) + h - 1 = \underline{O(h^2)} \\ \frac{1}{h^2}\Big(\big(\cancel{[y]_l} - 2h\cancel{[y']_l} + \frac{4h^2}{2!}[y'']_l - \frac{9h^3}{3!}[y''']_l \big) \\ \quad -2\big(\cancel{[y]_l} - h\cancel{[y']_l} + \frac{h^2}{2!}[y'']_l - \frac{h^3}{3!}[y''']_l \big) \\ \quad + \cancel{[y]_l} \\ \quad h^2[y]_l \Big) = \end{cases}$$

$$= \frac{1}{h^2}\left(h^2[y]_l - \frac{7h^3}{3!} \ldots + O(h^4)\right) = [y]_l + \underline{\underline{O(h)}}$$

Ответ: $O(h)$

№ 6.2.21

$y'' = ax + b \qquad a, b = const$

$y(0) = 0 \qquad y'(0) = 0 \qquad x \in [0, 1]$

$$\frac{-y_{l+2} + 4y_{l+1} - 5y_l + 2y_{l-1}}{h^2} = ax_l + b$$

$l = \overline{1, L-2} \qquad y_0 = 0 \qquad y_1 = bh^2/2 \qquad y_2 = 2bh^2 \qquad x_l = lh \quad l = \overline{0, L}$

$$\begin{cases} [y]_0 - 0 \\ [y]_1 - \dfrac{bh^2}{2} \\ \dfrac{-[y]_{l+2} + 4[y]_{l+1} - 5[y]_l + 2[y]_{l-1}}{h^2} - ax_l - b \\ y_2 - \dfrac{2bh^2}{1} \end{cases}$$

$$\overset{0}{[y]_0} + h\overset{0}{[y']_0} + \frac{h^2}{2!}\overset{b}{[y'']_0} + O(h^3) - \frac{bh^2}{2} = O(h^3)$$

$$[y]_0 + 2h[y']_0 + \frac{4h^2}{2!}\overset{b}{[y'']_0} + O(h^3) - 2bh^2 = O(h^3)$$

$$\frac{1}{h^2}\Big(-1\big(\cancel{[y]_l} + \cancel{2h[y']_l} + \frac{4h^2}{2!}[y'']_l + \frac{8h^3}{3!}[y''']_l$$

$$+4\big([y]_l + h\cancel{[y']_l} + \frac{h^2}{2!}[y'']_l + \frac{h^3}{3!}[y''']_l$$

$$-5\big(\cancel{[y]_l}\big)$$

$$+2\big([y]_l - \cancel{h[y']_l} + \frac{h^2}{2!}[y'']_l - \frac{h^3}{3!}[y''']_l\big)\Big)$$

$$-ax_l - b = \frac{1}{h^2}\left(h^2[y'']_l - \frac{3h^3}{3!}[y''']_l\right) - ax_l - b = [y'']_l - \underline{O(h)} - ax_l - b$$

$$y'' = ax + b$$

$$y = \frac{ax^3}{6} + \frac{bx^2}{2} + c_2 x + c_1 \qquad y(0) = 0 \Rightarrow c_1 = 0$$

$$y'(0) = 0 \Rightarrow c_2 = 0$$

$$y = \frac{ax^3}{6} + \frac{bx^2}{2}$$

$$[y]_h = \left\{ [y]_i = \frac{ax_i^3}{6} + \frac{bx_i^2}{2},\ i = \overline{0,L}\ hL = 1 \right\}$$ - сл

$$y^{(h)} = \{ y_i\ i\ \overline{0,L} \}$$

$$\begin{cases} y_0 = 0 \\ y_1 = \cancel{\ldots} + \frac{bh^2}{2} \\ y_2 = \cancel{\ldots}\ 2bh^2 \\ -y_{i+2} + 4y_{i+1} - 5y_i + 2y_{i-1} = ah^3 i + bh^2 \qquad i = \overline{3,L} \end{cases}$$

$$y_i = c_1 (Y_1)_i + c_2 (Y_2)_i + c_3 (Y_3)_i + Y_i$$

$$-(Y_1)_{i+2} + 4(Y_1)_{i+1} - 5(Y_1)_i + 2(Y_1)_{i-1} = 0 \mid : q^{i-1} \quad q^i = Y_i$$

$$-q^3 + 4q^2 - 5q + 2 = 0$$

$$q_1 = 2 \quad q_{23} = 1$$

общее:

$$\tilde{y}_i = c_1 \cdot 2^i + c_2 i + c_3$$

$Y_l = Al^3 + Bl^2$

подставляем в систему и получаем

$6Al - 6A + 2B = ah^3 l + bh^2$

$A = \frac{ah^3}{6}$ $\qquad B = \frac{ah^3 + bh^2}{2}$

$Y_l = \frac{ah^3}{6} l^3 + \frac{ah^2 + bh^2}{2} l^2$

$y_l = C_1 \cdot 2^l + C_2 l + C_3 + \frac{ah^3}{6} l^3 + \frac{ah^2 + bh^2}{2} l^2$

$\cancel{y_0 = 0 \qquad C_1 + C_2 + C_3 = 0 \qquad C_2 = -C_1}$

$\cancel{y_1 = \frac{bh^2}{2} \qquad 2C_1 + C_1 - C_1 + \frac{ah^3}{6} + \frac{ah^2 + bh^2}{2} = \frac{bh^2}{2}}$

$y_0 = 0 \qquad C_1 + C_3 = 0 \qquad C_3 = -C_1$

$y_1 = \frac{bh^2}{2} \qquad 2C_1 + C_2 + C_3 + \frac{ah^3}{6} + \frac{ah^2 + bh^2}{\cancel{2}} = \frac{bh^2}{\cancel{2}}$

$C_1 + C_2 = \frac{2}{3} ah^3 \qquad C_2 = \frac{2}{3} ah^3 - C_1$

$y_2 = 2bh^2 \qquad 4C_1 + 2C_2 + C_3 + \frac{ah^3}{6} \cdot 8 + \frac{ah^2 + bh^2}{\cancel{2}} 4 = \cancel{2bh^2}$

$4C_1 + 2C_2 + C_3 + \frac{ah^3}{3} \cdot 4 = 0$

$4C_1 - \frac{4}{3} ah^3 - 2C_1 - C_1 + \cancel{\frac{4}{3} ah^3} = 0$

$C_1 = 0$

~~[illegible]~~

$y_l^{(h)} = \frac{2}{3} ah^3 l + \frac{ah^3}{6} l^3 + \frac{ah^2 + bh^2}{2} l^2 = \frac{5ah^3}{6} l^3 + \frac{ah^2 + bh^2}{2}$

$$\| [y]_h - y^h \|_{F_h} = \max_{L} \left| \frac{5}{6} a h^3 L^3 + \frac{ah^2 + bh^2}{2} L^2 - \frac{ah^3}{6} L^3 - \frac{bh^2}{2} L^2 \right|$$

$$= \frac{4}{6} a h^3 L^3 + \frac{ah^2}{2} L^2 \quad = \underline{\underline{O(h^2)}}$$