№6.2.21

а) $y'' = ax + b \qquad y(0)=0 \quad y'(0)=0 \qquad x \in [0,1]$

б) $\dfrac{-y_{l+2} + 4y_{l+1} - 5y_l + 2y_{l-1}}{h^2} = ax_l + b \qquad l = \overline{1,L}$

$y_0 = 0$

$y_1 = bh^2/2$

$y_2 = 2bh^2$

$$\begin{cases} [y_0] - 0 \\ [y_1] - \dfrac{bh^2}{2} \\ [y_2] - 2bh^2 \\ \dfrac{-[y]_{l+2} + 4[y]_{l+1} - 5[y]_l + 2[y]_{l-1}}{h^2} - ax_l + b \end{cases}$$

$[y_0] - 0 = 0$

$[y]_1 - \dfrac{bh^2}{2} = [y]_0 + \dfrac{h^2}{2!}[y'']_0 + O(h^3) - \dfrac{bh^2}{2} = O(h^3)$

$[y]_2 - 2bh^2 = [y]_0 + \dfrac{(2h)^2}{2!}[y'']_0 + O(h^3) - 2bh^2 = O(h^3)$

$\dfrac{1}{h^2}\Big(-1\big([y]_l + \dfrac{(2h)^2}{2!}[y'']_l + 2h[y']_l + \dfrac{(2h)^3}{3!}[y''']_l + \dfrac{16h^4}{4!}[y^{IV}]_l\big)$

$+ 4\big([y]_l + h[y']_l + \dfrac{h^2}{2!}[y'']_l + \dfrac{h^3}{3!}[y''']_l + \dfrac{h^4}{4!}[y^{IV}]_l\big)$

$- 5[y]_l$

$+ 2\big([y]_l - h[y']_l + \dfrac{h^2}{2!}[y'']_l - \dfrac{h^3}{3!}[y''']_l + \dfrac{h^4}{4!}\big)\Big) - ahl - b =$

$= \dfrac{1}{h^2}\left(h^2[y'']_l + \underline{h^3[y''']_l}\right) - ahl - b \quad = O(h)$

$y'' = ax + b$

$y = \frac{ax^3}{6} + \frac{bx^2}{2} + c_2 x + c_1$

$y'(0) = 0 \Rightarrow c_2 = 0$

$y(0) = 0 \Rightarrow c_1 = 0$ след:

$y = \frac{ax^3}{6} + \frac{bx^2}{2}$ $[y]_h = \left\{ [y]_i = \frac{ax_i^3}{6} + \frac{bx_i^2}{2}, \ i = \overline{0, L} \atop hL = L \right\}$

$[y]_i = \frac{ah^3}{6} i^3 + \frac{bh^2}{2} i^2$

$$\begin{cases} y_0 = 0 \\ y_1 = bh^2/2 \\ y_2 = 2bh^2 \\ -y_{i+2} + 4y_{i+1} - 5y_i + 2y_{i-1} = ah^3 i + bh^2 \end{cases}$$

$y_i = \mu^i$

$-\mu^3 + 4\mu^2 - 5\mu + 2 = 0$

$\mu_1 = 2 \quad \mu_{2,3} = 1$ общее:

$\tilde{y}_i = C_1 \cdot 2^i + C_2 i \cdot 1^i + C_3 \cdot 1^i = C_1 \cdot 2^i + C_2 i + C_3$

частное:

$Y_i = Ai^3 + Bi^2 = \frac{ah^3}{6} i^3 + \frac{ah^3 + bh^2}{2} i^2$

$-(A(i+2)^3 + B(i+2)^2) + 4(A(i+1)^3 + B(i+1)^2) - 5Ai^3 - 5Bi^2 +$
$+ 2(A(i-1)^3 + B(i-1)^2) = 6Ai - 6A + 2B = ah^3 i + bh^2$

$$\begin{cases} 6A = ah^3 \\ -6A + 2B = bh^2 \end{cases} \qquad A = \frac{ah^3}{6} \qquad B = \frac{ah^3 + bh^2}{2}$$

$$y_i = C_1 \cdot 2^i + C_2 i + C_3 + \frac{ah^3}{6} i^3 + \frac{ah^3 + bh^2}{2} i^2$$

$$\begin{cases} y_0 = 0 & C_1 + C_3 = 0 \quad C_3 = -C_1 \\ y_1 = \frac{bh^2}{2} & 2C_1 + C_2 + C_3 + \frac{ah^3}{6} + \frac{ah^3 + \cancel{bh^2}}{\cancel{2}} = \frac{\cancel{bh^2}}{\cancel{2}} \end{cases}$$

$$C_1 + C_2 = -\frac{2}{3} ah^3$$

$$C_1 = -\frac{2}{3} ah^3 - C_2 \qquad C_2 = -\frac{2}{3} ah^3 - C_1$$

$$y_2 = 2bh^2 \qquad 4C_1 + 2C_2 + C_3 + \frac{8}{6} ah^3 + 2(ah^3 + bh^2) = 2b$$

$$4C_1 + 2C_2 + C_3 = -\frac{20}{6} ah^3$$

$$4C_1 - \frac{8}{6} ah^3 - 2C_1 - C_1 = -\frac{20}{6} ah^3$$

$$C_1 = -2ah^3 \qquad C_3 = 2ah^3 \qquad C_2 = \frac{4}{3} ah^3$$

$$y_i = -2ah^3 \cdot 2^i + \frac{4}{3} ah^3 i + 2ah^3 + \frac{ah^3}{6} i^3 + \frac{ah^3 + bh^2}{2} i^2$$

$$\| [y_h] - y^h \| =$$

$$= \max_i \left| -2ah^3 \cdot 2^i + \frac{4}{3} ah^3 i + 2ah^3 + \cancel{\frac{ah^3}{6} i^3} + \frac{ah^3}{2} i^2 + \cancel{\frac{bh^2}{2} i^2} - \cancel{\frac{ah^3}{6}} - \cancel{\frac{bh^2}{2} i^2} \right| = \max_i \left| -2ah^3 \cdot 2^i + \frac{4}{3} ah^3 i + 2ah^3 + \frac{ah^3}{2} i^2 \right|$$

$$\xrightarrow[i \to \infty]{} \infty$$