EECE5644 Spring 2020 – Take Home Exam 1

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1 Question 1

Minimum expected risk classification:
1)
$D = 1 \rightarrow x^T [\begin{smallmatrix} 0 & 9.47 \\ 9.47 & 0 \end{smallmatrix}] x + [\begin{smallmatrix} 2.104 & 0 \end{smallmatrix}] x - 2.77 > 0$
$D = 0 \rightarrow x^T [\begin{smallmatrix} 0 & 9.47 \\ 9.47 & 0 \end{smallmatrix}] x + [\begin{smallmatrix} 2.104 & 0 \end{smallmatrix}] x - 2.77 < 0$
2)
Github
3)
Minimum probability of error threshold value $\gamma = 4$;
$\tau = ln(\gamma) = 1.386$;
Estimate of the minimum probability of error P(error) = 0.0787;
Naive Bayesian:
1)
$D = 1 \rightarrow [\begin{smallmatrix} 0.4 & 0 \end{smallmatrix}] x - 2.77 > 0$
$D = 0 \rightarrow [\begin{smallmatrix} 0.4 & 0 \end{smallmatrix}] x - 2.77 < 0$
2)
Github
3)
Minimum probability of error threshold value $\gamma = 1.874$;
$\tau = ln(\gamma) = 0.6282$;
Estimate of the minimum probability of error P(error) = 0.1980;
Fisher Linear Discriminant Analysis:
1) Original sigmas:
Minimum probability of error threshold $\tau = 3.2227$;
Estimate of the minimum probability of error P(error) = 0.2046;
2) Identity matrix sigmas:
Minimum probability of error threshold $\tau = 3.2227$;
Estimate of the minimum probability of error P(error) = 0.2046;
3) Code: Github

2 Question 2

1) Github;
2)
$P(L=0)=0.35 \quad P(L=1)=0.65 \quad P_{00}=0.7 \quad P_{01}=0.3$
$P_{10}=0.15 \quad P_{11}=0.85$
$\mu_{10}=[\begin{smallmatrix}5\\1\end{smallmatrix}] \quad \mu_{11}=[\begin{smallmatrix}3\\-2\end{smallmatrix}] \quad \mu_{00}=[\begin{smallmatrix}-2\\1\end{smallmatrix}] \quad \mu_{01}=[\begin{smallmatrix}0\\1\end{smallmatrix}]$
$\mathbf{\Sigma}_{10}=\frac{1}{10}[\begin{smallmatrix}2&-2\\-2&5\end{smallmatrix}] \quad \mathbf{\Sigma}_{11}=\frac{1}{14}[\begin{smallmatrix}20&1\\1&5\end{smallmatrix}] \quad \mathbf{\Sigma}_{00}=\frac{1}{8}[\begin{smallmatrix}3&2\\2&3\end{smallmatrix}] \quad \mathbf{\Sigma}_{01}=\frac{1}{12}[\begin{smallmatrix}5&-1\\-1&10\end{smallmatrix}]$
Classification rule:
$ln[P_{10}*(2\pi)^{\frac{-n}{2}}det(\Sigma_{10})^{\frac{-1}{2}}e^{-0.5(x-\mu_{10})^T\Sigma_{10}^{-1}(x-\mu_{10})}+P_{11}*(2\pi)^{\frac{-n}{2}}det(\Sigma_{11})^{\frac{-1}{2}}e^{-0.5(x-\mu_{11})^T\Sigma_{11}^{-1}(x-\mu_{11})}]$
$-ln[P_{00}*(2\pi)^{\frac{-n}{2}}det(\Sigma_{00})^{\frac{-1}{2}}e^{-0.5(x-\mu_{00})^T\Sigma_{00}^{-1}(x-\mu_{00})}+P_{01}*(2\pi)^{\frac{-n}{2}}det(\Sigma_{01})^{\frac{-1}{2}}e^{-0.5(x-\mu_{01})^T\Sigma_{01}^{-1}(x-\mu_{01})}]$
$+0.619$
$>0 \rightarrow D=1$
$<0 \rightarrow D=0$
The empirical smallest probability of error achievable for my dataset is 0. However, the tail of normal distribution is never zero, although it's asymptotically approaching to zero, so in theory, as sample size approaches ∞, we would have some overlap and misclassifications. Therefore, the smallest theoretical probability of error is approaching 0.

3 Question 3

Classification rule:
$ln(p_1(x))-ln(p_0(x)) \lesseqgtr ln(\frac{q_0(\lambda_{10}-\lambda_{00})}{q_1(\lambda_{01}-\lambda_{11})})$
$ln(\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(x-2)^2})-ln(\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(x+2)^2}) \lesseqgtr ln(\frac{-0.5}{-0.5})$
$x>0 \rightarrow D=1$
$x<0 \rightarrow D=0$
Smallest probability of error:
$\int_{-\infty}^{0}P(L=1)P(x|L=1)dx+\int_{0}^{\infty}P(L=0)P(x|L=0)dx$
The first integral is to calculate the probability of label 1 sample with $x<0$. Since we are classifying all label with $x<0$ to be label 0, this is an error. The second integral is the same for calculating probability of label 0 sample with $x>0$. We multiply the priors to the PDF to scale it (weight).

4 Citation and Acknowledgment

For question 1, I used and "paraphrased" Prof. Erdogmus's code example from shared folder.

For question 2, I used and "paraphrased" one of the volunteer's code from the shared folder.

5 Code repo

Github: https://github.com/MicroRAsus/Machine-Learning-Pattern-Recognition-HW1