Wideband Measurement of the Dielectric Constant of an FR4 Substrate Using a Parallel-Coupled Microstrip Resonator

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***Abstract*—We have made a wideband measurement of the real part of the dielectric constant of flame retardant #4 epoxy (FR4), a common high-frequency printed-circuit-board insulator. We designed a novel test circuit, an electrically long parallel-coupled microstrip resonator, which was etched on a 0.014-in FR4 substrate, manufactured by NELCO, Melville, NY. We used a computer model of the resonator to extract the dielectric constant at the frequencies of zeroes in its measured transmission response. By adjusting the model's dielectric constant, we tuned the frequency of each zero to match the measured frequency, yielding the dielectric constant at that frequency. To validate our method and results, we present a simple, but original proof that the frequencies of zeroes in the resonator's transmission response are insensitive to input and output mismatches. Additionally, we compare the measured and predicted response of a two-stub filter designed with our measured data. The fabricated filter's measured return loss and insertion loss from 3 to 12 GHz are within 1% of the predictions of Agilent Technology's Momentum.**

***Index Terms*—Dielectric materials, measurement, microstrip resonators, permittivity measurement, printed circuits.**

I. Introduction

FLAME-RETARDANT #4 epoxy (FR4) is a low-cost dielectric material that finds use as a substrate for RF and microwave printed circuit boards (PCBs). Its dielectric constant is known to vary with frequency and manufacturer [1]. FR4 data sheets generally do not list dielectric-constant data over a wide frequency range, and we found only one set of broadband data in the literature [1]. Unfortunately, these data are presented in a relative sense only (normalized to unity), not well validated, and the vendors are not identified. Further to this, four different measurement techniques were used to obtain the data. A simple means to measure the real part of the dielectric constant is desirable, particularly if it can serve as a process control monitor during production.

Many methods for measuring the dielectric constant of materials have been developed and used successfully. For a PCB material such as FR4, a practical approach is to fabricate a circuit having easy-to-measure characteristics that can be used to determine the material's dielectric constant. If such a circuit is modeled accurately with computer-aided design (CAD) software, one can determine the substrate's dielectric constant by comparing the predictions of the software with the circuit's measured characteristics. The extracted dielectric-constant data can then be used to design other circuits.

This type of empirical/analytical approach has been demonstrated by a number of researchers in the microwave field. Das *et al.* used two microstrip lines of unequal length to measure the effective dielectric constant of microstrip [2]. With a computer model of microstrip, they extracted the substrate dielectric constant and were able to achieve a measurement accuracy of 1% over a broad bandwidth. Their method required care in assembling the test fixture, long microstrip lines, and well-matched and repeatable coaxial transitions according to Lee and Nam [3]. Shimin [4], and Verma and Verma [5] used a microstrip patch antenna as the test circuit, and by comparing the resonant frequency predicted by an analytical model with the measured resonant frequency, they determined the dielectric constant of the substrate. For the best results, the substrate had to be 3×–4× larger than the patch. Akhavan and Mirshekar-Syahkal replaced the patch with a microstrip fed slot antenna to overcome some of the limitations of the resonant patch method [6]. In both cases, a different test circuit was required for each frequency of interest. Bernard and Gautray used a ring resonator fabricated on alumina as their test circuit [7]. They placed a test sample of the material of interest on top of the ring resonator. The ring's resonant frequency was perturbed by the sample, enabling the authors to determine the dielectric constant of the material using an analytical model of the ring. Measurements of several substrates were within 15% of those from a cavity resonator. Similarly, Kantor used microstrip, stripline, and disk resonators to determine the dielectric constant of several microwave PCB materials [8]. Yue *et al.* measured the characteristic impedance of the stripline, and determined the dielectric constant of the substrate from equations for the impedance [9]. Their technique required a precision coaxial load to terminate one end of the stripline and a full two-port calibration of the vector network analyzer making the measurements. Gruszczynski and Zaradny made measurements of a sample of dielectric of fixed width, metallized on both sides [10]. The primary source of error in their technique was also the coaxial transition.

Each of the above techniques, to a varying degree, depends on having well-matched coaxial transitions attached to the substrate sample under test. With increasing frequency, such transitions become difficult to produce, and it is at higher frequencies that accurate knowledge of the dielectric constant of most substrates is most critical and often is not known. A measurement technique that is insensitive to transition mismatch is desirable. Toward that end, Amey and Curilla [11] and Peterson and Drayton [12] used the transmission response of microstrip and

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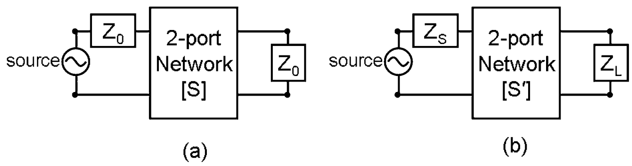

Fig. 1. Two-port network situated between a source and load. (a) With impedances Z_0. (b) With impedances Z_S and Z_L.

coplanar lines with series stubs to extract the dielectric constant of the substrate. Peterson and Drayton demonstrated empirically that their measurement is insensitive to impedance mismatch at the transitions. Another advantage of the stub is that a single circuit has multiple transmission zeroes over a wide frequency band, with each zero yielding a value of the substrate dielectric constant. A limitation of the coplanar version is that higher order modes are excited at the tee junction.

In this paper, we extend the work of Peterson and Drayton, first by presenting in Section II a simple proof that theoretically validates their observation that the frequencies of the transmission zeroes of a passive two-port circuit are independent of port mismatch. In Section III, we describe an alternative to the tee circuit, the parallel-coupled resonator, which inherently is well matched. We use this resonator to measure the real part of the dielectric constant of NELCO FR4 over a broad range of frequencies. In modeling our test circuit, we take advantage of the high level of accuracy that commercial circuit simulators can achieve. In particular, we use Agilent Technology's Advanced Design System (ADS), which includes a standard circuit simulator, based on analytical models, and Momentum, which is based on the method of moments (MoM). We know the dielectric constant of FR4 sufficiently well to design the test circuit. We then fabricate it, measure its insertion response accurately, and compare the data with the predictions of ADS. Due to our confidence in the simulator, we can attribute any difference between the measured and predicted performance primarily to the error in our knowledge of the dielectric constant. With a relatively high-Q circuit element such as our microstrip resonator, we can accurately adjust the dielectric constant in the simulator until its prediction matches the data at the zero frequencies. In Section IV, we use our measured FR4 dielectric constant data to design an evaluation circuit. We perform a precision thru-reflect-line (TRL) calibration to enable us to measure the S-parameters of the circuit at its microstrip inputs, and compare the results with the predictions. Our measured and predicted resonant frequency agreement is within 1%, which is excellent, considering the variations in etch tolerance, metallization thickness, substrate thickness, and dielectric constant typical of most PCB manufacturing processes.

II. Theory

Fig. 1 shows a generic two-port network embedded between a microwave source and load. We follow the analysis of Ha [13], and assign the two-port network a scattering matrix **S**, normalized to the port impedance Z_0 at which the S-parameters were determined [see Fig. 1(a)]. **S'** is the generalized scattering matrix of the same two-port network situated between a source and load having impedances Z_S and Z_L, whose real components are positive for all frequencies in the band of interest [see Fig. 1(b)].

We can write the transmission response or insertion loss as [12]

$$|S'_{21}|^2 = |S_{21}|^2 \left[\frac{(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22})-\Gamma_S \Gamma_L S_{12} S_{21}|^2} \right]. \tag{1}$$

If the source and load impedances are equal to Z_0, the two-port is perfectly matched, $|\Gamma_S| = |\Gamma_L| = 0$, and $|S'_{21}|^2 = |S_{21}|^2$.

Now let us assume $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$, and that $|S_{21}|^2$ has a zero at a frequency f_0. At f_0, the bracketed term in (1) becomes

$$\left[\frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22})|^2} \right]$$

which is finite valued. Thus, $|S'_{21}|$ has a zero at the same frequency f_0 as $|S_{21}|$. At frequencies away from f_0, the denominator of the bracketed term in (1) is nonzero. Therefore, the only zeroes that appear in $|S'_{21}|$ are those appearing in $|S_{21}|$, and we can select a two-port circuit with transmission zeroes whose frequencies are dependent on the substrate dielectric constant. If we build and test such a circuit, the frequencies of those zeroes will be insensitive to port mismatches. A calibration of the test equipment should not even be necessary, as verified empirically by Peterson and Drayton [12]. We can use an accurate model of the circuit to extract the value of the dielectric constant at each measured zero frequency.

III. Test Circuit Design and Measurement

A. Test Circuit

Circuits fabricated on FR4, a relatively lossy material, typically have passbands that do not extend above 6 GHz, but they may have reject requirements at higher frequencies. Thus, it would be useful to have accurate dielectric-constant data from approximately 2 to 12 GHz. We know that FR4's dielectric constant varies slowly over that frequency. If we design a test circuit with half a dozen transmission zeroes over that bandwidth, we will have sufficient data to interpolate values at other frequencies with good accuracy. Fig. 2 shows such a circuit, i.e., a microstrip parallel-coupled resonator. This particular example has zeroes in transmission starting at approximately 2.7 GHz, and repeating approximately every 2.7 GHz. To select the resonator dimensions, we assumed the dielectric constant of the FR4 substrate is 4.5 for all frequencies. Fig. 3 plots the insertion loss of the resonator as predicted by ADS's circuit simulator and by Momentum. All Momentum analyses used a mesh with at least 15 cells/wavelength at the highest frequency of simulation. Momentum's edge mesh feature was enabled also. We generated a photo-mask and printed the filter on 14-mil FR4. We confirmed the filter dimensions to be within 0.5 mil of the design and adjusted our model's dimensions accordingly. The only important circuit dimension is the resonator length, which, along with the dielectric constant of the material, sets the zero frequencies. The

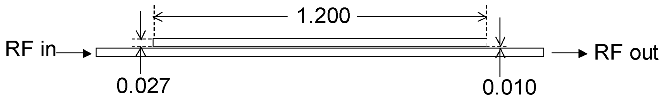

Fig. 2. Microstrip parallel-coupled resonator. Dimensions are in inches. FR4 substrate thickness $= 0.014$ in. Metallization thickness $= 0.007$ in (1/2-oz copper).

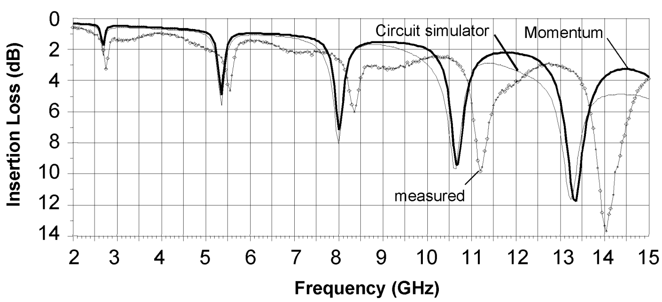

Fig. 3. Insertion loss of microstrip parallel-coupled resonator—ADS circuit simulator, Momentum, and measurement. Simulations use 4.5 for FR4 dielectric constant.

separation of the resonator and main transmission line only affects the depth of the transmission zero at each frequency.

Our test setup consisted of a Hewlett-Packard 8510 vector network analyzer, a Wiltron 3680 K Universal Test Fixture with two K-connector coaxial input ports, and a ground-plane backed FR4 substrate metallized with the test circuit shown in Fig. 2. Since calibration is not critical, we only calibrated the analyzer with a K-connector coaxial calibration. We then placed the test circuit in the test fixture and measured its transmission response over frequency.

Fig. 3 plots the measured insertion loss, and it is obvious that our assumed value of 4.5 for the dielectric constant is in error, with the error increasing with increasing frequency.

B. Dielectric-Constant Computation

To extract the correct frequency-dependent dielectric constant, we adjust manually its value in our ADS circuit simulator and Momentum models at each of the measured reject frequencies until the predicted zero matches the measured zero. We then know the dielectric constant at the reject frequency null. Fig. 4 shows an example at 11.21 GHz. In this case, a dielectric constant of 4.00 in the circuit simulator and 4.03 in Momentum matched the frequencies of the zeroes predicted by the models to the measured results. The values differ slightly because the two analytical methods are different.

We extracted the real part of the dielectric constant in this manner at every measured zero frequency through 16.7 GHz, and the results are summarized in Table I. The rise in dielectric constant above 14 GHz, though surprising, has been observed by others [1].

Each set of data can be fit to a third-order polynomial. The polynomial for the circuit simulator, which can be inserted directly into the ADS MSUB block, is

$$\varepsilon_r(CS) = 0.0002462 \times f^3 - 0.006278 \times f^2 + 0.02455 \times f + 4.1752 \quad (2)$$

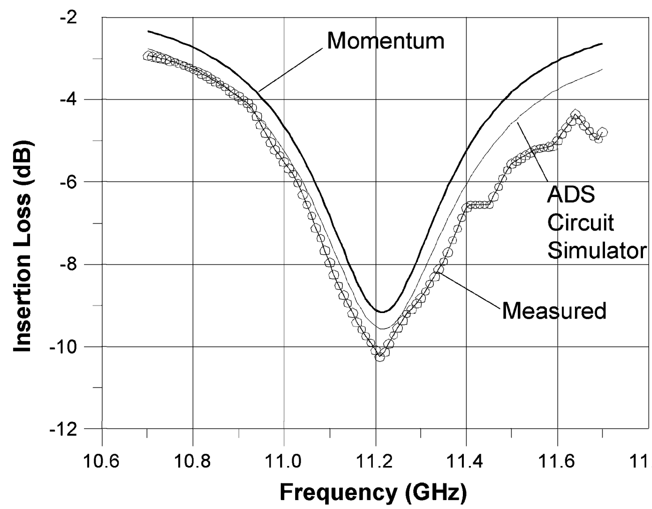

Fig. 4. Adjustment of substrate dielectric constant (ADS—4.00, MoM—4.03) to match predicted and measured insertion loss at 11.21 GHz.

TABLE I
DIELECTRIC CONSTANT OF FR4 VERSUS FREQUENCY FOR ADS'S CIRCUIT SIMULATOR AND MOMENTUM. FR4 MANUFACTURER: NELCO

Measurement frequency (GHz)	Calculated dielectric constant	
	Circuit Sim	Momentum
2.76	4.20	4.22
5.54	4.16	4.16
8.36	4.09	4.10
11.21	4.00	4.03
14.04	3.97	4.00
16.73	3.98	4.06

where f is the frequency in gigahertz.

After designing a preliminary circuit with the circuit simulator, one should perform an analysis in Momentum, which models the circuit more accurately. Since Momentum does not allow parameterization of the dielectric constant as a function of frequency, one must analyze the circuit over frequency bands narrow enough such that the dielectric-constant variation is small. For instance, if we design a circuit to operate from 3 to 8 GHz, we might use two frequency bands based on the data in Table I for analysis in Momentum, say, from 3 to 5.5 GHz and from 5.5 to 8 GHz. Over these bands, the dielectric-constant variation will be no more than 0.06, approximately 1(1/2)%. For those designers who want to interpolate the Momentum data in Table I, we have generated a third-order fit

$$\varepsilon_r(\mathrm{MoM}) = 0.0002529 \times f^3 - 0.00576 \times f^2 + 0.01646 \times f + 4.211. \quad (3)$$

It is important to keep in mind that (2) and (3) and the data in Table I may not be valid for FR4 produced by vendors other than NELCO. New data should be measured.

IV. VALIDATION CIRCUIT

To confirm the accuracy of our dielectric-constant data, we designed a microstrip two-stub reject filter on FR4. This filter was designed to pass the band at 5.7–5.9 GHz while rejecting signals at 3.3 and 11.5 GHz. The design was optimized with

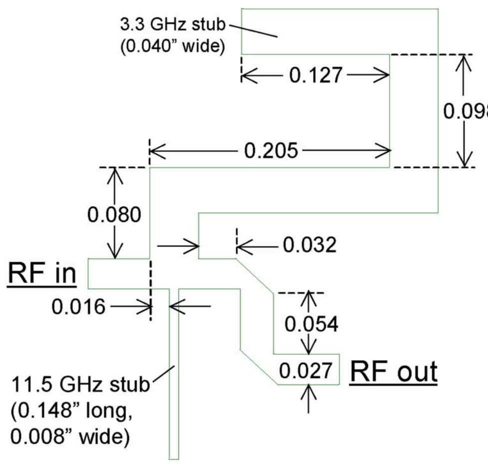

Fig. 5. Microstrip two-stub reject filter for validating the measured dielectric constant of FR4. All dimensions are in inches. Substrate: NELCO 14-mil FR4.

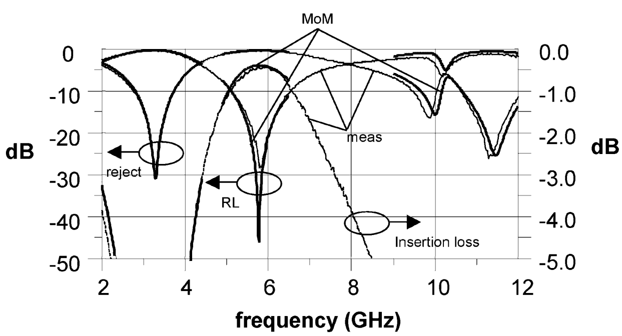

Fig. 6. Microstrip reject filter—comparison of measured and predicted (Momentum) rejection, return loss, and insertion loss.

ADS's circuit simulator using (2) for the dielectric constant. It was then further adjusted with Momentum, using (3). The layout of the filter is shown in Fig. 5.

We fabricated the filter along with TRL calibration standards covering the 2–12-GHz frequency range. With these standards, we deembedded our Wiltron test fixture's coax-to-microstrip transitions and microstrip lines up to the input and output ports of the filter. As shown in Fig. 6, the measured insertion loss and return loss are within 1% of the performance predicted by Momentum.

V. Conclusion

FR4's known variability is best managed with a circuit-board process control monitor. The efficient shape and noncritical test conditions of our parallel-coupled resonator make it a good candidate. Its insertion response can be an important part of a specification provided to a circuit-board vendor. These resonators can be placed on the edge of or between the circuits on a standard panel. After the panel has been processed, one can measure the frequency response of the filter to determine if the dielectric constant of the substrate is sufficiently close to the desired value by comparing the frequencies of the transmission zeroes with the specification. The verification test can be used to decide whether or not to separate and assemble the production circuit-boards, which may include costly surface-mount components.

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