An introduction to Prolog

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February 2, 2017

- Prolog
- 2 Functions

Flow control

Other features

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A first glimpse at Prolog

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- Well suited for symbolic, non-numeric computation. Good for dealing with objects and relations.
- Let us start with facts (ground atoms) for some relations (predicates).

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 father (juancarlos, elena).
Their mother is Sofia:
 mother (sofia, felipe).
 mother (sofia, cristina).
 mother (sofia, elena).
Felipe and Letizia have two children:
```

```
father (felipe, leonor).
father (felipe, sofia2).
mother (letizia, leonor).
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```

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- What would these queries mean?

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?- father(X,leonor).
?- mother(sofia,X).
?- father(X,Y).
?- mother(X,Y), father(Y,leonor).
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• How would you check whether Cristina and Elena have the same father?

 The comma means conjunction. These queries are logically equivalent:

```
?- mother(X,Y), father(Y,leonor).
?- father(Y,leonor), mother(X,Y).
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although their computation is different, as we will see later.

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Who has a father and a mother?

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Notice that the two '_' are different irrelevant variables.

• We can "give name" to queries using rules. For instance, for:

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we can define a new predicate grandmother using:

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grandmother (X, Z) := mother(X, Y), father (Y, Z).
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Rule head Rule body

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• We can use it now in queries:

```
?- grandmother(X, leonor).
```

• We may have several rules to define a predicate.

If prolog can't unify the first conjunction of objectives (body rule) for any value of X,Y,Z. Prolog do backtrack and jumps to the next body rule that defines the predicate grandmother in which mother(Y,Z) is included

 We may have several rules to define a predicate. For instance, my mother's mother is also my grandmother:

```
grandmother(X, Z) :- mother(X, Y), father(Y, Z).
grandmother(X, Z) :- mother(X, Y), mother(Y, Z).
```

to obtain solutions to ?- grandmother (X,Y) . we can apply any of these rules.

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parent (X,Y):- father (X,Y).
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We can use disjunction ';' for rules with same head

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parent (X,Y):- father (X,Y); mother (X,Y).
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• Exercises: who are Felipe's parents? Redefine grandmother with a single rule using the parent relation.

Aquí quedé 12/03/2025



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female(cristina). female(elena).
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but we can also derive it from mother

```
female(X) :- mother(X, \_).
```

• Exercise: define the sister relation.

Adding rules

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```
sister(X,Y) :- parent(Z,X), parent(Z,Y), female(X).
?- sister(felipe,X).
?- sister(leonor,X).
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Adding rules

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?- sister(leonor,X).
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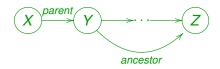
 Problem: Leonor is sister of herself! We should specify that they are different:

```
sister(X,Y) :- parent(Z,X), parent(Z,Y),
female(Y), X = Y.
```

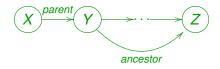
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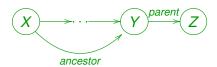


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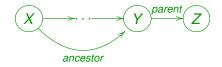


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ancestor(X, Y) :- parent(X, Y).
ancestor(X, Z) :- parent(X, Y), ancestor(Y, Z).
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• Another alternative can be:

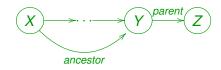


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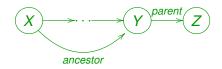
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In principle, this program is equivalent to:

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but Prolog further introduces an evaluation ordering that, for instance, causes query ?- ancestor(X, juancarlos) to iterate forever.

So, how does this work? Take

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 X=sofia, Y=leonor. This is like having the rule instance:

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 As matching succeeded, we replace our initial goal by the rule body parent (sofia, leonor), which becomes our new goal.

 We try then to match parent (sofia, leonor) with some rule head. This predicate has two rules

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- However, father is extensional (only facts), and this fact is not included in the program. So, our goal fails.
- A failure implies backtracking to the last matching, and looking for new matches.

• So we "reconsider" the last deleted goal parent (sofia, leonor) and try to match another rule

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parent (X, Y): - father (X, Y). (failed)
parent (X, Y): - mother (X, Y).
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 succeeds for Y=felipe (more matchings are possible).

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- Matching parent (sofia, Y) with parent (X', Y') :father (X', Y'). is possible under replacement X'=sofia,
 Y'=Y. This leads to new goal father (sofia, Y) that fails.
- Matching parent (sofia, Y) with parent (X', Y') :mother (X', Y'). leads to new goal mother (sofia, Y) that
 succeeds for Y=felipe (more matchings are possible).
- Important: assignment Y=felipe affects our whole list of goals.
 That is, ancestor (Y, leonor) becomes ancestor (felipe, leonor).

Matching ancestor (felipe, leonor) with ancestor (X, Y)
 :- parent (X, Y) . leads to goal parent (felipe, leonor).

- Matching ancestor (felipe, leonor) with ancestor (X, Y)
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- Finally, matching parent (felipe, leonor) with parent (X, Y)
 :- father (X, Y) . leads to new goal father (felipe, leonor) that succeeds.

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 father (felipe, leonor) that succeeds. Prolog answers Yes!

- 1 Prolog
- 2 Functions

- Flow control
- Other features

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 We can use function symbols to pack some data together as a single structure. Example:

```
born(juancarlos, f(5,1,1938)).
born(felipe, f(30,1,1968)).
born(letizia, f(15,9,1972)).
born(sofia, f(2,11,1938)).

later(f(_,_,Y), f(_,_,Y1)) :- Y>Y1.
later(f(_,M,Y), f(_,M1,Y)) :- M>M1.
later(f(D,M,Y), f(D1,M,Y)) :- D>D1.

birthday(X,d(D,M)) :- born(X,f(D,M,_)).
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later(f(,M,Y), f(,M1,Y)) :- M>M1.
later(f(D,M,Y), f(D1,M,Y)) :- D>D1.
birthday (X, d(D, M)) := born(X, f(D, M, )).
```

Predicate > is predefined for arithmetic values.

Some examples of queries:

Which is Juan Carlos' date of birth?

?- born(juancarlos,X).

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 - ?- born(juancarlos, X).
- Is Felipe older than Letizia?
 - ?- born(felipe, X), born(letizia, Y), later(Y, X).

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 - ?- born $(X, f(_{,,,}Y))$, born $(Z, f(_{,,,}Y))$, X = Z.

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- Find two people that were born in the same year
 - ?- born $(X, f(_, _, Y))$, born $(Z, f(_, _, Y))$, X = Z.
- Which is Sofia's birthday? ?- birthday (sofia, X).

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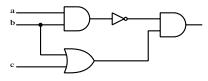
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- Note that, in principle, functions are not evaluated. They are just a way to build data structures.
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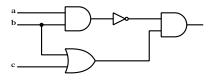
• As in First Order Logic, we call terms to any combination of functions, constants and variables. In fact, a constant c is a 0-ary functor c/0.

• Example: we can represent a digital circuit.



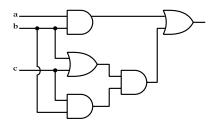
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• Example: we can represent a digital circuit.



and (not (and (a,b)), or (b,c))

Exercise: try to represent this circuit



• Arithmetic operators are also (infix) functors. The term 2+3*4 is not equal to 4*3+2 or 14.

We can also define our own functors using the op directive.

```
:- op(X, Y, Z).
```

means we declare operator z with precedence number x (higher = less priority) and associativity y.

- Associativity can be:
 - ▶ infix operators: xfx xfy yfx
 - ▶ prefix operators: fx fy
 - postfix operators: xf yf

where:

- f: is the functor position
- x: argument of strictly lower precedence
- y: argument of lower or equal precedence

For instance, the fact:

```
equivalent (not (and (A, B)), or (not (A), not (B))).

can be written in a more readable way:
```

```
:- op(800, xfx, <==>).
:- op(700, xfy, v).
:- op(600, xfy, &).
:- op(500, fy, not).
not (A & B) <==> not A v not B.
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```

- Try the following ?- F=(not a v b & c), F=(H v G).
- Note that = > < :- , are predefined operators. Predicate current_op/3 shows the currently defined operators.

Exercise 1

Build a predicate eval/5 that computes the output of any circuit for 3 variables so that eval(A, B, C, Circuit, X) returns the output of Circuit in X for values a=A, b=B and c=C.

The predicate must also allow returning the models of the circuit (combinations of values that yield a 1).

Try with the two previous circuits.

Examples:

```
?- eval(1,0,0, a & ( not b v c) ,X).
X = 1.
?- eval(A,B,C, a v not b,1).
A = 1, B = 1;
A = 0, B = 0;
A = 1, B = 0;
```

• How are functors handled in the goal satisfaction algorithm?

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 When searching a goal, we see whether it matches a rule head.
- To see how it works, we can use the built in =/2 Prolog predicate.
 Try the following:

```
?- f(X,b)=f(a,Y).
?- f(X,b)=f(X,Y).
?- f(f(Y),b)=f(X,Y).
?- f(f(Y),b)=f(a,Y).
```

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- Given a set of expressions E, we compute a disagreement set searching from left to right the first different symbol and taking the corresponding subexpression.

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- Given a set of expressions E, we compute a disagreement set searching from left to right the first different symbol and taking the corresponding subexpression.
- For instance, given p(f(X), Y) and p(f(g(a, Z), f(Z))) we get the disagreement set $\{X, g(a, Z)\}$.

 If two atoms can be unified, they have an MGU that can be computed as follows:

```
\begin{split} \sigma &:= [\ ]; \\ \text{while } |E| > 1 \ \{ \\ D &:= \text{disagreement set of } E; \\ \text{if } D \text{ contains an } X \text{ and a term } t \text{ not containing } X \ \{ \\ E &:= E[X/t]; \\ \sigma &:= \sigma \cdot [X/t]; \\ \text{else return 'not unifiable';} \\ \} \end{split}
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```

- Example $E = \{f(f(Y), b), f(X, Y)\}$. Then $D = \{f(Y), X\}$ and we can replace X by f(Y). E becomes $\{f(f(Y), b), f(f(Y), Y)\}$.
- The new disagreement is $D = \{b, Y\}$. After replacing $E[Y/b] = \{f(f(b), b)\}$ and the algorithm stops $\sigma = [X/f(Y)][Y/b]$.

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list(1, list(2, list(3, list(4, null))))
```

 Prolog has a predefined operator '[|]'/2 and a predefined constant [] so that a term like

```
'[|]'(1,'[|]'(2,'[|]'(3,'[|]'(4,[]))))
```

can be simply abbreviated as [1,2,3,4]

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```
?- member(c, [a,b,c,d,c]).
```

?- member(a,X).

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• Program predicate append (L1, L2, L3)

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```
append([],L,L). append([X|L1],L2,[X|L3]):-append(L1,L2,L3).
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?- append([a,b],L,[a,b,c,d,e]).
```

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• Use append to find the prefix P and suffix S of a given element X in a list L. For instance, with X=wed and L= [sun, mon, tue, wed, thu, fri, sat], we should get P=[sun, mon, tue] and S=[thu, fri, sat].

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- In the same list, find the predecessor and successor weekdays to some day X.

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- Use previous predicates to define perm(L, L2) so that L2 is an arbitrary permutation of L.
- Define predicate flatten (L1, L2) that removes nested lists putting all constants at a same level in a single list. Example:

```
?- flatten([[a,b],[c,[d]]],L2).
```

$$L2 = [a,b,c,d]$$

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4 D > 4 A > 4 B > 4 B > B

- 1 Prolog
- 2 Functions

Flow control

Other features

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• The cut predicate written! behaves as follows:

$$H: - B_1, \ldots, B_n, !, B_{n+1}, \ldots, B_m.$$

When ! is reached, it succeeds but ignores any remaining choice for B_1, \ldots, B_n .

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Example: the program

$$\max(X, Y, X) :- X>=Y.$$

 $\max(X, Y, Y) :- X$

can be replaced by

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assuming that it is called with an unbounded third variable. Otherwise, a guery max(3,1,1) will succeed.

This second alternative overcomes that problem

```
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X > = Y, !, M = X

; M = Y.
```

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```
max(X,Y,M):-
X>=Y,!,M=X
; M=Y.
```

• Another example:

```
p(1).

p(2):-!.

p(3).

try the queries

?-p(X).

?-p(X),p(Y).

?-p(3).

?-p(X),!,p(Y).
```

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- In some cases, it is really necessary for a reasonable solution to a programming problem. Example: add a non-existing element as head of a list. If existing, leave the list untouched.

```
add(X,L,L) := member(X,L),!.
 add(X,L,[X|L]).
```

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Example: all birds fly, excepting penguins.

```
bird(a). bird(b). bird(c). penguin(b). fly(X) :- bird(X), \ penguin(X).
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bird(a). bird(b). bird(c). penguin(b).
fly(X) :- bird(X), \+ penguin(X).
```

Floundering problem: be careful with unbound variables inside negation. The query ?- fly(X). will fail if using rule fly(X):- \+ penguin(X), bird(X).

Predicate repeat

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- This means that anything that fails afterwards, will return to repeat forever.
- Its effect can only be canceled by a cut!

```
writelist(L) :-
  repeat, (member(X,L), write(X), fail; !).
```

- 1 Prolog
- 2 Functions

Flow control

4 Other features

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 \bullet Predicate ${\tt is}$ evaluates an arithmetic expression. We can use:

```
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+ - * / ** (power) // (integer division) mod (modulo).
```

• We can make comparisons of numeric values using:

```
> < >= =< =;= =\=
```

• Examples:

```
gcd(X,X,X) :- !.
gcd(X,Y,D) :- X>Y,!,X1 is X-Y,gcd(X1,Y,D).
gcd(X,Y,D) :- X<Y,gcd(Y,X,D).
length([],0).
length([_|L],N):-length(L,M),N is M+1.</pre>
```

Exercise 3

Define predicate $set_nth0 (N, L1, X, L2)$ so that the element of list L1 at position N (starting from 0) is replaced by X to produce list L2.

Example:

```
?- set_nth0(3,[a,b,c,d,e,f],z,L2).
L2=[a,b,c,z,e,f].
```

Exercise 4

We have a list of 9 elements that capture the content of a 3×3 grid. The positions in the list corresponds to the grid positions:

| 0 | 1 | 2 |
|---|---|---|
| 3 | 4 | 5 |
| 6 | 7 | 8 |

Define predicate nextpos(X,D,Y), so that Y is the adjacent position to X following direction D varying in $\{u,d,l,r\}$.

Example:

```
?- nextpos(4,u,X).
X=1.
?- nextpos(4,1,X).
X=3.
```

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• write (X) writes a term on the standard output; tab (N) writes N spaces; nl writes a newline character.

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Input/output

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- Similarly, tell (Filename) changes standard output to Filename. When finished, we invoke predicate told.
- put (C) puts character with code C in the standard output.
- get0 (C) gets a character code from standard input. get (C) is similar but ignoring blank or non-printable characters.

Assert/retract

- We can modify the database of facts and rules in a dynamic way.
 - ▶ assert (T) includes new fact/rule T.
 - ▶ asserta(T) includes new fact/rule T in the beginning.
 - ▶ assertz(T) includes new fact/rule T in the end.
 - ► retract (T) retracts fact/rule T. It fails when not possible (the fact did not match to any existing one).
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 - retractall(T) like retract but retracts all matching facts or rules.
- Some Prolog implementations require that predicates are declared as dynamic.

```
:- dynamic user/1.
user(1).
user(2).
?- asserta(user(0)).
?- user(X).
```

Assert/retract

We can use assert/retract to create a "global variable"

```
:- dynamic mycounter/1.
mycounter(0).
increment(X):-
    retract (mycounter (C)),
    D is C+X,
    assert (mycounter (D)).
?- mycounter(C).
C=0.
?- increment(5), mycounter(C), increment(10).
C=5.
?- mycounter(C).
C = 15.
```

Testing the type of terms

- var (X) true when X is an uninstantiated variable
- nonvar (X) true when X is not a variable or is already instantiated
- atom(X) true when X is a symbolic atom
- integer (X) true when X is an integer number
- float (X) true when X is a floating point number
- number (X) true when X is a numeric atom (either integer or float)
- atomic (X) true when X is atomic (either atom or number)

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- Symbolic atoms can contain special characters by using simple quote: mother ('Juana la Loca', 'Carlos I').
- The use of double quotes "Carlos I" stands for a list of ASCII codes [67, 97, 114, 108, 111, 115, 32, 73].
- name (A, L) transforms atom A into a list of ASCII codes or vice versa. Examples:

```
?- name('Carlos I',L).
L = [67, 97, 114, 108, 111, 115, 32, 73]
?- append("Hello ","World !",L), name(A,L).
L = [72, 101, 108, 108, 111, 32, 87, 111, 114|...],
A = 'Hello World !'
```

Any ASCII code for a character c can be retrieved by using 0 'c.
 For instance:

```
?- name(A, [ 0'a, 0'\$, 0'., 0'[]).
A = 'a\$.['
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```

 concat_atom(L, A) concatenates a list of atoms into a new atom. Example:

```
?- concat_atom(['Hello ','World ','!'],A).
A = 'Hello World !'
```

 The special equiality predicate X = . . L unifies term X with a list L=[F,A1,A2,...] where F is the main functor of X and and A1,A2,... its arguments.

The special equiality predicate X = ..L unifies term X with a list L=[F,A1,A2,...] where F is the main functor of X and and A1,A2,... its arguments. Examples
 ?- f(a,b) = .. L.

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L = [f, a, b]
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The special equiality predicate X = ...L unifies term X with a list L=[F,A1,A2,...] where F is the main functor of X and and A1,A2,... its arguments. Examples
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$$L = [f, a, b]$$

?- $T = ...[+,3,4]$.

$$T = 3+4$$

The special equiality predicate X = ...L unifies term X with a list L=[F,A1,A2,...] where F is the main functor of X and and A1,A2,... its arguments. Examples
 ?- f(a,b) = ...L.

```
L = [f, a, b]
?- T=..[+,3,4].
```

process([],[]):-!.

T = 3+4

 Process a list of terms so that the numeric arguments of unary functors are increased in one.

```
process([X|Xs],[Y|Ys]):-
   X =.. [F,A], number(A),!, A1 is A+1,
   Y=..[F,A1], process(Xs,Ys).
process([X|Xs],[X|Ys]):- process(Xs,Ys).
```

 Predicate call allows calling other predicates handled as arguments.

- Predicate call allows calling other predicates handled as arguments.
- Example: apply some function to a list of numbers

```
double(X,Y) :- Y is 2*X.
minus(X,Y) :- Y is -X.
map([],_,[]).
map([X|Xs],P,[Y|Ys]) :- call(P,X,Y), map(Xs,P,Ys).
?- map([1,3,6],double,L).
?- map([1,3,6],minus,L).
```

- Predicate call allows calling other predicates handled as arguments.
- Example: apply some function to a list of numbers

double (X,Y) := Y is 2*X.

?- map([1,3,6], minus, L).

```
minus(X,Y) :- Y is -X.
map([],_,[]).
map([X|Xs],P,[Y|Ys]) :- call(P,X,Y), map(Xs,P,Ys).
?- map([1,3,6],double,L).
```

• We can also use = . . to build the term to be called:

```
map([],_,[]).
map([X|Xs],P,[Y|Ys]) :-
    T=..[P,X,Y], T, map(Xs,P,Ys).
```

• Predicate findall(T,G,L) collects in list L all the instantiations for term T that satisfy goal G

- Predicate findall(T,G,L) collects in list L all the instantiations for term T that satisfy goal G
- Get a list with all the ancestors of leonor

```
?- findall( X, ancestor(X, leonor), L).
```

 Example: convert a list of elements [a,b,c,d] into a list of duplicated pairs

```
?- findall((X,X), member(X,[a,b,c,d]), L).
```