Microwave & Optical Communications 18ECC302J

Session 3

Microwave Tubes

Klystron amplifier

Microwave Tubes

High Frequency Limitations of Conventional Tubes

- ✓Conventional Tubes can’t work above 1GHz.
- ✓Due to:

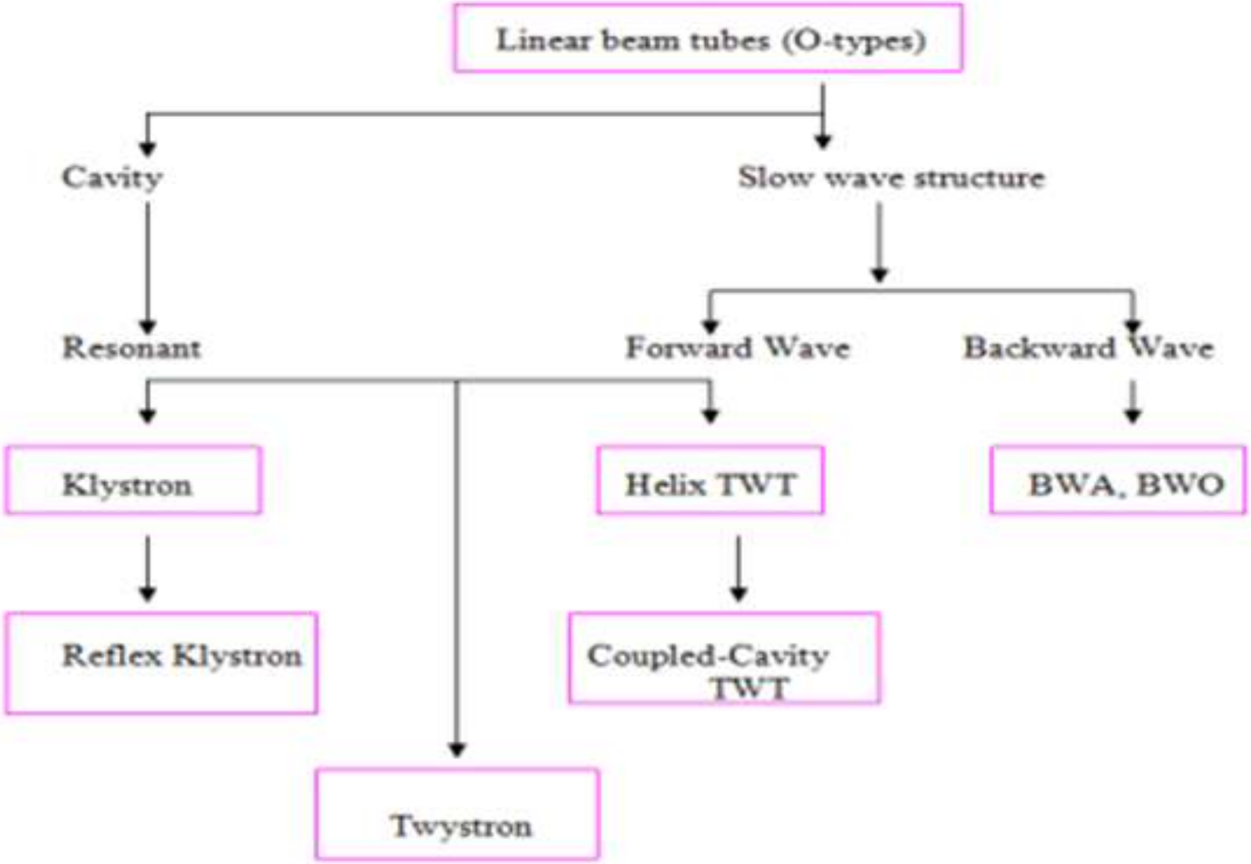
1. Lead inductance
2. Inter electrode capacitance
3. Transit angle effect
4. Gain bandwidth product
5. Skin Effects
6. Radiation Loss
7. Dielectric Loss.

- ✓High frequency tubes two types:
    1. Linear beam Type (O-Type)
    2. Cross field Tube (M-Type)

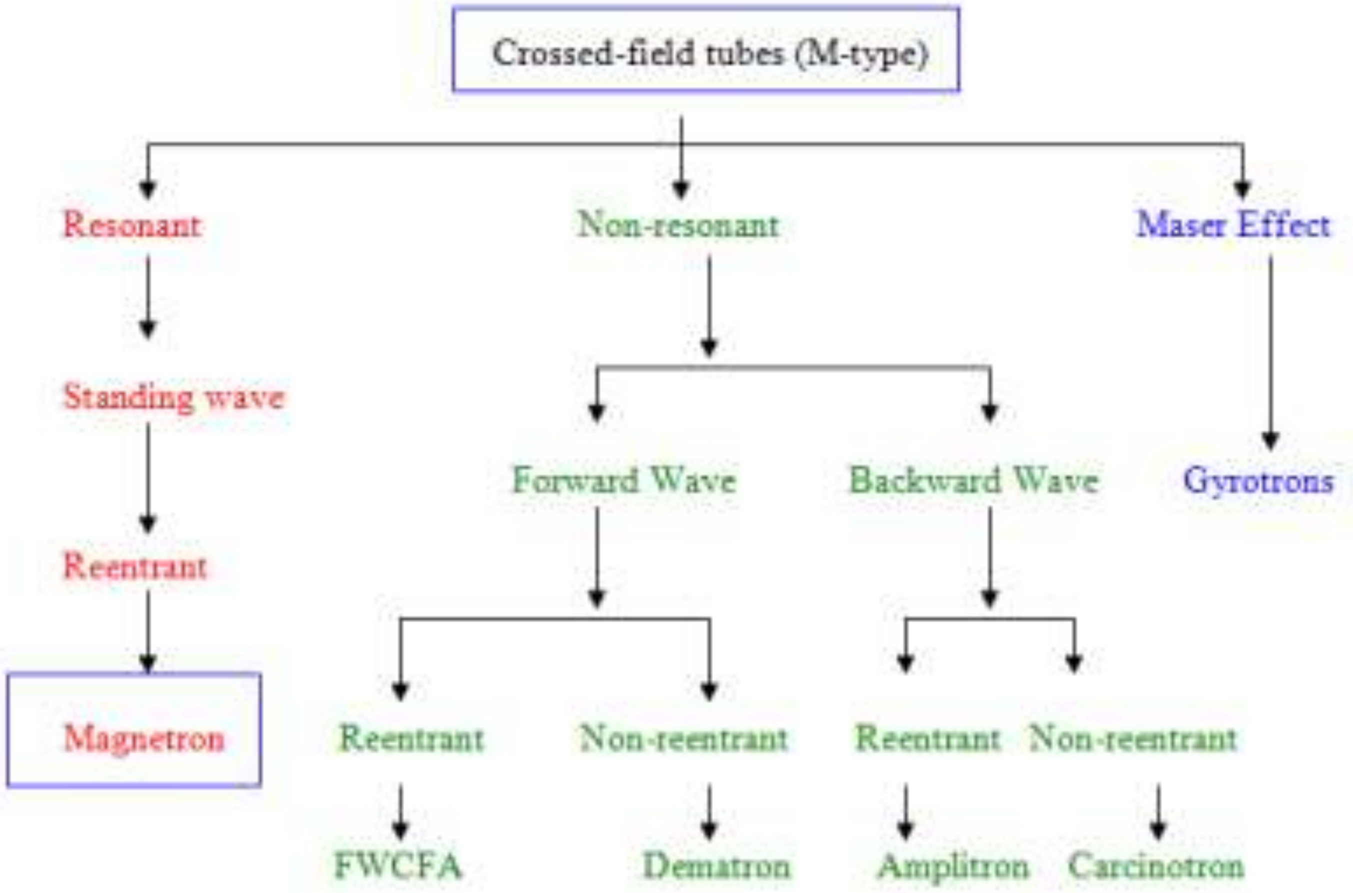
Microwave Tubes

- Linear Beam Tubes
- Cross Field Tubes

❖ Linear Beam Tubes



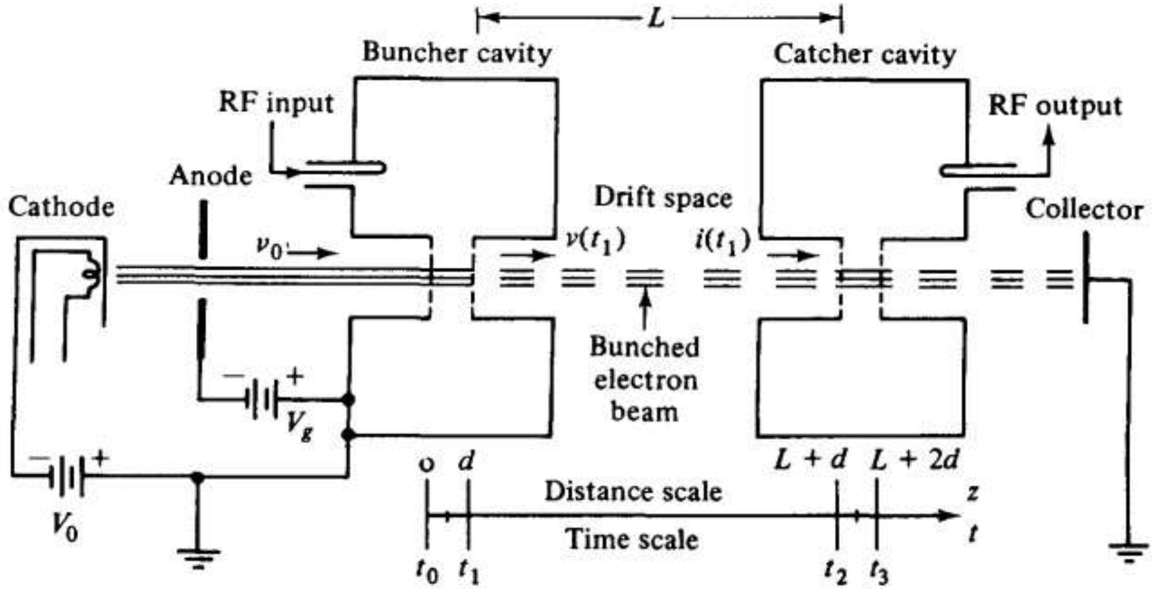
❖Cross Field Tubes



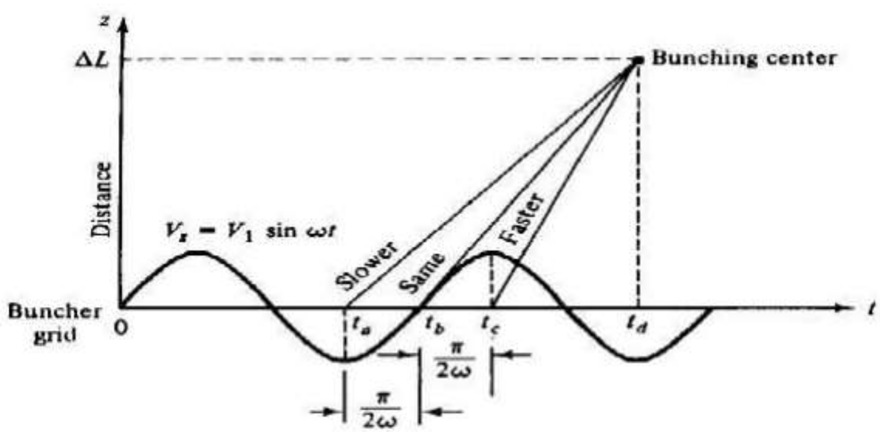
Klystron Amplifiers

Two Cavity Klystron Amplifier

- ✓ 500 kW, CW Power, 30 MW pulse power at 10 GHz, power gain 30 dB and efficiency 40%



Applegate Diagram



Velocity Modulation Process & Efficiency Derivation

- By supplying dc voltage Vo, before entering the buncher grid their velocity is uniform: $v_0 = \sqrt{2eV_0/m} = 0.593\times10^6\sqrt{V_0}$ ---------(1)
- When Microwave Signals are applied at input terminal, gap voltage between buncher grids: $V_s = V_1 \sin(\omega t)$ ---------------------(2)
- V_1 is the amplitude of the signal and $V_1 << V_0$.

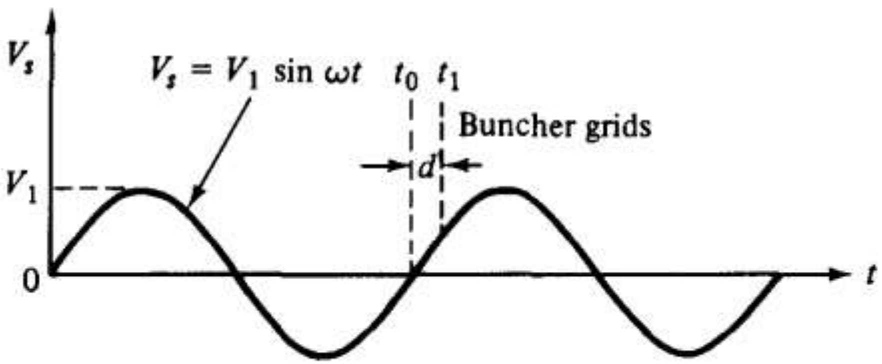


Figure 9-2-6 Signal voltage in the buncher gap.

- The average transit time through the buncher gap distance d:

$$\tau \approx d/v_o = t_1 - t_0 \text{ --------------------(3)}$$

- Average gap transit angle :

$$\theta_g = \omega\tau \approx \omega d/v_0 = \omega(t_1 - t_0) \text{ -----------(4)}$$

Continue.....

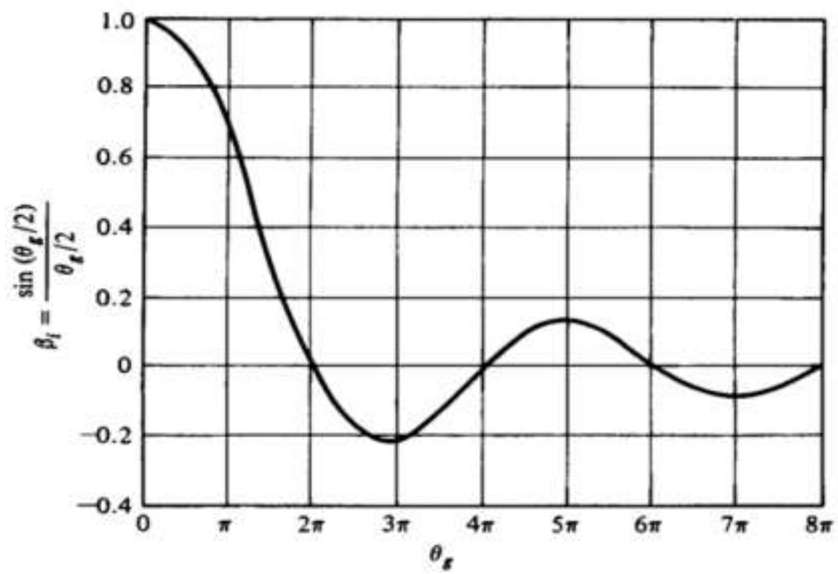
- Te average microwave voltage in buncher gap can be found in following ways:
.

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\,dt = -\frac{V_1}{\omega\tau}\left[\cos(\omega t_1) - \cos(\omega t_0)\right]$$

$$= V_1 \frac{\sin(\theta_g/2)}{(\theta_g/2)} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) ----------------(5)$$

- beam coupling coefficient of the input cavity gap and denoted by β_i
- By increasing the gap transit angle θ_g

decrease the coupling between the electron beam and the buncher cavity, so the velocity modulation of the beam for a given microwave signal is decreased.



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- At buncher cavity, the input signal is superimposed on the DC voltage Vo, so the velocity of electrons, exiting from the buncher cavity can be written as:

$$V_1 = \sqrt{\frac{2e}{m}\left\{V_0 + \beta_i V_1\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}} = \sqrt{\frac{2e}{m}\left\{1 + \frac{\beta_i V_1}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}} ----(6)$$

$\frac{\beta_i V_1}{V_0}$ known as depth of velocity modulation

- Since $\beta_i < 1$, and $V_1 << V_0, \beta_i V_1 \ll 1$, using binomial expansion of equation (6)

$$v(t_1) = \sqrt{\frac{2eV_0}{m}}\left\{1 + \frac{\beta_i V_1}{2V_o}\sin\left(\omega t_o + \frac{\theta_g}{2}\right)\right\} ------(7)$$

- Substituting the velocity of electron into equation (7)

$$v(t_1) = v_o\left\{1 + \frac{\beta_i V_1}{2V_o}\sin\left(\omega t_o + \frac{\theta_g}{2}\right)\right\} ------(8)$$

- This equation is known as velocity modulation and can be written as

$$v(t_1) = v_o\left\{1 + \frac{\beta_i V_1}{2V_o}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right\} ------(9)$$

Bunching Process

• Since $t_c > t_b > t_a$, ΔL , can be estimated such that it satisfied the following condition:

$$\Delta L = v_o\left(t_d - t_b\right) = v_{\min}\left(t_d - t_a\right) = V_{\max}\left(t_d - t_c\right) ------(1)$$

• under these condition, all electron leaving the cavity between t_a and t_c, will arrive at a distance from the buncher cavity at the same time t_d . So equation (1) can be written as :

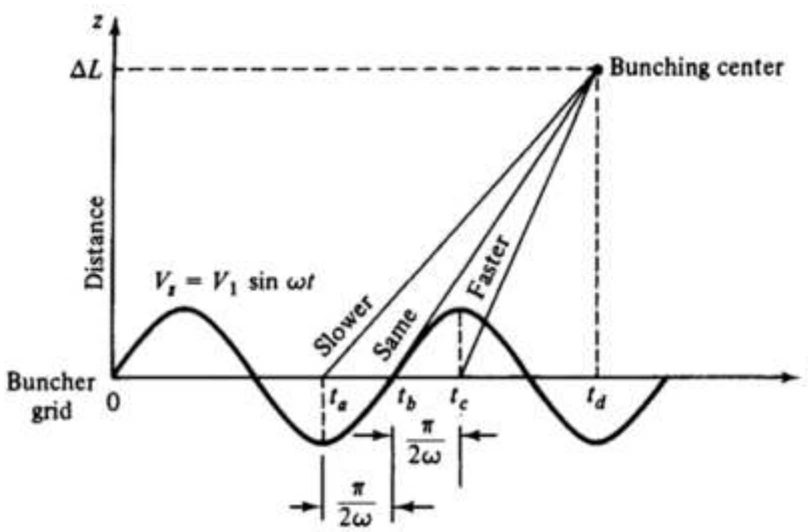
$$\Delta L = v_o\left(t_d - t_b\right) = v_{\min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) = V_{\max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) -----(2)$$

•From equation of velocity modulation:

$$v_{\max} = v_o\left(1 + \frac{\beta_i V_1}{2V_o}\right) ------(3)$$

$$v_{\min} = v_o\left(1 - \frac{\beta_i V_1}{2V_o}\right) ------(4)$$

•Substitute these equation in Equation (2)



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$$\Delta L = v_o\left(1+\frac{\beta_i V_1}{2V_o}\right)\left(t_d - t_b - \frac{\pi}{2\omega}\right) = v_o\left(t_d - t_b\right) + \left\{-v_o\frac{\pi}{2\omega} + v_o\frac{\beta_i V_1}{2V_o}\left(t_d - t_b\right) - v_o\frac{\beta_i V_1}{2V_o}\frac{\pi}{2\omega}\right\} ---(5)$$

$$\Delta L = v_o\left(1-\frac{\beta_i V_1}{2V_o}\right)\left(t_d - t_b + \frac{\pi}{2\omega}\right) = v_o\left(t_d - t_b\right) + \left\{v_o\frac{\pi}{2\omega} - v_o\frac{\beta_i V_1}{2V_o}\left(t_d - t_b\right) - v_o\frac{\beta_i V_1}{2V_o}\frac{\pi}{2\omega}\right\} ---(6)$$

- Comparing equation (2) with (5) and (6):

$$t_d - t_b = \frac{2\pi V_0}{2\omega\beta_i V_1} \pm \frac{\pi}{2\omega} = \frac{\pi V_0}{\omega\beta_i V_1} \pm \frac{\pi}{2\omega} \approx \frac{\pi V_0}{\omega\beta_i V_1} \text{ as } \omega \gg \pi ----(7)$$

- Substitute equation (7) into equation (2)

$$\Delta L = v_o\left(t_d - t_b\right) = v_o\frac{\pi V_0}{\omega\beta_i V_1} ------(8)$$

- Within the drift region, electrons moves with a velocity $v(t_1)$, if the catcher cavity is placed at a distance L from the buncher cavity, the transit time of the electron can be written as

$$T = t_2 - t_1 = \frac{L}{v(t_1)} = \frac{L}{v_o}\left\{1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right\}^{-1} = T_0\left\{1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right\}^{-1} ----(9)$$

Continue….

- Using binomial expansion,

DC Transit time

$$T = T_0\left\{1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right\} - - - - -(10)$$

- In terms of radians, the expression can be written as:

$$\omega T = \omega t_2 - \omega t_1 = \omega T_0 - \omega T_0 \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) = \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) - - - - -(11)$$

•Where $\theta_0 = \omega T_0 = \dfrac{\omega L}{v_0} = 2\pi N$, DC transit angle between cavity. N is the number of electrons transit cycle in drift space, and

$$X \equiv \frac{\beta_i V_1}{2V_0}\theta_0 - - - - - -(b)$$

is bunching parameter of a klystron.

▪ Assume that, dQ_o charge is passed through the buncher cavity gap at the interval of dt_o

$$dQ_o = I_0 dt_o = i_2\left|dt_2\right| - - - - -(a)$$

▪ The instant at which the electrons arrives at catcher cavity is

$$t_2 = t_1 + T_0\left\{1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right\} = t_0 + \tau + T_0\left\{1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\} - - - - -(12)$$

Continue….

- Alternatively

$$\underbrace{\boxed{\omega t_2-\left(\theta_0+\frac{\theta_g}{2}\right)}}_{\textbf{Cather cavity arrival angle}}=\overbrace{\boxed{\left(\omega t_0+\frac{\theta_g}{2}\right)}}^{\textbf{Buncher cavity departure angle}}-X\sin\left(\omega t_0+\frac{\theta_g}{2}\right)------(13)$$

- Differentiating equation (12) with respect to t_0

$$\omega\frac{dt_2}{dt_0}=\omega-X\omega\cos\left(\omega t_0+\frac{\theta_g}{2}\right)=dt_0\left\{1-X\cos\left(\omega t_0+\frac{\theta_g}{2}\right)\right\}=i_2\left(t_0\right)dt_0\left\{1-X\cos\left(\omega t_0+\frac{\theta_g}{2}\right)\right\}--(14)$$

- The Current arriving at catcher cavity is given by

$$i_2\left(t_0\right)=I_0\Big/1-X\cos\left(\omega t_0+\frac{\theta_g}{2}\right)----(15)$$

- In terms of t_2

$$i_2\left(t_2\right)=I_0\Big/1-X\cos\left(\omega t_2-\theta_0-\frac{\theta_g}{2}\right)----(16)$$

$$t_2=t_0+\tau+T_0,\omega t_2=\omega t_0+\omega\tau+\omega T_0=\omega t_0+\theta_g+\theta_0$$

Continue….

• Since bunches are formed at periodic intervals, the beam current in the catcher cavity is also a periodic waveform. Therefore the current expanded in the fourier series:

$$i_2 = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(n\omega t_2\right) + b_n \sin\left(n\omega t_2\right)\right] ------(17)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} i_2 d\,\omega t_2 -------(18)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_2 \cos\left(n\omega t_2\right) d\left(\omega t_2\right) -------(19)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_2 \sin\left(n\omega t_2\right) d\left(\omega t_2\right) -------(20)$$

•Substitute equation (a) and (11) in equation (18, 19, 20)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} i_2 d\left(\omega t_2\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} I_0 d\left(\omega t_2\right) = \frac{I_0}{2\pi}\int_{-\pi}^{\pi}\left[\omega t_0\right]_{-\pi}^{\pi} = I_0 ---(21)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} I_0 \cos\left[\left(n\omega t_0 + n\theta_g + n\theta_0\right) + nX \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d\left(\omega t_0\right) ---(22)$$

Continue….

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} I_0 \sin\left[\left(n\omega t_0 + n\theta_g + n\theta_0\right) + nX\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d\left(\omega t_0\right) --- (23)$$

Its involves sine and cosine function, each term of integrand contains an infinite number of terms of Bessel function.

$$\cos\left[nX\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] = 2J_0\left(nX\right) + 2\left|J_2\left(nX\right)\cos\left\{2\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right|$$

$$+\left[J_4\left(nX\right)\cos\left\{4\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right] + 2\left[J_6\left(nX\right)\cos\left\{6\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right] --- (24)$$

$$\sin\left[nX\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] = 2\left|J_1\left(nX\right)\cos\left\{\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right|$$

$$+2\left[J_3\left(nX\right)\cos\left\{3\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right] + 2\left[J_5\left(nX\right)\cos\left\{5\left(\omega t_0 + \frac{\theta_g}{2}\right)\right\}\right] --- (25)$$

Substitute equation (24, 25) into equation (22) (23).

Continue….

$$a_n = 2I_0 J_n(nX)\cos\left(n\theta_g + n\theta_0\right) - -(26)$$

$$b_n = 2I_0 J_n(nX)\sin\left(n\theta_g + n\theta_0\right) - -(27)$$

•The final expression for

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX)\cos\left[n\omega\left(t_2 - \tau - T_0\right)\right] - - - (28)$$

•The fundamental current of the beam current at the catcher cavity has magnitude

$$I_f = 2I_0 J_1(X) - - - - - - (29)$$

•The fundamental component I_f has its maximum amplitude at $X = 1.841$

The Optimum distance L at which the maximum fundamental component of current occurs is

$$L_{Optimum} = 1.841\frac{2V_0 v_0}{\beta_i V_0 \omega} = 3.62\frac{V_0 v_0}{\beta_i V_0 \omega} - - - - - (30)$$

• Comparing equation (8) and (30) $\dfrac{\Delta L}{L_{Optimum}} = \dfrac{\pi V_0 v_0}{\beta_i V_0 \omega}\dfrac{\beta_i V_0 \omega}{3.62 V_0 v_0} = 0.85$

• i.e. approximately 15% less than $L_{Optimum}$.

Output Power and Beam Loading

❖ The fundamental component of induced microwave current in catcher cavity is given by:

$$i_{2,ind} = \beta_0 i_2 = 2\beta_0 I_0 J_1(X)\cos\{\omega(t_2 - \tau - T_0)\} ----(31)$$

❖ β_0 is the beam coupling coefficient of the catcher cavity. If the buncher and catcher cavity are identical $\beta_0 = \beta_i$ and the magnitude of the fundamental component of current induced in the catcher cavity can be expressed as:

$$i_{2,ind} = \beta_0 I_2 = 2\beta_0 I_0 J_1(X) ----(32)$$

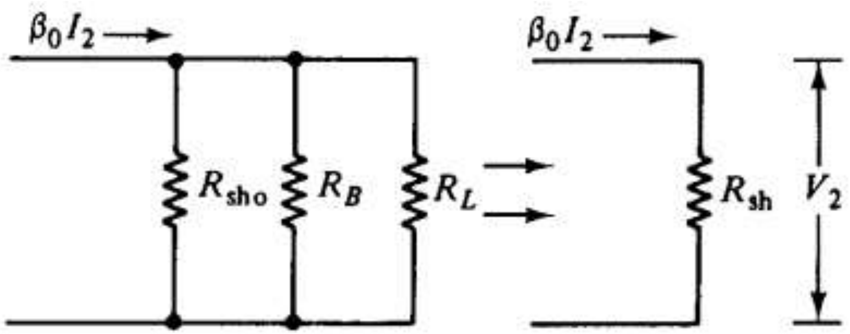

Figure 9-2-13 Output equivalent circuit.

❖ R_{sho} = wall resistance of the catcher cavity

R_B = beam loading resistance, R_L external load and R_{sh} is effective shunt resistance

❖The output power delivered to the load and catcher cavity is

$$P_{out} = (\beta_0 I_2)^2 R_{sh}/2 = \beta_0 I_2 V_2(X) ---(33)$$

Klystron Efficiency

❖ The efficiency of the klystron can be expressed as:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\beta_0 I_2 V_2}{2I_0 V_0} ------------------------(34)$$

if coupling is perfect, $\beta_0 = 1$, $V_2 = V_0$, and maximum beam current approches,

$$I_{2,\max} = 2I_0 J_1(1.841) = 1.164 I_0 ------(35)$$

Thus maximum efficiency, $\eta_{\max} = \dfrac{\beta_0 I_2 V_2}{2I_0 V_0} = \dfrac{1.164 I_0 V_0}{2I_0 V_0} \approx 0.58$

❖Therefore the maximum efficiency of the klystron is 58%. In practice 15-30%.

Mutual Conductance of a Klystron Amplifier

➢ The ratio of induced output current to input voltage.

$$|G_m| \equiv \frac{I_{2,ind}}{V_1} = \frac{2\beta_0 I_0 J_1(X)}{V_1} ------(36)$$

➢From equation (b)

$$V_1 = \frac{2V_0 X}{\beta_i \theta_0} ------(37)$$

Mutual Conductance of a Klystron Amplifier

DC Beam Conductance

$$|G_m| = \frac{2\beta_0\beta_i\theta_0 J_1(X)}{2X}\frac{I_0}{V_0} = \frac{\beta_0\beta_i\theta_0 J_1(X)}{X}G_0 ------(38)$$

❖ Assume $\beta_0 = \beta_i$

$$\frac{|G_m|}{G_0} = \beta_0^2\theta_0\frac{J_1(X)}{X} -------(39)$$

$$\frac{|G_m|}{G_0} = \beta_0^2 J_1(1.841)\theta_0 / 1.841 = 0.316\beta_0^2\theta_0 ---(40)$$

❖ The voltage gain of the klystron amplifier is defined as

DC Beam Resistance

$$A_v \equiv \frac{|V_2|}{|V_1|} = \frac{\beta_0 I_2 R_{sh}}{V_1} = \frac{\beta_0^2\theta_0}{R_0}\frac{J_1(X)}{X}R_{sh} ------(41)$$

❖ $$A_v = g_m R_s -----(42)$$

Numerical Example

A two-cavity klystron amplifier has the following parameters:

$$V_0 = 1000 \text{ V} \qquad R_0 = 40 \text{ k}\Omega$$
$$I_0 = 25 \text{ mA} \qquad f = 3 \text{ GHz}$$

Gap spacing in either cavity: $d = 1$ mm
Spacing between the two cavities: $L = 4$ cm
Effective shunt impedance, excluding beam loading: $R_{sh} = 30$ kΩ

a. Find the input gap voltage to give maximum voltage V_2.
b. Find the voltage gain, neglecting the beam loading in the output cavity.
c. Find the efficiency of the amplifier, neglecting beam loading.
d. Calculate the beam loading conductance and show that neglecting it was justified in the preceding calculations.

Solution

a. For maximum V_2, $J_1(X)$ must be maximum. This means $J_1(X) = 0.582$ at $X = 1.841$. The electron velocity just leaving the cathode is

$$v_0 = (0.593 \times 10^6)\sqrt{V_0} = (0.593 \times 10^6)\sqrt{10^3} = 1.88 \times 10^7 \text{ m/s}$$

The gap transit angle is

$$\theta_g = \omega \frac{d}{v_0} = 2\pi (3 \times 10^9) \frac{10^{-3}}{1.88 \times 10^7} = 1 \text{ rad}$$

The beam-coupling coefficient is

$$\beta_i = \beta_0 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \frac{\sin(1/2)}{1/2} = 0.952$$

Continue

The dc transit angle between the cavities is

$$\theta_0 = \omega T_0 = \omega \frac{L}{v_0} = 2\pi(3 \times 10^9)\frac{4 \times 10^{-2}}{1.88 \times 10^7} = 40 \text{ rad}$$

The maximum input voltage V_1 is then given by

$$V_{1\,max} = \frac{2V_0 X}{\beta_i \theta_0} = \frac{2(10^3)(1.841)}{(0.952)(40)} = 96.5 \text{ V}$$

b. The voltage gain is found as

$$A_v = \frac{\beta_0^2 \theta_0}{R_0} \frac{J_1(X)}{X} R_{sh} = \frac{(0.952)^2(40)(0.582)(30 \times 10^3)}{4 \times 10^4 \times 1.841} = 8.595$$

c. The efficiency can be found as follows:

$$I_2 = 2I_0 J_1(X) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1 \times 10^{-3} \text{ A}$$

$$V_2 = \beta_0 I_2 R_{sh} = (0.952)(29.1 \times 10^{-3})(30 \times 10^3) = 831 \text{ V}$$

$$\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2I_0 V_0} = \frac{(0.952)(29.1 \times 10^{-3})(831)}{2(25 \times 10^{-3})(10^3)} = 46.2\%$$

Continue

d. Calculate the beam loading conductance (refer to Fig. 9-2-13). The beam loading conductance G_B is

$$G_B = \frac{G_0}{2}\left(\beta_0^2 - \beta_0 \cos \frac{\theta_g}{2}\right) = \frac{25 \times 10^{-6}}{2}[(0.952)^2 - (0.952)\cos(28.6^\circ)]$$

$$= 8.8 \times 10^{-7} \text{ mho}$$

Then the beam loading resistance R_B is

$$R_B = \frac{1}{G_B} = 1.14 \times 10^6\ \Omega$$

Multi Cavity Klystron