Unit III - Detection Of Signals In Noise

S1- Detection Criteria ,
S2-Probabilities of Detection and False Alarm

DETECTION CRITERIA

1. Neyman pearson observes   Probability of error

Type I error - false alarm → fixed.

Type II error - missed detection → error is Minimized (max. prob. of detection)

2. likelihood Ratio Receiver

$\uparrow L_r(V) = \dfrac{P_{sn}}{P_n}$ → prob. density fn. with signal
→ - " " without signal

3. Inverse probability Receiver

→ optimum Receivers - detection & info. extraction

4. sequential observer / sequential detection

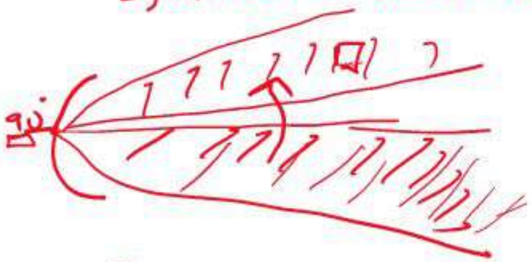
* fewer pulses on more pulse response

when SNR ↑, detection at fewer pulses

Three choices

(i) sample due to signal & noise } → detection
(ii) " " noise alone
(iii) cannot be determined
↳ another observation

* phased array radar



* Adv
save tx power
reduce revisit time

sequential detection → two stages

↳ radar tx. series of pulses in a particular direction
with lower threshold (false alarm)

→ if no threshold crossing → move position
if threshold crossing happening → Energy.

* power saving of 3 ~ 4 dB compare to uniform scanning

Probabilities of Detection and False Alarm

- It shows how to find the minimum signal-to-noise ratio required to achieve a specified probability of detection and probability of false alarm.
- The signal-to-noise ratio is needed in order to calculate the maximum range of a radar using the radar range equation.

Envelope Detector

- Figure 2.3 shows a portion of a super heterodyne radar receiver with IF amplifier of bandwidth B_{IF}, second detector, video amplifier with bandwidth B_V, and a threshold where the detection decision is made.
- An envelope detector requires that the video bandwidth $B_V >= B_{IF}/2$ and the IF center frequency $f_{IF} > B_{IF}$

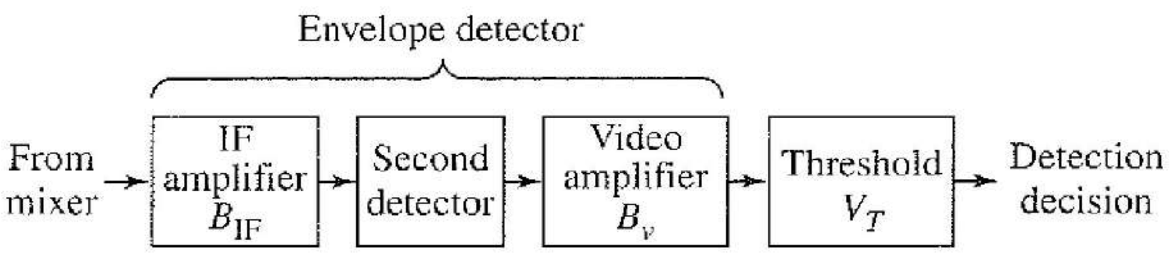


Figure 2.3 Portion of the radar receiver where the echo signal is detected and the detection decision is made.

Probabilities of False Alarm

Figure 2.4 illustrates the occurrence of false alarms. The average time between cros ings of the decision threshold when noise alone is present is called the *false-alarm tim* T_{fa}, and is given by

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k \qquad [2.2$$

where T_k is the time between crossings of the threshold V_T by the noise envelope. Tl

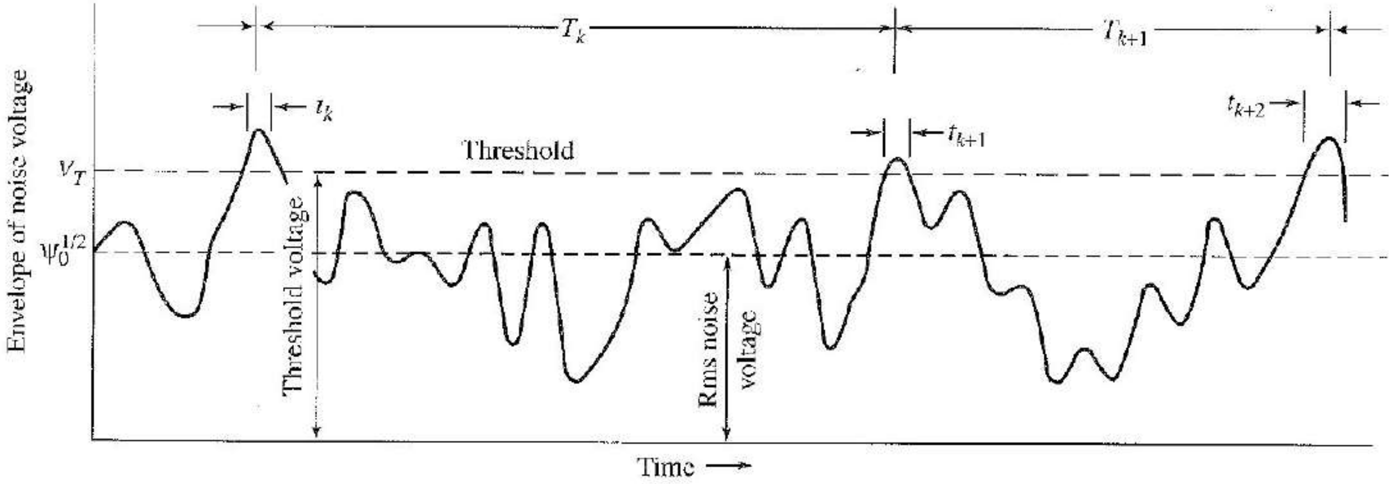


Figure 2.4 Envelope of the receiver output with noise alone, illustrating the duration of false alarms and the time between false alarms.

Probabilities of False Alarm

probability of false alarm. The false-alarm probability can be expressed in terms of false-alarm time by noting that the false-alarm probability P_{fa} is the ratio of the time the envelope is actually above the threshold to the total time it could have been above the threshold, or

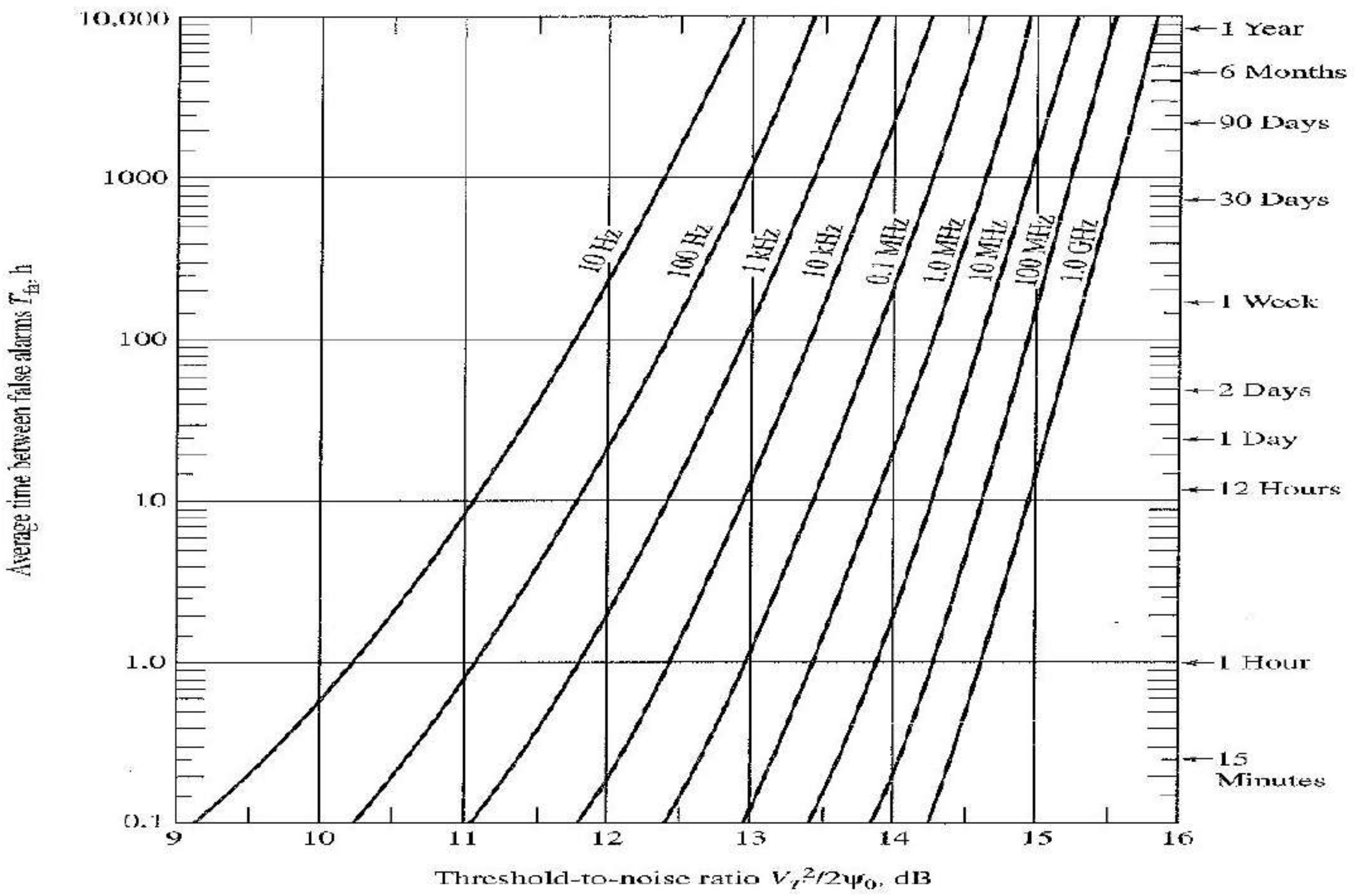
$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{av}}{\langle T_k \rangle_{av}} = \frac{1}{T_{fa} B} \qquad [2.25]$$

where t_k and T_k are shown in Fig. 2.4, and B is the bandwidth of the IF amplifier of the radar receiver. The average duration of a threshold crossing by noise $\langle t_k \rangle_{av}$ is approximately the reciprocal of the IF bandwidth B. The average of T_k is the *false-alarm time*, T_{fa}. Equating Eqs. (2.23) and (2.25) yields

$$T_{fa} = \frac{1}{B} \exp\left(\frac{V_T^2}{2\psi_0} \right) \qquad [2.26]$$

A plot of T_{fa} as a function of $V_T^2/2\Psi_0$ is shown in Fig. 2.5. If, for example, the bandwidth

Average time between false alarms as a function of the threshold level V_T and the receiver bandwidth B



The exponential relationship between the false-alarm time T_{fa} and the threshold level V_T [Eq. (2.26)] results in the false-alarm time being sensitive to small variations in the threshold. For example, if the bandwidth were 1 MHz, a value of $10 \log(V_T^2/2\Psi_0) = 13.2$ dB results in a false-alarm time of about 20 min. A 0.5-dB decrease in the threshold to 12.7 dB decreases the false-alarm time by an order of magnitude, to about 2 min.

If the threshold is set slightly higher than required and maintained stable, there is little likelihood of false alarms due to thermal noise. In practice, false alarms are more likely to occur from clutter echoes (ground, sea, weather, birds, and insects) that enter the radar and are large enough to cross the threshold. In the specification of the radar's false-alarm time, however, clutter is almost never included, only receiver noise.

Although the crossing of the threshold by noise is called a false alarm, it is not necessarily a *false-target report.* Declaration of a target generally requires more than one detection made on multiple observations by the radar (Sec. 2.13). In many cases, establishing the track of a target is required before a target is declared as being present. Such criteria can allow a higher probability of false alarm for each detection; hence, the threshold can be lowered to improve detection without obtaining excessive false-target reports. In the present chapter, however, most of the discussion relating to the radar equation concerns a detection decision based on a single crossing of the threshold.

Probability of detection

Probability of Detection So far, we have discussed only the noise input at the radar receiver. Next, consider an echo signal represented as a sinewave of amplitude A along with gaussian noise at the input of the envelope detector. The probability density function of the envelope R at the video output is given by[9]

$$p_s(R) = \frac{R}{\Psi_0} \exp\left(-\frac{R^2 + A^2}{2\Psi_0}\right) I_0\left(\frac{RA}{\Psi_0}\right) \qquad [2.27]$$

where $I_0(Z)$ is the modified Bessel function of zero order and argument Z. For large Z, an asymptotic expansion for $I_0(Z)$ is

The probability of detecting the signal is the probability that the envelope R will exceed the threshold V_T (set by the need to achieve some specified false-alarm time). Thus the probability of detection is

$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR \tag{2.29}$$

When the probability density function $p_s(R)$ of Eq. (2.27) is substituted in the above, the probability of detection P_d cannot be evaluated by simple means. Rice[9] used a series approximation to solve for P_d. Numerical and empirical methods have also been used.

The expression for P_d, Eq. (2.29), along with Eq. (2.27), is a function of the signal amplitude A, threshold V_T, and mean noise power Ψ_0. In radar systems analysis it is more convenient to use signal-to-noise power ratio S/N than $A^2/2\Psi_0$. These are related by

$$\frac{A}{\Psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}\ (\text{rms signal voltage})}{\text{rms noise voltage}}$$

$$= \left(2\,\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

The probability of detection P_d can then be expressed in terms of S/N and the ratio of the threshold-to-noise ratio $V_T^2/2\Psi_0$. The probability of false alarm, Eq. (2.23) is also a function of $V_T^2/2\Psi_0$. The two expressions for P_d and P_{fa} can be combined, by eliminating the threshold-to-noise ratio that is common to each, so as to provide a single expression relating the probability of detection P_d, probability of false alarm P_{fa}, and the signal-to-noise ratio S/N. The result is plotted in Fig. 2.6.

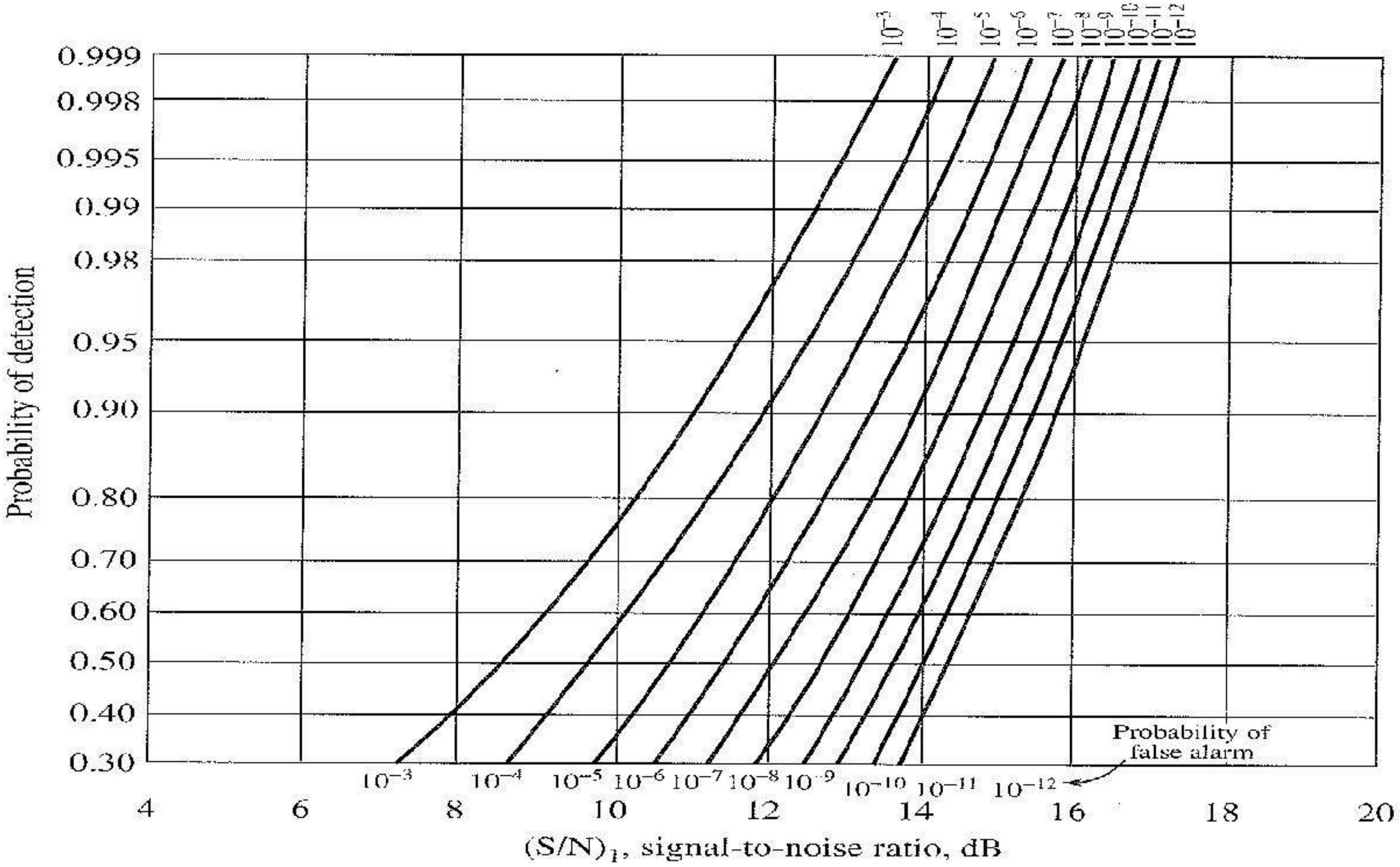
Albersheim[11,12] developed a simple empirical formula for the relationship between S/N, P_d, and P_{fa}, which is

$$S/N = A + 0.12AB + 1.7B \qquad [2.30]$$

where

$$A = \ln\,[0.62/P_{fa}] \quad \text{and} \quad B = \ln\,[P_d/(1 - P_d)]$$

Probability of detection for a sinewave in noise
as a function of the signal-to- noise (power) ratio and the probability of false alarm.



The signal-to-noise ratio in the above is a numeric, and not in dB; and ln is the natural logarithm. Equation (2.30) is said to be accurate to within 0.2 dB for P_{fa} between 10^{-3} and 10^{-7}, and P_d between 0.1 and 0.9. (It is probably suitable for rough calculations for even greater values of P_d and lower values of P_{fa}.) From such an expression or from a graph such as Fig. 2.6, the minimum signal-to-noise ratio required for a particular probability of detection and a specified probability of false alarm can be found and entered into the radar range equation. The above applies for a single pulse. The case for multiple pulses is given later.

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*

Unit III - Detection Of Signals In Noise

S2 Matched Filter Receiver, Derivation of Matched filter frequency response

DETECTION OF RADAR SIGNALS IN NOISE

The two basic operations performed by radar are

(I) ***detection*** of the presence of reflecting objects, and

(2) ***extraction*** of information from the received waveform to obtain such target data as position, velocity, and perhaps size.

The operations of detection and extraction may be performed separately and in either order, although a radar that is a good detection device is usually a good radar for extracting information, and vice versa.

Matched-Filter Receiver

- **A** network whose frequency-response function maximizes the output peak-signal-to-mean noise (power) ratio is called a ***matched filter.***
- The frequency-response function, denoted ***H(f***), expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.
- The magnitude I ***H (***f)I of the frequency-response function is the receiver amplitude pass band characteristic. If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio.

- On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lowered signal-to-noise ratio.
- Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer.
- The rule of thumb quoted in pulse radar practice is that the receiver bandwidth B should be approximately equal to the reciprocal of the pulse width.
- The receiver frequency-response function, for purposes of this discussion, is assumed to apply from the antenna terminals to the output of the IF amplifier. (The second detector and video portion of the well-designed radar superheterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.)

- Narrow banding is most conveniently accomplished in the IF. The bandwidths of the RF and mixer stages of the normal superheterodyne receiver are usually large compared with the IF bandwidth.
- Therefore the frequency-response function of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.
- Thus we need only obtain the frequency-response function that maximizes the signal-to-noise ratio at the output of the IF. The IF amplifier may be considered as a filter with gain.

- For a received waveform s(t) with a given ratio of signal energy E to noise energy N_0 (or noise power per hertz of bandwidth)
- North showed that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise (power) ratio for a fixed input signal-to-noise (energy) ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f t)\, dt$ = voltage spectrum (Fourier transform) of input signal

$S^*(f)$ = complex conjugate of $S(f)$

t_1 = fixed value of time at which signal is observed to be maximum

G_a = constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be Gaussian.
- The filter whose frequency-response function is given by above equation has been called the North filter, the conjugate filter, or more usually the matched filter. It has also been called the Fourier transform criterion.

-

- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift exp $(-j2\pi f t_1)$. This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay.
- **A** time delay is necessary in the specification of the filter for reasons of physical reliability since there can be no output from the filter until the signal is applied. The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ (and a phase spectrum $\exp[-j\O_s(f)]$.
- The matched filter frequency-response function may similarly be written in terms of its amplitude and phase spectra $|H(f)|$ and

  $\exp[-j\O_m(f)]$.

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_1]\}$$

$$|H(f)| = |S(f)|$$

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

The matched filter may also be specified by its impulse response $h(t)$, which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft)\, df \tag{10.4}$$

Physically, the impulse response is the output of the filter as a function of time when the input is an impulse (delta function). Substituting Eq. (10.1) into Eq. (10.4) gives

$$h(t) = G_a \int_{-\infty}^{\infty} S^*(f) \exp[-j2\pi f(t_1 - t)]\, df \tag{10.5}$$

Since $S^*(f) = S(-f)$, we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t_1 - t)]\, df = G_a s(t_1 - t) \tag{10.6}$$

Derivation of the matched-filter characteristic

- The frequency-response function of the matched filter has been derived by a number of authors using either the calculus of variations or the Schwartz inequality.

We wish to show that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

when the input noise is stationary and white (uniform spectral density). The ratio we wish to maximize is

$$R_f = \frac{|s_o(t)|^2_{max}}{N} \tag{10.7}$$

where $|s_o(t)|_{max}$ = maximum value of output signal voltage and N = mean noise power at receiver output. The ratio R_f is not quite the same as the signal-to-noise ratio which has been considered previously in the radar equation. [Note that the peak power as used here is actually the peak *instantaneous* power, whereas the peak power referred to in the discussion of the radar equation in Chap. 2 was the average value of the power over the duration of a pulse of sine wave. The ratio R_f is *twice* the average signal-to-noise power ratio when the input signal $s(t)$ is a rectangular sine-wave pulse.] The output voltage of a filter with frequency-response function $H(f)$ is

$$|s_o(t)| = \left| \int_{-\infty}^{\infty} S(f)H(f) \exp(j2\pi f t)\, df \right| \tag{10.8}$$

where $S(f)$ is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df \tag{10.9}$$

where N_0 is the input noise power per unit bandwidth. The factor $\frac{1}{2}$ appears before the integral because the limits extend from $-\infty$ to $+\infty$, whereas N_0 is defined as the noise power per cycle of bandwidth over positive values only. Substituting Eqs. (10.8) and (10.9) into (10.7) and assuming that the maximum value of $|s_o(t)|^2$ occurs at time $t = t_1$, the ratio R_f becomes

$$R_f = \frac{\left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t_1) \, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df} \tag{10.10}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^* P \, dx \int Q^* Q \, dx \geq \left| \int P^* Q \, dx \right|^2 \tag{10.11}$$

The equality sign applies when $P = kQ$, where k is a constant. Letting

$$P^* = S(f) \exp(j2\pi f t_1) \quad \text{and} \quad Q = H(f)$$

and recalling that

$$\int P^* P \, dx = \int |P|^2 \, dx$$

we get, on applying the Schwartz inequality to the numerator of Eq. (10.10),

$$R_f \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df \int_{-\infty}^{\infty} |S(f)|^2 \, df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 \, df}{\frac{N_0}{2}} \tag{10.12}$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |S(f)|^2 \, df = \int_{-\infty}^{\infty} s^2(t) \, dt = \text{signal energy} = E \tag{10.13}$$

Therefore we have

$$R_f \leq \frac{2E}{N_0} \tag{10.14}$$

The frequency-response function which maximizes the peak-signal-to-mean-noise ratio R_f may be obtained by noting that the equality sign in Eq. (10.11) applies when $P = kQ$, or

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1) \tag{10.15}$$

where the constant k has been set equal to $1/G_a$.

The interesting property of the matched filter is that no matter what the shape of the input-signal waveform, the maximum ratio of the peak signal power to the mean noise power is simply twice the energy E contained in the signal divided by the noise power per hertz of bandwidth N_0. The noise power per hertz of bandwidth, N_0, is equal to $kT_0 F$ where k is the Boltzmann constant, T_0 is the standard temperature (290 K), and F is the noise figure.

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*

Unit III - Detection Of Signals In Noise

S-3 Automatic Detection and
Constant-False-Alarm Rate Receivers

Automatic Detection

- Automatic detection is the name applied to the part of the radar that performs the operations required for the detection decision without operator intervention.
- In many respects, automatic detection requires much better receiver design than when an operator makes the detection decision.
- Operators can recognize and ignore clutter echoes and interference that would limit the recognition abilities of some automatic devices.
- An operator might have better discrimination capabilities than automatic methods for sorting clutter and interference;
- but the automatic, computer-based decision devices can operate with far greater number of targets than an operator can handle.

Automatic Detection

Automatic detection of radar signals involves the following:

- Quantization of the radar coverage into range, and maybe angle, resolution cells.
- Sampling of the output of the range-resolution cells with at least one sample per cell,more than one sample when practical.
- Analog-to-digital conversion of the analog samples.
- Signal processing in the receiver to remove as much noise, clutter echoes, and interference as practicable before the detection decision is attempted.
- Integration of the available samples at each resolution cell.

Constant false-alarm rate

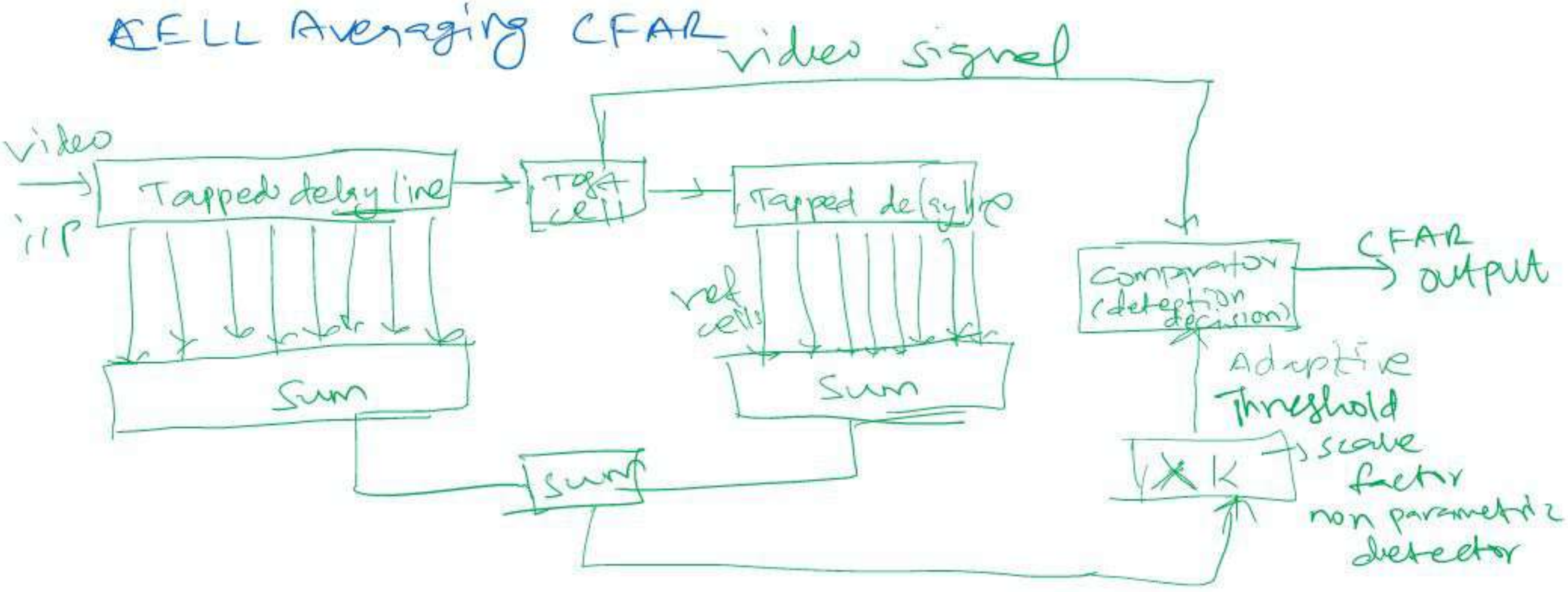
- Constant false-alarm rate (CFAR) circuitry to maintain the false-alarm rate when the receiver cannot remove all the clutter and interference.
- Clutter map to provide the location of clutter so as to ignore known clutter echoes.
- Threshold detection to select target echoes for further processing by an automatic tracker or other data processor.
- Measurement of range and angle after the detection decision is made.

Constant-False-Alarm Rate Receivers

Constant false Alarm Rate Receiver

* Threshold value (internal noise)
  ↳ due to clutter echoes & noise
  amp ↑ false alarm ↑
* To avoid clutter echo & noise proposing CFAR
* Adaptive threshold
* In case of no MTI

Cell Averaging CFAR



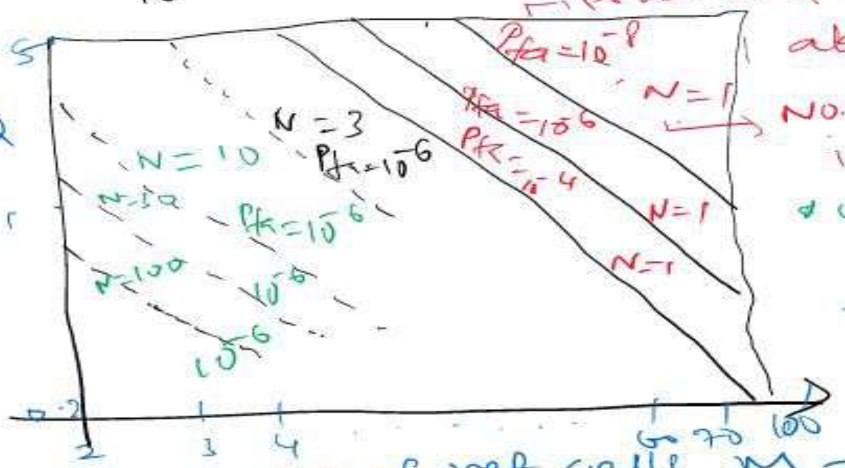
CFAR Loss

* If Ref cells ↑ loss ↓
* 16 - 20 ref cells

ref cell $\xleftarrow{0.5}$ test cell $\xrightarrow{0.5 \text{ mill}}$ ref

& 10   & 10

Theorical CFAR Loss (-dB) ↑



prob. of false alarm error

→ No. of pulses integrated

* when M ↑, Loss ↓,

No. of ref cells M →

$$\text{CFAR Loss} = \frac{\text{SNR required when CFAR is employed}}{\text{SNR required for fixed threshold}}$$

$$\text{CFAR Loss (dB)} = -\frac{5}{M} \log P_{fa} \quad \text{— (Nitzberg)}$$

Clutter edges

Clutter edges :→ threshold is getting lowered

- GTO CFAR
  - ↳ compare ref. cell O/P
    - ↳ threshold is defined
- It provides additional loss of 0.1 to 0.3 dB

Effects of multiple Targets

Effect of multiple targets

* Due to multiple target, threshold ↑
* J cells → multiple target with high amp
  * discard o/p of J cells

(i) * This is called censored-mean level detector

(ii) ordered statistic CFAR
→ M cells → 16 cells
→ order → smallest to largest
1 to 16 → × K → factor
↳ 10th cells value is multiplied with scaling factor

(iii) employ log video to suppress echoes

Range Resolution

Range resolution

* target → 0.8 pulse width } separation b/w two targets
→ 2.5 pulse width }

P.W ↑ due to spill over energy by adjacent cells
→ Antenna beems.

Non parametric detectors

* To choose 'K' factor
* when clutter statistics are unknown
* large CFAR loss

Clutter map
→ azimuthal cells on a rectangular grid

other forms of SKR
→ siebert CFAR → Avg noise level
→ Hard limiter
→ log FTC

Constraints

Constraints:

- lowers the prob. detector
- loss SNR
- poor range resolution
- spoofing

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*

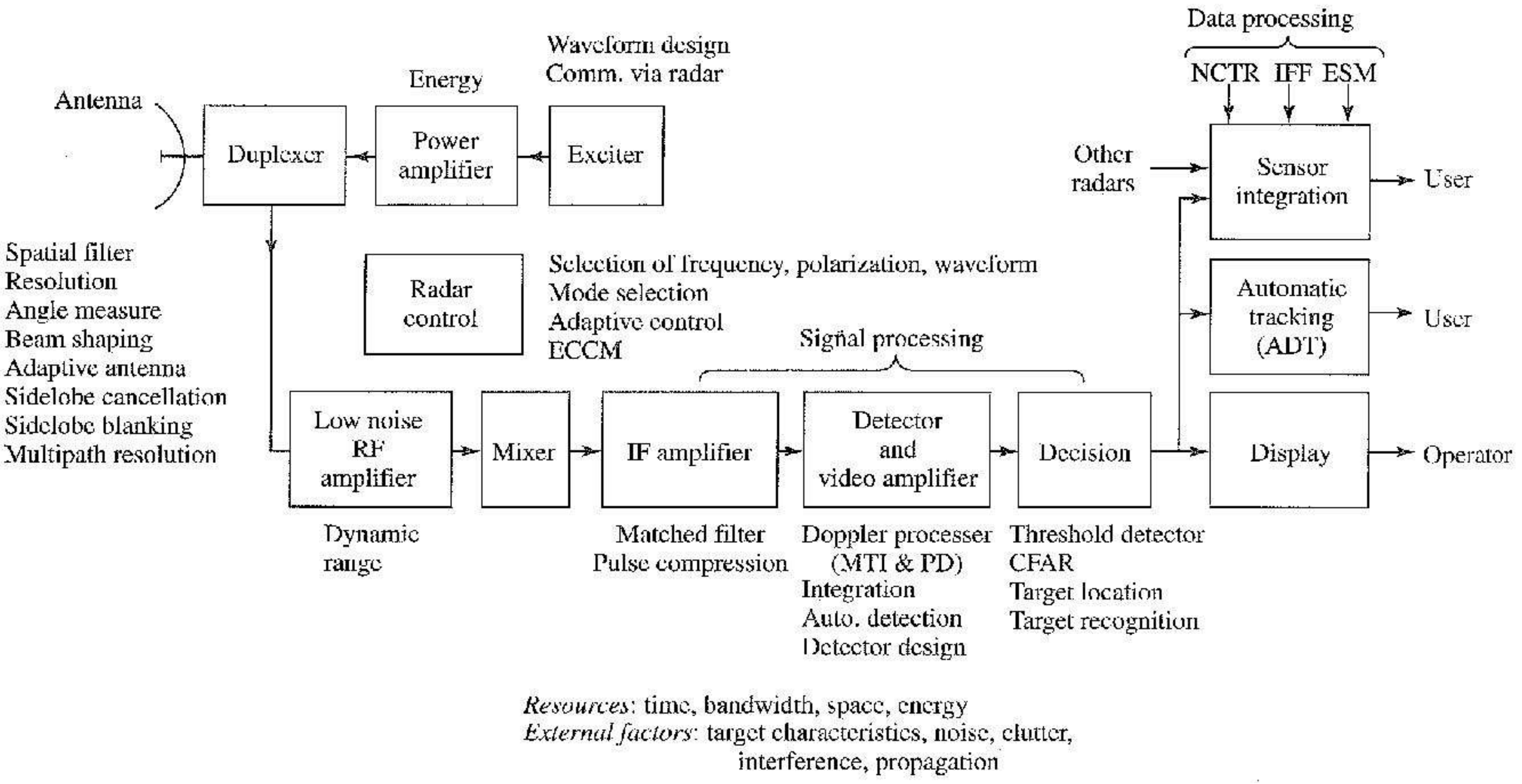
18ECE221T – Module 3 – Session 4

Signal Management

Signal Management

- Everything associated with the waveforms and their processing that is required by a radar
- It deals with design of suitable waveforms

Signal Management – Block Diagram



1. Signal Processing

- Matched Filter – to maximize the signal-to-noise at the output of the radar receiver, for detectability of signals.
- Detector/Integrator – processing of number of pulses received
- Clutter reduction –
- CFAR – to maintain constant false alarm rate
- Electromagnetic Compatibility – Elimination of Interference
- Electronic counter-countermeasures (ECCM) – to reduce or eliminate the effectiveness of jamming or deception

2. Data Processing

- To get further information about the target
  - Target Location – in range, angle, and radial velocity
  - Target Trajectory – or target track
  - Target recognition – to differentiate ship from plane
  - Weapon control – use the output of radar to control the guidance of missiles.

3. Waveform Design

- Waveform detection depends what radar wants to do? Detection in presence of noise, clutter, interference and ECCM
- Factors
  1. Antenna design
  2. Automatic Radar Control
  3. Sensor Integration

4. Resources & Constraints

Resources

1. Energy – Power * Time
2. Bandwidth
3. Time – time required to measure Doppler frequency
4. Space – Physical aperture area required for an antenna

Constraints

5. Unable to get Sl. No. 4 to the requirement
6. Microwave frequency atmospheric attenuation
7. Atmospheric refraction
8. Size, space and weight

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*

18ECE221T – Module 3 – Session 5

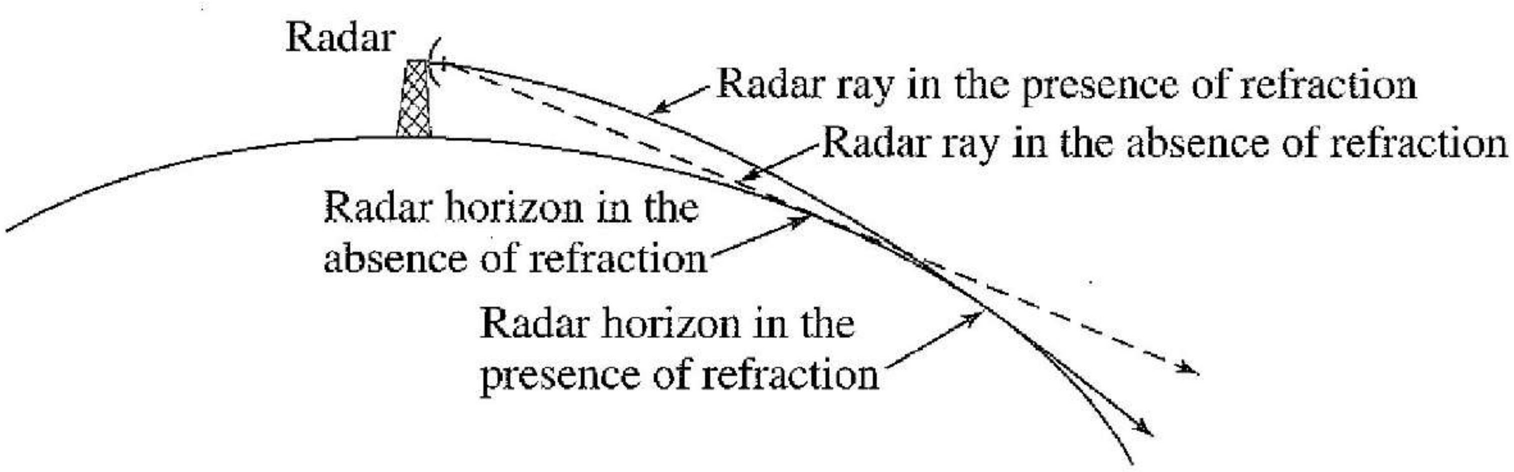
Propagation Radar Waves- Atmospheric Refraction -
Standard propagation, Non
Standard propagation

Atmospheric Refraction

- Radar waves travel in straight lines in free space. Propagation in the earth's atmosphere however, is not in free space.
- The atmosphere is not uniform; hence, it causes electromagnetic waves to be bent, or refracted.

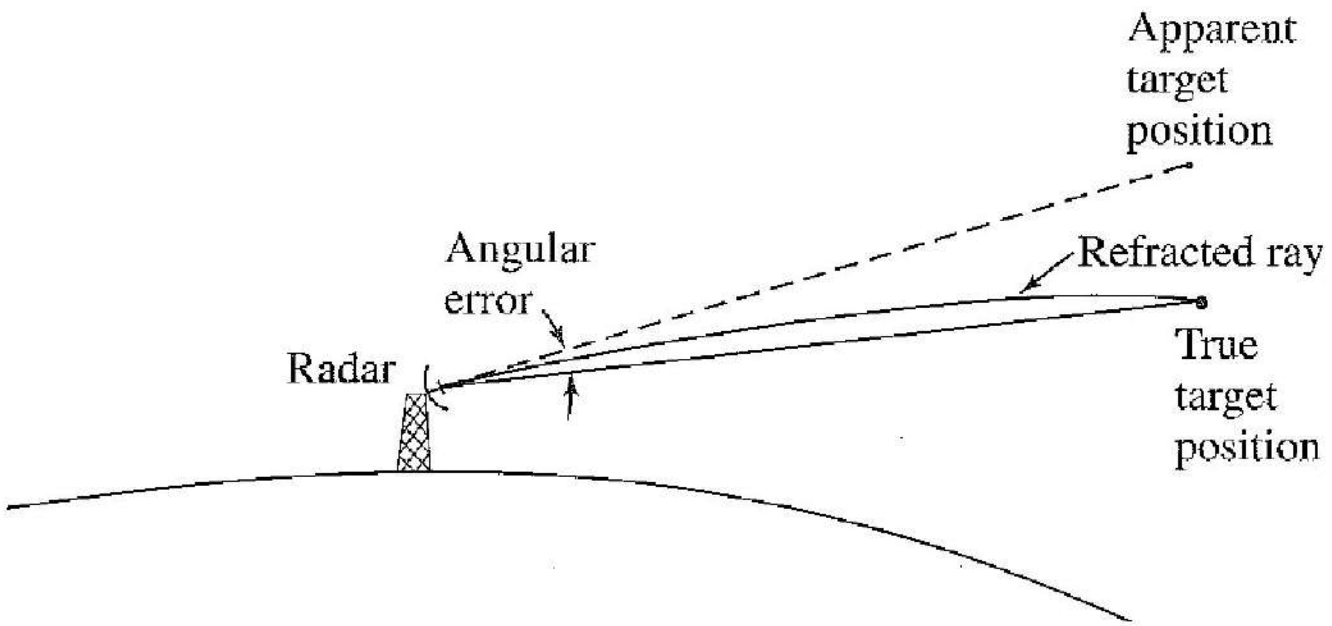
Extension of the radar horizon due to refraction of radar rays by the atmosphere

- Normally the effect of bending caused by atmospheric refraction is favorable, in that it causes the radar horizon to be extended and increases the coverage of a radar beyond the geometrical horizon



Angular error caused by atmospheric refraction.

- On the other hand, the bending of the rays by the atmosphere can introduce an error in the measurement of the elevation angle.



Standard Propagation

- **Refractivity**
  - Effective Earth Radius
  - Distance to the horizon
- **Radar Measurement Errors due to refraction**

Refractivity

- Refractivity Refraction of radar waves in the atmosphere is due to the variation of the velocity of propagation with altitude.
- The index of refraction (n=1.000350 at earth surface) is a measure of the velocity of propagation and is defined as the velocity in free space divided by the velocity in the medium (atmosphere).

At microwave frequencies, the refractivity N for air is given by the empirical relation[14,15]

$$N = (n - 1) \cdot 10^6 = \frac{77.6}{T}\left[p + \frac{4810e}{T}\right] \qquad [8.15]$$

where

p = barometric pressure, mbar (1 mm Hg = 1.3332 mbar)
e = partial pressure of water vapor, mbar
T = absolute temperature, K

- Atmospheric refractivity depends on the pressure, temperature, and water vapor.
- Of these, water vapor is the most important at microwave frequencies.
- It strongly affects the speed of microwave propagation.
- Temperature variations are more significant than pressure variations.

Effective Earth Radius

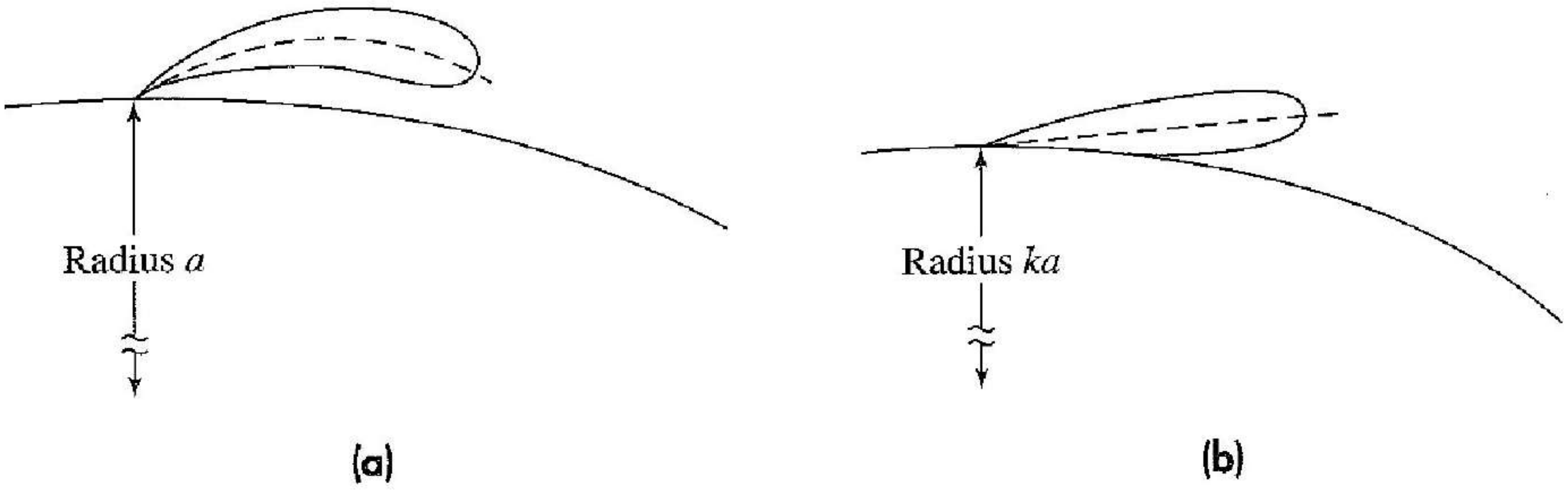


Figure 8.8 (a) Bending of the antenna beam due to refraction by the earth's atmosphere; (b) shape of the beam in the equivalent-earth representation with radius *ka*.

Distance to the Horizon

Distance to the Horizon The distance d to the horizon from a radar antenna at a height h may be shown from simple geometrical considerations to be

$$d = \sqrt{2kah} \tag{8.17a}$$

where ka is the effective earth's radius and the height h above the surface is assumed to be small compared to the real earth's radius a. For a four-thirds earth, this relationship becomes

$$d \text{ (nautical miles)} = 1.23\sqrt{h(\text{ft})} \tag{8.17b}$$

or,

$$d \text{ (km)} = 4.12\sqrt{h(\text{m})} \tag{8.17c}$$

Radar Measurement Errors due to refraction

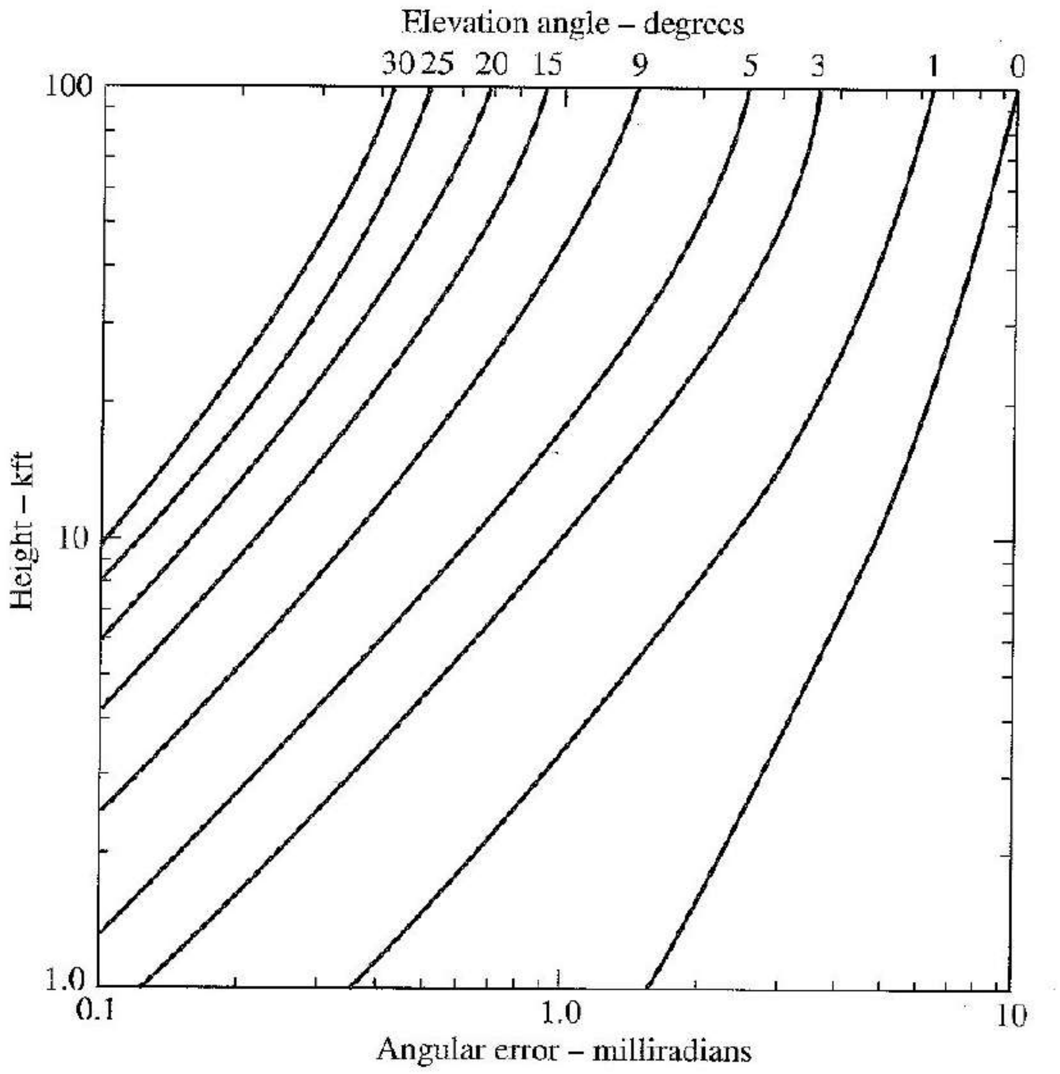


Figure 8.11 Calculated angle error (abscissa) due to atmospheric refraction for a standard atmosphere as a function of elevation angle and target height (ordinate.) (After Shannon.[25])

A correction for the range error due to refraction, suggested by G. Robertshaw,[31] is given by the expression

$$R_c \cong 0.42 + 0.0577\, R_t\left(\frac{N_s}{h}\right)^{1/2} \tag{8.19}$$

where

R_c = range correction, m
R_t = radar range, km
N_s = surface refractivity in N-units
h = radar altitude in kft

Non standard Propagation

- Evaporation Ducts
- Surface based Ducts
- Elevated Ducts
- Consequences of Ducted propagation on Radar Performance

Non standard Propagation

- Refractive effects can be much more complex than described by the standard exponential model, and can cause significant changes in radar propagation. Such conditions are known as anomalous, or nonstandard, propagation.
- Normal refraction occurs when the refractive gradient with height, dN/dh, is between 0 and —79 N units per km of height.
- Rays that are initially horizontal will then follow the curvature of the earth.
- Under such conditions, the radar range is significantly increased and detection beyond the radar horizon can result Refractive gradients between —79 and —157 N/km result in what is called **super refraction**.

- When the gradients exceed — 157 N/km, the curvature of the propagating ray exceeds the curvature of the earth and ducts can form that trap the radar energy.
- The trapped energy within the duct can propagate to ranges well beyond the normal horizon.
- If the refractive gradient were to increase with height, instead of the more usual decrease, the propagating rays would curve upward and the radar range would decrease as compared to normal conditions. This is called **subrefraction.**

Table 8.1 Summary of Refractive Propagation Conditions

Refractive Condition	Gradient: N units per km
Subrefraction	Positive gradient
No refraction (uniform atmosphere)	0
Standard refraction (4/3rd earth radius)	-39
Normal refraction	0 to -79
Superrefraction	-79 to -157
Trapping, or ducting	-157 to $-\infty$

Evaporation Ducts

- The air in contact with the sea surface is usually saturated with water vapor so that its relative humidity is almost 100 percent.
- The air several meters above the sea surface is not usually saturated so there will be a decrease in humidity from the surface value to the ambient value determined by the general meteorological conditions well above the surface.
- The rapid decrease of water vapor causes a rapid decrease of refractivity that results in the formation of a low-lying duct that traps the radar energy so that it propagates close to the sea surface.
- Ducting can cause the radar ranges for targets at or near the sea surface to be considerably greater than the free-space range.

Multiple mode Propagation

- If the duct height is large enough, more than one mode can be propagated. (A mode is a configuration of electric and magnetic field distribution, similar to the modes of propagation in a conventional waveguide.)
- Multiple propagation modes have two consequences:

(1) the signal strength will not vary uniformly through out the duct and

(2) there can be more than one antenna height suitable for low-loss propagation.

Surface based Ducts

- Surface-based ducts are formed when the upper air is exceptionally warm and dry compared with the air at the surface.
- Over land, a surface-based duct can be caused by the radiation of heat from the earth on clear nights, especially in the summer when the ground is moist.
- The earth loses heat and its surface temperature falls, but there is little or no change in the temperature of the upper atmosphere.
- This leads to a temperature inversion at the ground and a sharp decrease in moisture with height.
- Thus over land, ducting is most noticeable at night and usually disappears during the warmest part of the day.
- Another cause of surface-based ducts is the movement (advection) of warm dry air, from land, over cooler bodies of water

Elevated Ducts

- The base of an elevated duct lies above the surface of the earth. Elevated ducts can occur in the trade wind regions between the mid-ocean high-pressure cells and the equator.
- Meteorological conditions necessary for an elevated duct are similar to those for a surface-based duct. Under the proper conditions, one can turn into the other.
- Although enhanced propagation can occur when the target and the radar are properly located with respect to the duct, it is also possible to obtain reduced or no coverage above or below the duct, compared to that expected with a standard atmosphere.
- This lack of coverage due to the duct is called a radar hole.

Consequences of Ducted propagation on Radar Performance

- Radar Holes which prevent a radar from seeing targets in some direction
- The loss of detectability caused by radar holes can affect airborne radars as well as ground-based and shipborne radars.
- Ducted propagation can make possible the detection of unwanted clutter echoes at long ranges that might otherwise not be detected in a normal atmosphere.
- Also, multiple-time-around clutter echoes that arrive from beyond the maximum unambiguous range might not be eliminated if the doppler processing employs pulse-to-pulse staggered repetition periods

- In areas of the world where surfaced-based ducts can significantly extend the radar horizon for a large portion of the time, the greatly increased clutter echoes that are obtained can seriously degrade the performance of a radar not designed to cope with it.
- It should be designed with a large dynamic range so as to avoid receiver saturation by large clutter echoes, the radar should have additional MTI or pulse Doppler improvement factor to eliminate the larger than normal clutter, and the radar waveforms and processing should be designed to cancel multiple-time-around clutter echoes that originate from long ranges.
- The last mentioned is accomplished by using a constant prf (instead of pulse-to-pulse prfs) with processing that uses the required number of "fill pulses.“
- (Fill pulses are those given zero weight in the digital MTI processor (that is, they are discarded) so as to eliminate pulse repetition intervals that do not have multiple-time-around clutter. “)

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*

18ECE221T – Module 3 – Session 6

Ambiguity Diagram

Ambiguity diagram

The ambiguity diagram represents the response of the matched filter to the signal for which it is matched as well as to doppler-frequency-shifted (mismatched) signals. Although it is seldom used as a basis for practical radar system design, it provides an indication of the limitations and utility of particular classes of radar waveforms, and gives the radar designer general guidelines for the selection of suitable waveforms for various applications.

The output of the matched filter was shown in Sec. 10.2 to be equal to the cross correlation between the received signal and the transmitted signal [Eq. (10.18)]. When the received echo signal from the target is large compared to noise, this may be written as

$$\text{Output of the matched filter} = \int_{-\infty}^{\infty} s_r(t)s^*(t - T'_R)\,dt \tag{11.46}$$

where $s_r(t)$ is the received signal, $s(t)$ is the transmitted signal, $s^*(t)$ is its complex conjugate, and T'_R is the estimate of the time delay (considered a variable). Complex notation is assumed in Eq. (11.46). The transmitted signal expressed in complex form is $u(t)e^{j2\pi f_0 t}$, where $u(t)$ is the

Ambiguity diagram

complex-modulation function whose magnitude $|u(t)|$ is the envelope of the real signal, and f_0 is the carrier frequency. The received echo signal is assumed to be the same as the transmitted signal except for the time delay T_0 and a doppler frequency shift f_d. Thus

$$s_r(t) = u(t - T_0)e^{j2\pi(f_0 + f_d)(t - T_0)} \tag{11.47}$$

(The change of amplitude of the echo signal is ignored here.) With these definitions the output of the matched filter is

$$\text{Output} = \int_{-\infty}^{\infty} u(t - T_0)e^{j2\pi(f_0 + f_d)(t - T_0)}[u(t - T'_R)e^{j2\pi f_0(t - T'_R)}]^* \, dt$$

$$= \int_{-\infty}^{\infty} u(t - T_0)u^*(t - T'_R)e^{j2\pi(f_0 + f_d)(t - T_0)} \, e^{-j2\pi f_0(t - T'_R)} \, dt \tag{11.48}$$

Ambiguity diagram

It is customary to set $T_0 = 0$ and $f_0 = 0$, and to define $T_0 - T'_R = -T'_R = T_R$. The output of the matched filter is then

$$\chi(T_R, f_d) = \int_{-\infty}^{\infty} u(t)u^*(t + T_R)e^{j2\pi f_d t}\, dt \tag{11.49}$$

In this form a positive T_R indicates a target beyond the reference delay T_0, and a positive f_d indicates an incoming target.[13] The squared magnitude $|\chi(T_R, f_d)|^2$ is called the *ambiguity function* and its plot is the *ambiguity diagram.*

The ambiguity diagram has been used to assess the properties of the transmitted waveform as regards its target resolution, measurement accuracy, ambiguity, and response to clutter.

Properties

maximum value: $|\chi(T_R,f_d)|_{\max}{}^2 = |\chi(0,0)|^2 = (2E)^2$ [6.42]

symmetry relation: $|\chi(-T_R,-f_d)|^2 = |\chi(T_R,f_d)|^2$ [6.43]

behavior on T_R axis: $|\chi(T_R,0)|^2 = |\int u(t)u^*(t+T_R)\,dt|^2$ [6.44]

behavior on f_d axis: $|\chi(0,f_d)|^2 = |\int u^2(t)e^{j2\pi f_d t}\,dt|^2$ [6.45]

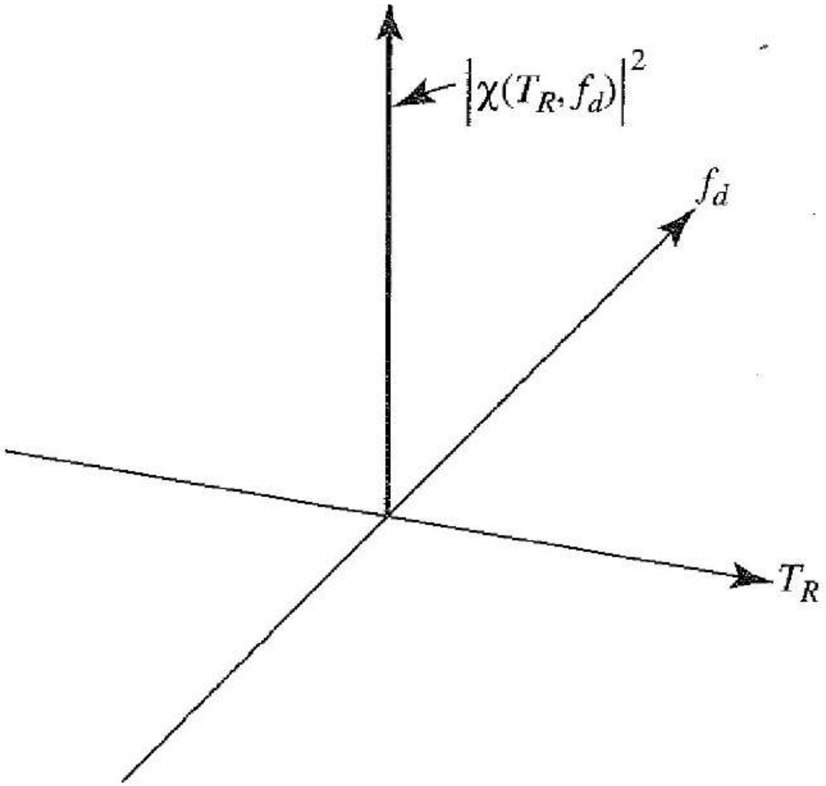
volume under surface: $\iint |\chi(T_R,f_d)|^2\,dT_R\,df_d = (2E)^2$ [6.46]

Properties

Equation (6.42) states that the maximum value of the ambiguity function occurs at the origin, which is the true location of the target when the doppler shift $f_d = 0$. Its maximum value is $(2E)^2$, where E is the energy contained in the echo signal. Equation (6.43) is a symmetry relation. Equation (6.44) is the form of the ambiguity function on the time-delay axis. It is the square of the autocorrelation function of $u(t)$. Equation (6.45) describes the behavior on the frequency axis and is the square of the inverse fourier transform of $[u(t)]^2$. The total volume under the ambiguity diagram is given by Eq. (6.46) and is a constant, also equal to $(2E)^2$. (All limits in the above equations go from $-\infty$ to $+\infty$.)

Ideal ambiguity

Ideal ambiguity diagram. If there were no theoretical restrictions, the ideal ambiguity diagram would consist of a single peak of infinitesimal thickness at the origin and be zero everywhere else, as shown in Fig. 11.7. The single spike eliminates any ambiguities, and its infinitesimal

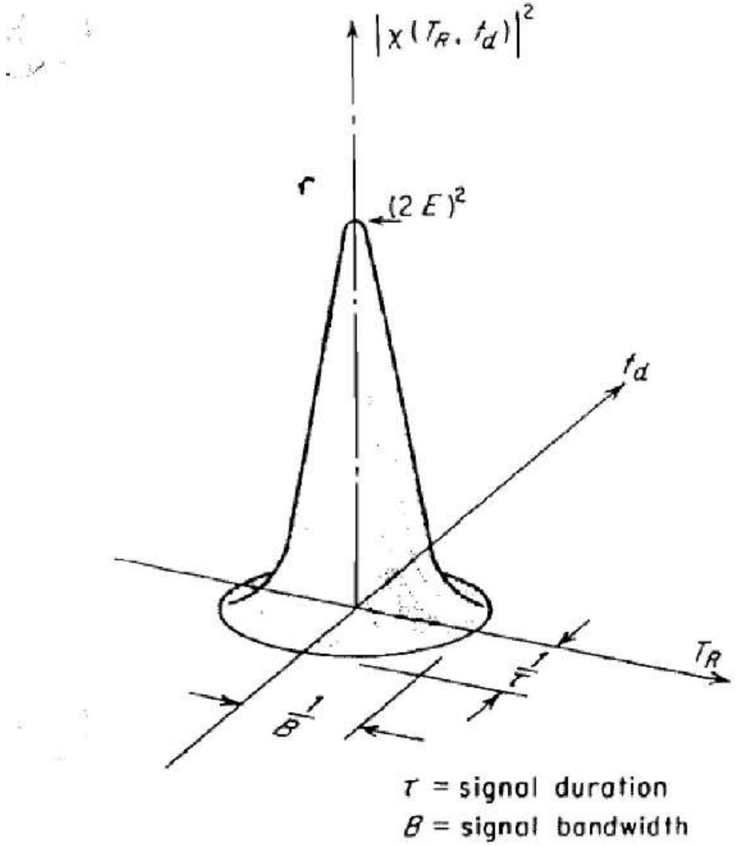


Ideal ambiguity

thickness at the origin permits the frequency and the echo delay time to be determined simultaneously to as high a degree of accuracy as desired. It would also permit the resolution of two targets no matter how close together they were on the ambiguity diagram. Naturally, it is not surprising that such a desirable ambiguity diagram is not possible. The fundamental properties of the ambiguity function prohibit this type of idealized behavior. The two chief restrictions are that the maximum height of the $|\chi|^2$ function be $(2E)^2$ and that the volume under the surface be finite and equal $(2E)^2$. Therefore the peak at the origin is of fixed height and the function encloses a fixed volume. A reasonable approximation to the ideal ambiguity diagram might appear as in Fig. 11.8. This waveform does not result in ambiguities since there is only one peak, but the single peak might be too broad to satisfy the requirements of accuracy and resolution. The peak might be narrowed, but in order to conserve the volume under its surface, the function must be raised elsewhere. If the peak is made too narrow, the requirement for a constant volume might cause peaks to form at regions of the ambiguity diagram other than the origin and give rise to ambiguities. Thus the requirements for accuracy and ambiguity may not always be possible to satisfy simultaneously.

Ideal ambiguity

Figure 11.8 An approximation to the ideal ambiguity diagram, taking account the restrictions imposed by the requirement for a fixed value of $(2E)^2$ at the origin and a constant volume enclosed by the $|\chi|^2$ surface.



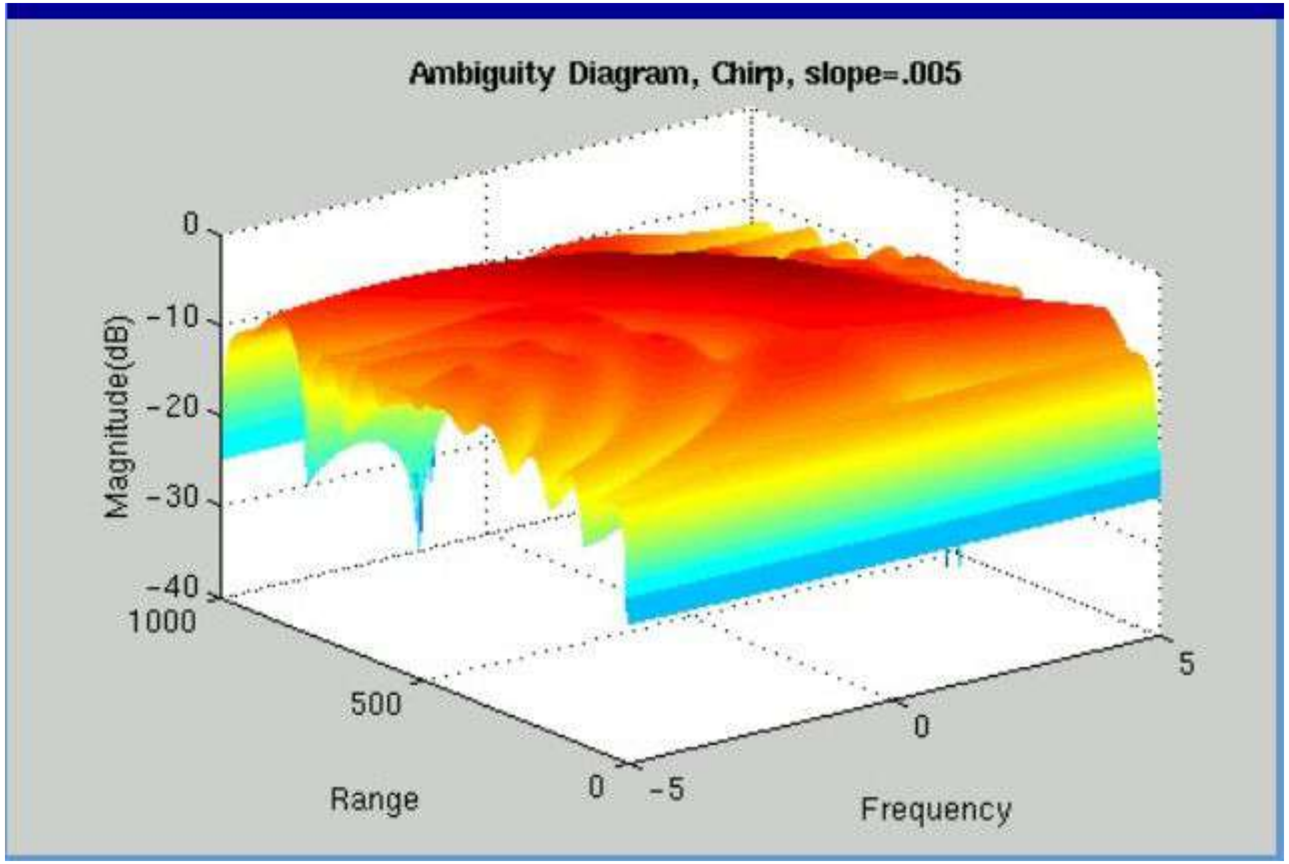
Ideal ambiguity

The ambiguity diagram in three dimensions may be likened to a box of sand. The total amount of sand in the box is fixed and corresponds to a fixed signal energy. No sand can be added, and none can be removed. The sand may be piled up at the center (origin) to as narrow a pile as one would like, but its height can be no greater than a fixed amount $(2E)^2$. If the sand in the center is in too narrow a pile, the sand which remains might find itself in one or more additional piles, perhaps as big as the one at the center.

The optimum waveform is one which has the desired ambiguity diagram for a given amount of "sand" (energy). The usual pulse radar or the usual CW radar, as we shall see, does not result in an ideal diagram. To produce an ambiguity diagram such as that shown in Fig. 11.8, the transmissions must be noiselike.

The synthesis of the waveform required to satisfy the requirements of accuracy, ambiguity, and resolution as determined by the ambiguity diagram is a difficult task. The usual design procedure is to compute the ambiguity diagram for the more common waveforms and to observe its behavior. Because of the limitations of synthesis, the ambiguity diagram has been more a measure of the suitability of a selected waveform than a means of finding the optimum waveform.

Ambiguity diagram



Waveform Design and the Ambiguity Diagram

- The waveform transmitted by a radar can affect

(1) target detection,

(2) measurement accuracy,

(3) resolution,

(4) ambiguities, and

(5) clutter rejection.

- The ambiguity diagram may be used to assess qualitatively how well a particular waveform achieves these capabilities.

Waveform Design and the Ambiguity Diagram

Detection:

- The ambiguity diagram is seldom used to assess the detection capability of a particular waveform, except to note if the signal contains sufficient energy.

Accuracy:

- The accuracy with which the range and radial velocity can be measured is indicated by the main response at the origin.
- The width along the time axis determines the range (time delay) accuracy, and the width along the frequency axis determines the radial-velocity accuracy.

Waveform Design and the Ambiguity Diagram

Resolution:

- The width of the central response also determines the resolution ability of a radar waveform in range and radial velocity.
- In order to resolve closely spaced targets, the central response must be isolated. There cannot be extraneous peaks near the main response that would mask the echo from a nearby target.

Ambiguity:

- Ambiguities occur in radar measurements when the waveform is not continuous.
- For example, a pulse train is not continuous; hence, such a waveform can produce ambiguities in range (time delay) and in velocity (frequency).

Waveform Design and the Ambiguity Diagram

- **Clutter Attenuation**

  - Resolution in range and velocity can enhance the target signal echo relative to nearby distributed clutter echoes.

  - The ambiguity diagram can indicate the ability of a waveform to reject clutter by superimposing the locations of the clutter echoes (as stored in a clutter map) on the TR,fd plane of the ambiguity diagram.

  - If the radar is to have good clutter rejection, the ambiguity diagram should have little or no response in regions of high clutter echoes.

  - The short-pulse waveform, for example, will reduce stationary clutter that is extensive in range since the ambiguity function for this waveform has little response on the time (range) axis.

Reference

- *Merrill I. Skolnik," Introduction to Radar Systems", 3rd Edition Tata Mc Graw-Hill 2008*