Signals & Systems

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Chapter 8 Communication Systems

Communication:
Transmission / receiving a signal

Modulation:
The general process of embedding an information-bearing signal into a second signal.

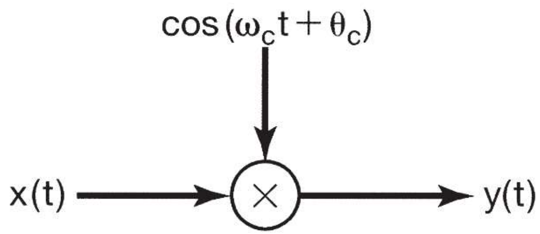
Amplitude modulation (AM):
Info of the input signal is embedded in the amplitude of the modulation output

Frequency modulation (FM):
Info of the input signal is embedded in the frequency of the modulation output.

Demodulation:
Extracting the information-bearing signal from the modulation output.

Multiplexing:
Making possible the simultaneous transmission of more than one signal.

Sect. 8.1 Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation



- Amplitude Modulation

$$y(t) = x(t)c(t)$$

$x(t)$: modulating signal (the input of modulation)

$c(t)$: carrier signal with carrier frequency ω_c

$y(t)$: modulated signal (the output of modulation)

$$c(t) = e^{j(\omega_c t + \theta_c)} \quad \text{or} \quad c(t) = \cos(\omega_c t + \theta_c).$$

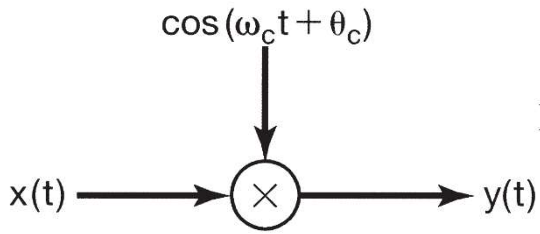
choose $\theta_c = 0$, so that the modulated signal is $y(t) = x(t)e^{j\omega_c t}$.

denoting the Fourier transforms of *x*(*t*), *y*(*t*), and *c*(*t*), respectively,

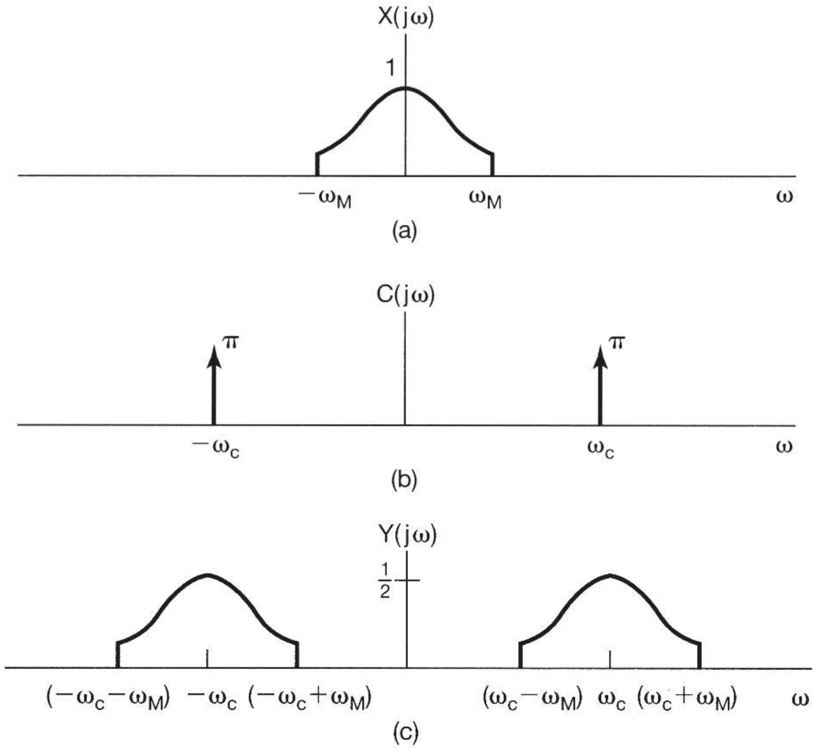
$$Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)C(j(\omega - \theta))d\theta.$$

- Amplitude Modulation (cont'd)

$$C(j\omega) = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)], \quad \theta_c = 0$$



$$Y(j\omega) = \frac{1}{2}[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)].$$



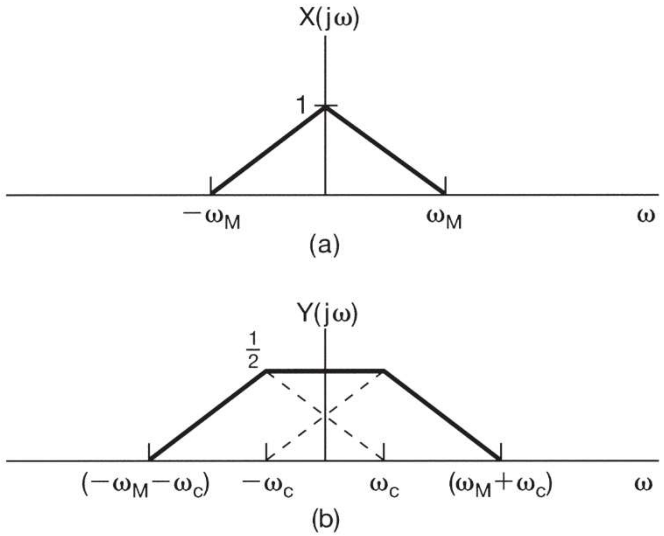
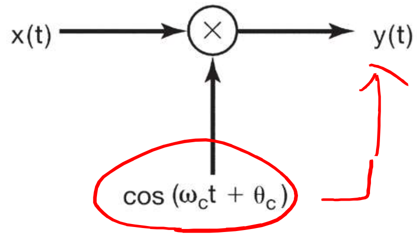


Figure 8.5 Sinusoidal amplitude modulation with carrier $\cos \omega_c t$ for which $\omega_c = \omega_M/2$: (a) spectrum of modulating signal; (b) spectrum of modulated signal.

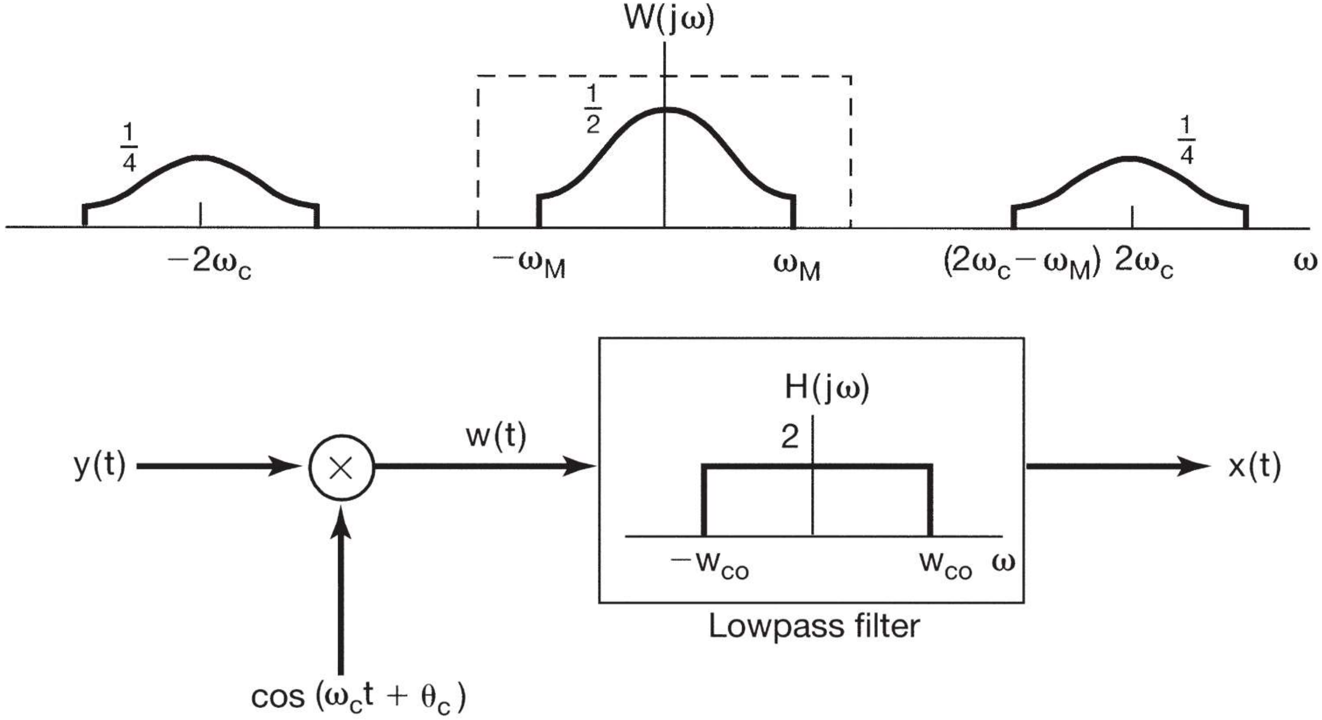
Sect. 8.2 Demodulation for Sinusoidal AM



(a)

- Synchronous Demodulation of $y(t) = x(t)\cos\omega_c t.$

$$w(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t.$$



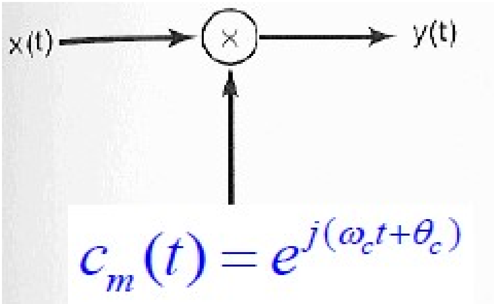
(b)

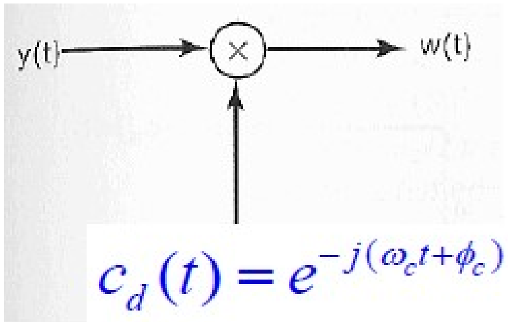
Sect. 8.2 Demodulation for Sinusoidal AM

- Synchronous or Asynchronous Demodulation

$$y(t) = e^{j(\omega_c t+\theta_c)}x(t),$$

$$w(t) = e^{-j(\omega_c t+\phi_c)}y(t), \qquad w(t) = e^{j(\theta_c-\phi_c)}x(t).$$

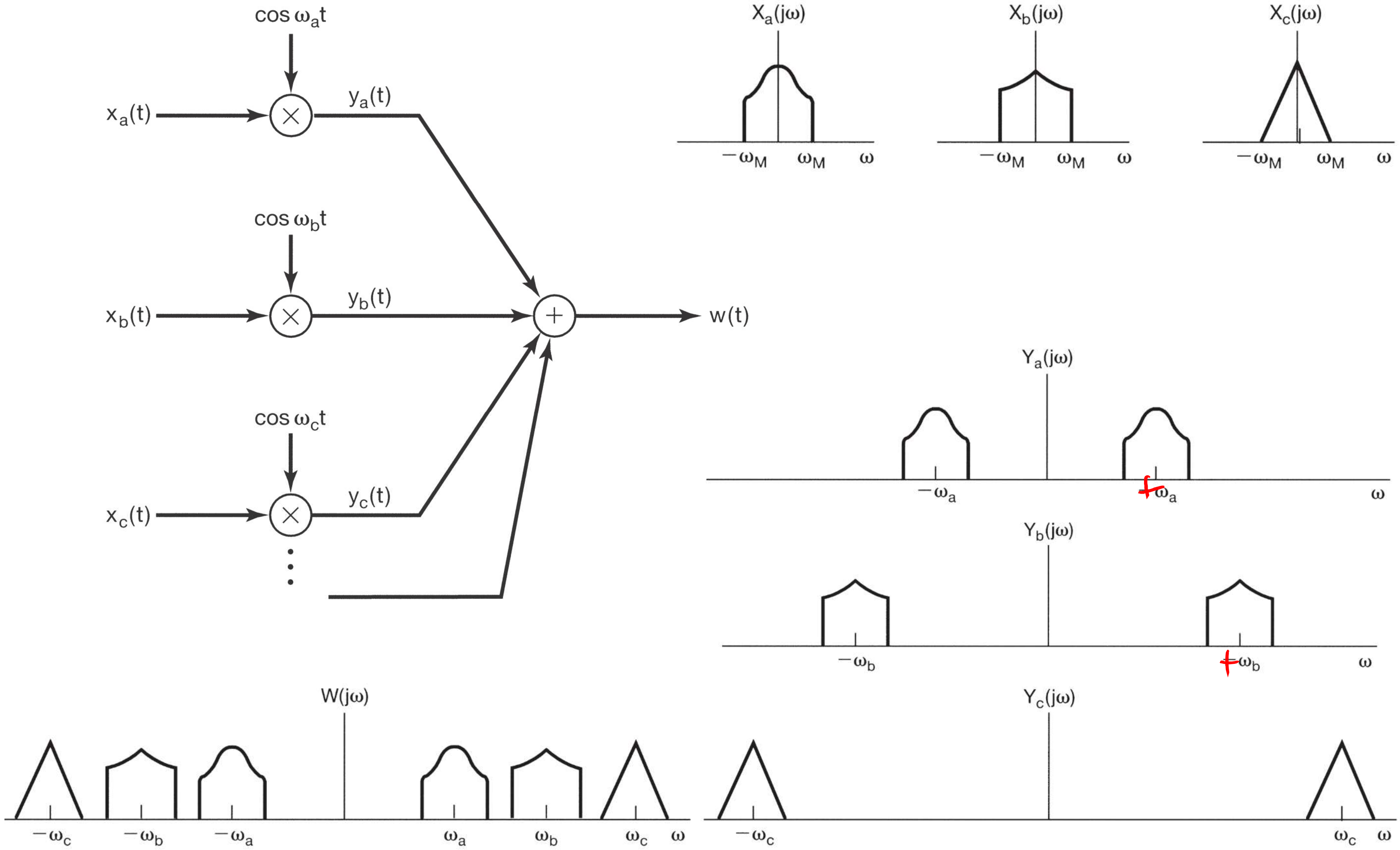




$\Rightarrow$ Only ensure $|x(t)| = |\omega(t)|$

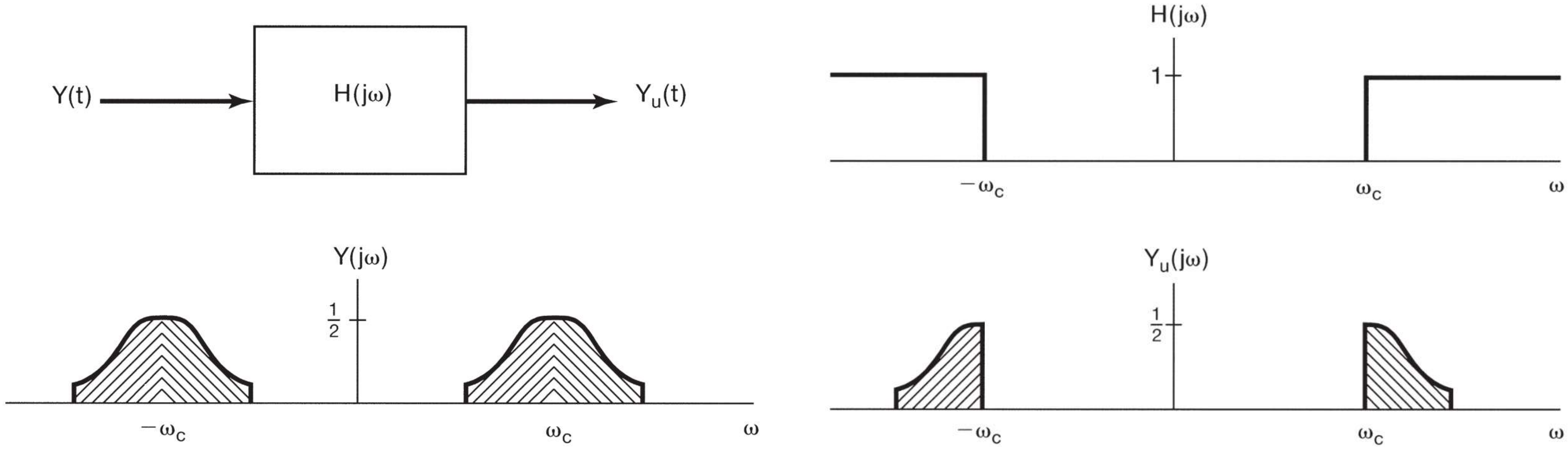
$\Rightarrow$ If $x(t) > 0$, we get $x(t) = |\omega(t)|$

Sect. 8.3 Frequency Division Multiplexing



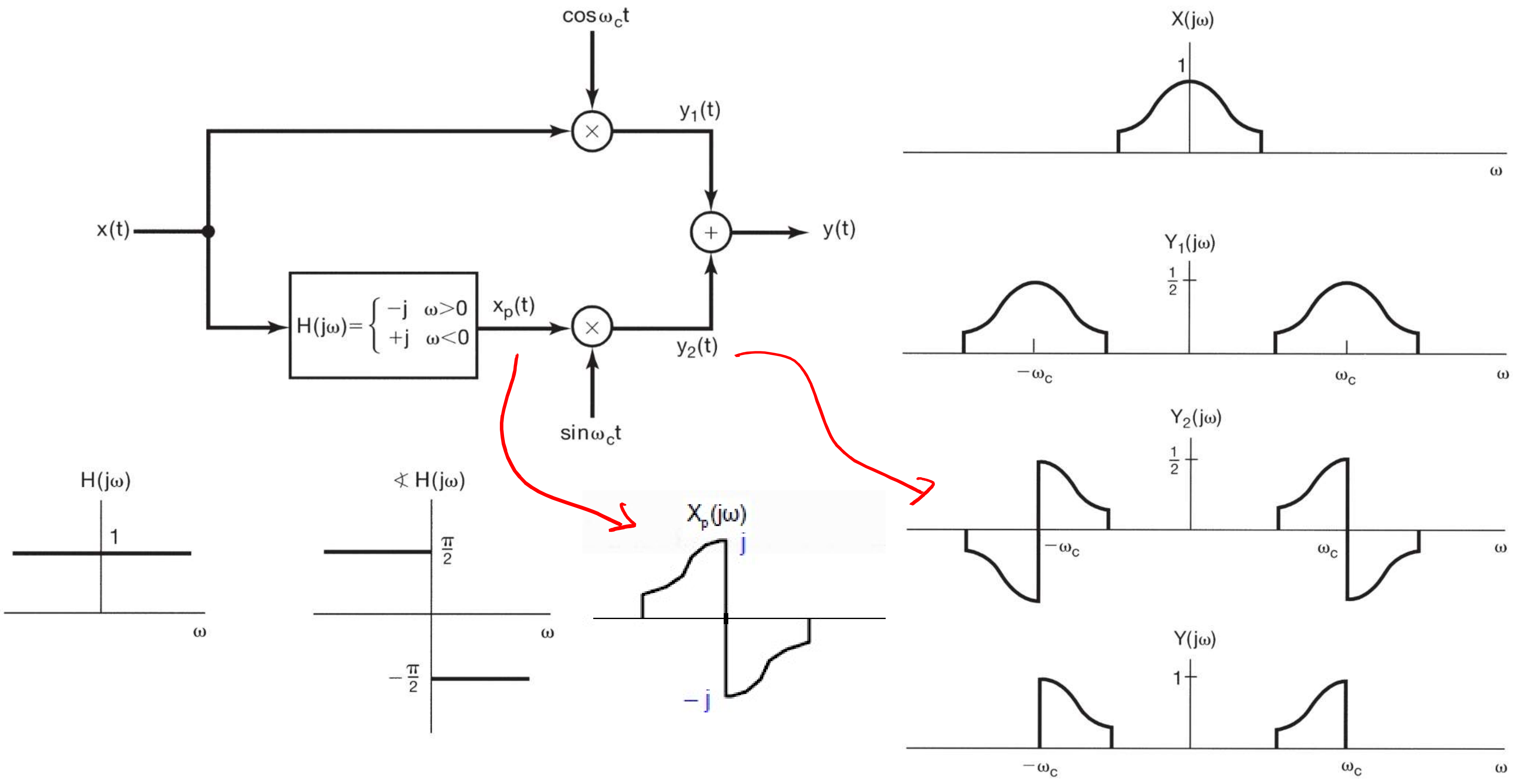
Sect. 8.4 Single-Sideband Sinusoidal AM

- Generating Sidebands Using **Ideal HPF**

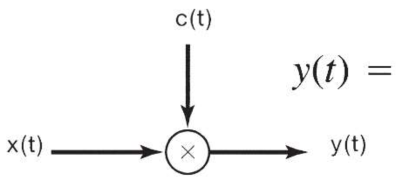


Sect. 8.4 Single-Sideband Sinusoidal AM

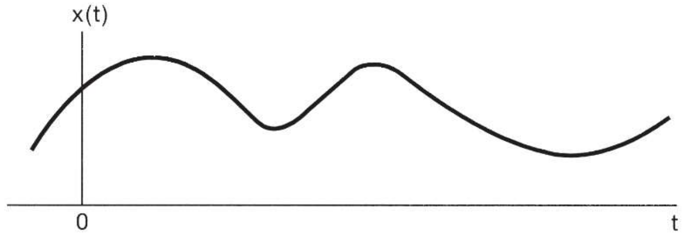
- Generating Sidebands Using **Phase Shifting**

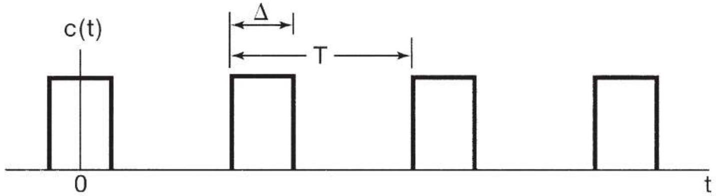


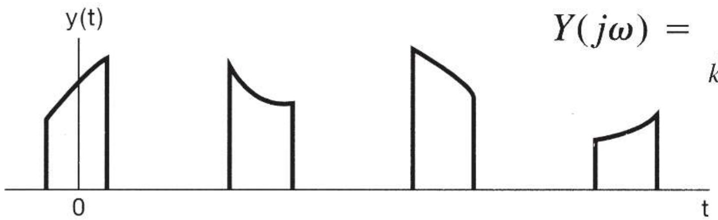
Sect. 8.5 AM with a Pulse-Train Carrier

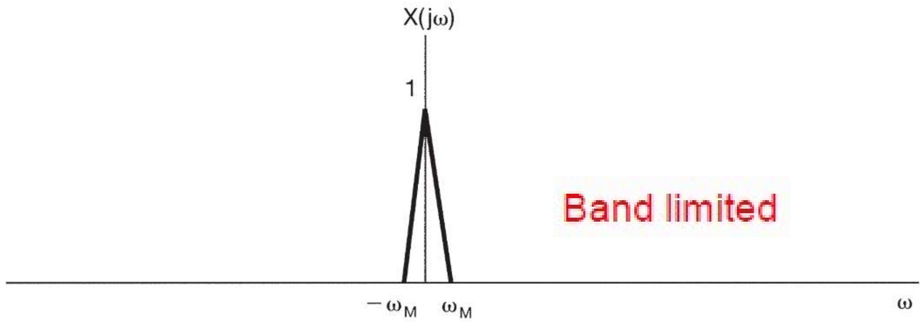


$$y(t) = x(t)c(t);$$







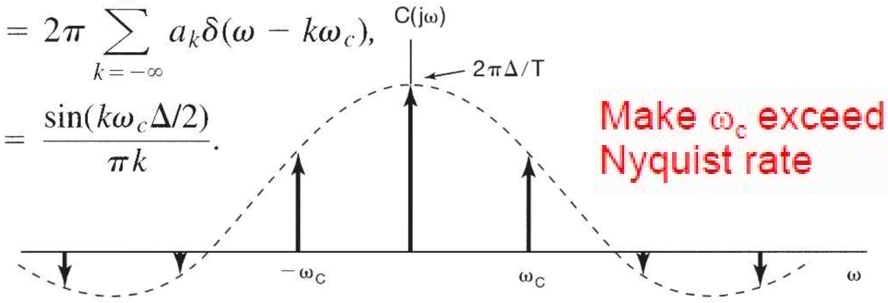


Band limited

(a)

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c),$$

$$a_k = \frac{\sin(k\omega_c \Delta/2)}{\pi k}.$$

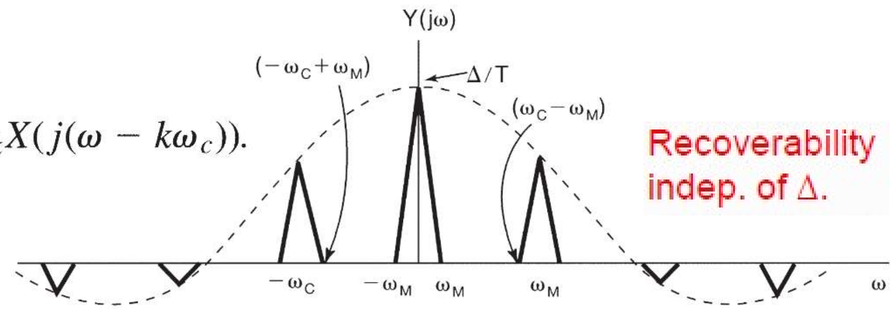


Make ω_c exceed Nyquist rate

(b)

(b)

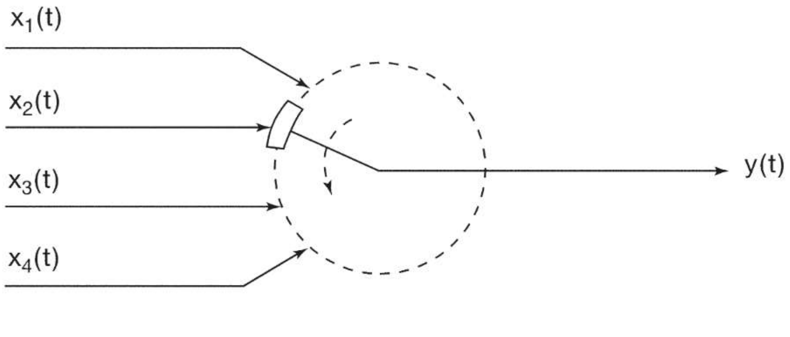
$$Y(j\omega) = \sum_{k=-\infty}^{+\infty} a_k X(j(\omega - k\omega_c)).$$



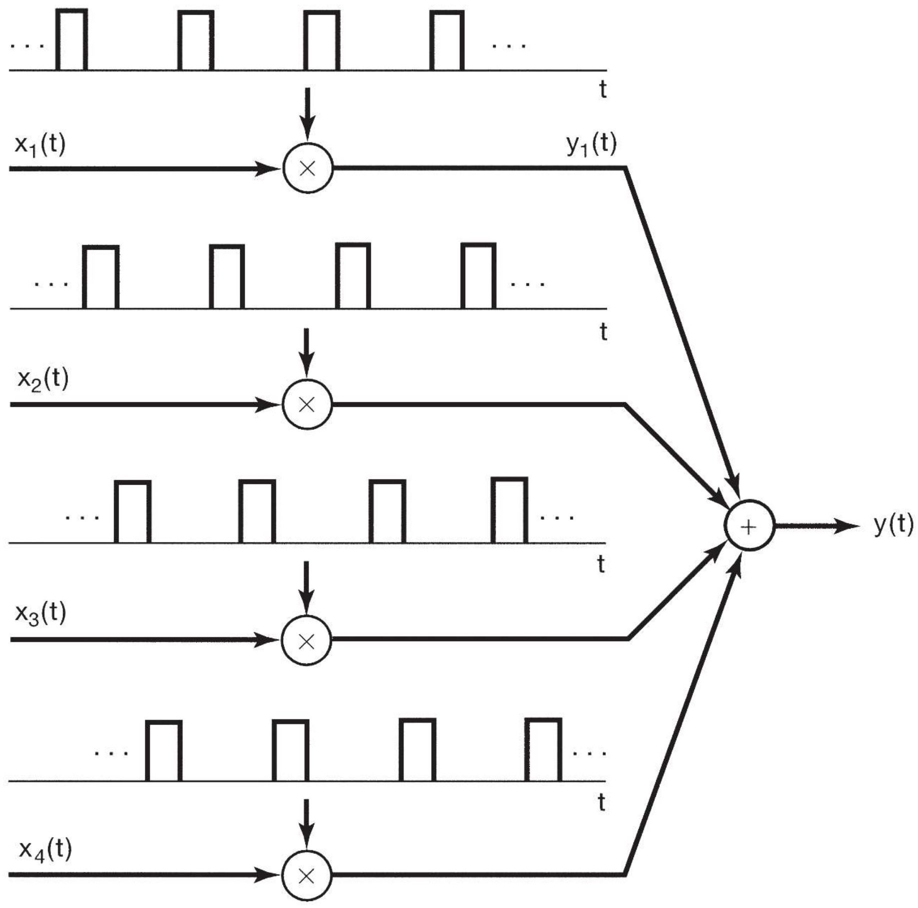
Recoverability indep. of Δ.

(c)

8.5.2 Time-Division Multiplexing (TDM)



(a)



(b)

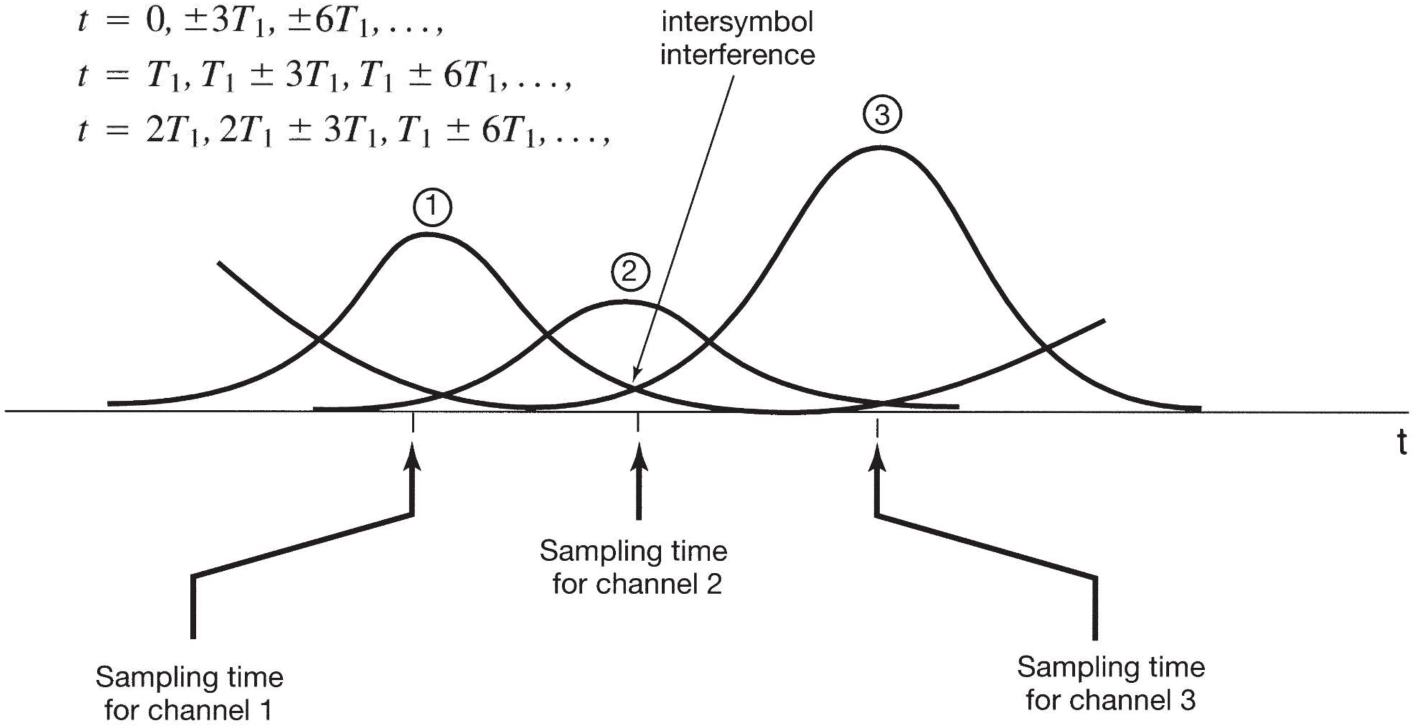
Sect. 8.6 Pulse Amplitude Modulation

- Intersymbol Interference in PAM Systems

$$y(t) = Ax_1(t), \qquad t = 0, \pm 3T_1, \pm 6T_1, \ldots,$$

$$y(t) = Ax_2(t), \qquad t = T_1, T_1 \pm 3T_1, T_1 \pm 6T_1, \ldots,$$

$$y(t) = Ax_3(t), \qquad t = 2T_1, 2T_1 \pm 3T_1, T_1 \pm 6T_1, \ldots,$$



Filtering due to non-ideal frequency response of the channel causes a smearing of the pulses, which can cause the received pulses to overlap in time. This is referred to as intersymbol interference.

Sect. 8.7 Sinusoidal Frequency Modulation

- Frequency Modulation
  - Modulating signal controls the **frequency** of a sinusoidal carrier.
  - For AM, the peak **amplitude** of the envelope of the modulated signal can have a large dynamic range.
  - For FM, a constant envelope is generated for the modulated signal. This means that an FM transmitter can always have a better quality than AM reception.
  - But, the price to pay…the **bandwidth**!

Sect. 8.7 Sinusoidal Frequency Modulation

- **Angle Modulation** $c(t) = A\cos(\omega_c t + \theta_c) = A\cos\theta(t),$

  - **Phase Modulation**

    Use the modulating signal $x(t)$ to vary the phase θ_c

$$y(t) = A\cos\theta(t) = A\cos[\omega_c t + \theta_c(t)]$$

$$\theta_c(t) = \theta_0 + k_p x(t)$$

$$\frac{d\theta(t)}{dt} = \omega_c + k_p \frac{dx(t)}{dt}$$

$$\boxed{\theta(t) = \omega_c t + \theta_0 + k_p x(t),}$$

  - **Frequency Modulation**

    Use the modulating signal $x(t)$ to vary the derivative of the angle

$$y(t) = A\cos(\theta(t))$$

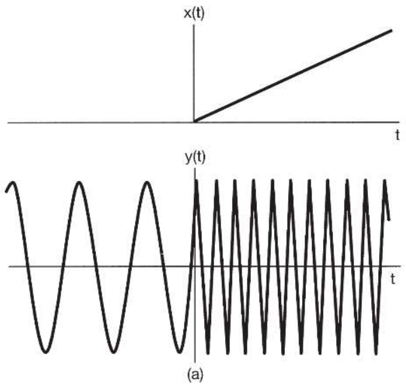
$$\frac{d\theta(t)}{dt} = \omega_c + k_f x(t)$$

$$\boxed{\theta(t) = \omega_c t + \theta_0 + k_f \int x(t)}$$

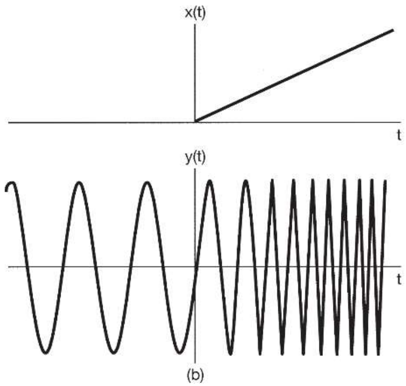
Sect. 8.7 Sinusoidal Frequency Modulation

- **Phase & Frequency Modulation**

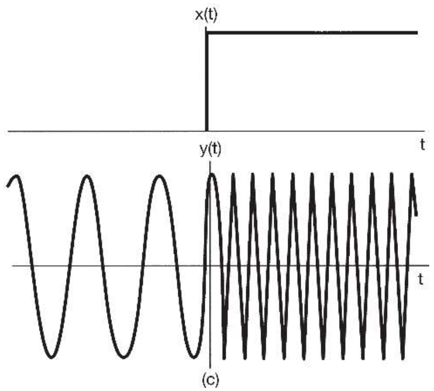
Phase modulation



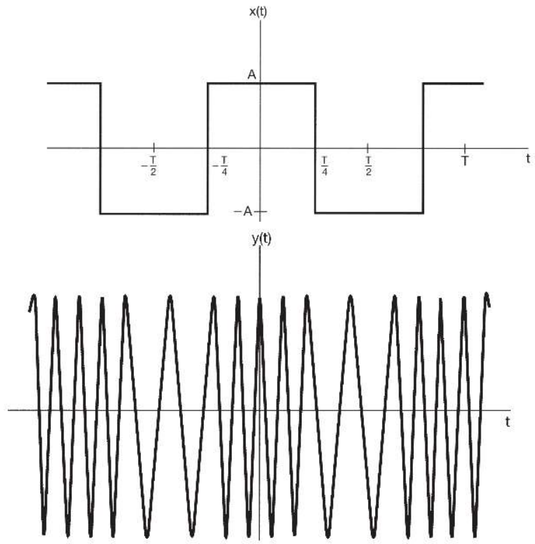
Frequency modulation



Frequency modulation



Frequency modulation



$$\boxed{\theta(t) = \omega_c t + \theta_0 + k_p x(t),}$$

$$\boxed{\theta(t) = \omega_c t + \theta_0 + k_f \int x(t)}$$

$$y(t) \;=\; r(t)\cos[(\omega_c + \Delta\omega)t] + r\left(t - \frac{T}{2}\right)\cos[(\omega_c - \Delta\omega)t],$$

Sect. 8.7 Sinusoidal Frequency Modulation

- Instantaneous Frequency

$$y(t) = A\cos\theta(t), \qquad \omega_i(t) = \frac{d\theta(t)}{dt}.$$

- Phase Modulation:

$$\omega_i = \frac{d\theta(t)}{dt} = \omega_c + k_p \frac{dx(t)}{dt}$$

- Frequency Modulation:

$$\omega_i = \frac{d\theta(t)}{dt} = \omega_c + k_f x(t)$$

Sect. 8.7 Sinusoidal Frequency Modulation

- Narrowband FM

$$x(t) = A\cos\omega_m t.$$

$$\omega_i(t) = \omega_c + k_f A\cos\omega_m t,$$

$$\begin{aligned}\omega_i(t) &= \frac{d\theta(t)}{dt} \\ &= \omega_c + k_f A\cos\omega_m t \\ &= \omega_c + \Delta\omega\cos(\omega_m t)\end{aligned}$$

$$\Delta\omega \triangleq k_f A$$

$$\begin{aligned}y(t) &= \cos\ \omega_c t + k_f \int x(t)dt \\ &= \cos\left(\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t + \theta_0\right) \\ &= \cos\left(\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right)\end{aligned}$$

let $\theta_0 = 0$

Sect. 8.7 Sinusoidal Frequency Modulation

- Narrowband FM (cont'd) $y(t) = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right].$

$$y(t) = \cos(\omega_c t + m\sin\omega_m t)$$

m: modulation index

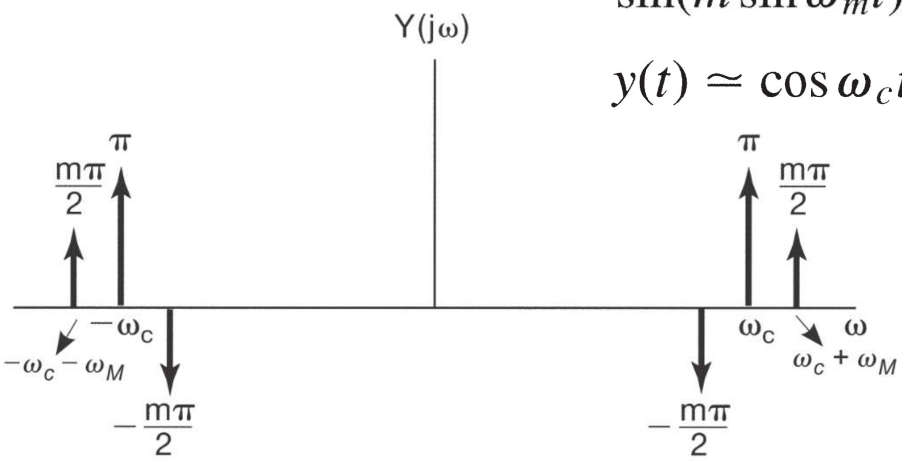
$$y(t) = \cos\omega_c t\cos(m\sin\omega_m t) - \sin\omega_c t\sin(m\sin\omega_m t).$$

When m is small ($<< \pi/2$), it becomes narrowband FM and

$$\cos(m\sin\omega_m t) \simeq 1,$$

$$\sin(m\sin\omega_m t) \simeq m\sin\omega_m t,$$

$$y(t) \simeq \cos\omega_c t - m(\sin\omega_m t)(\sin\omega_c t).$$

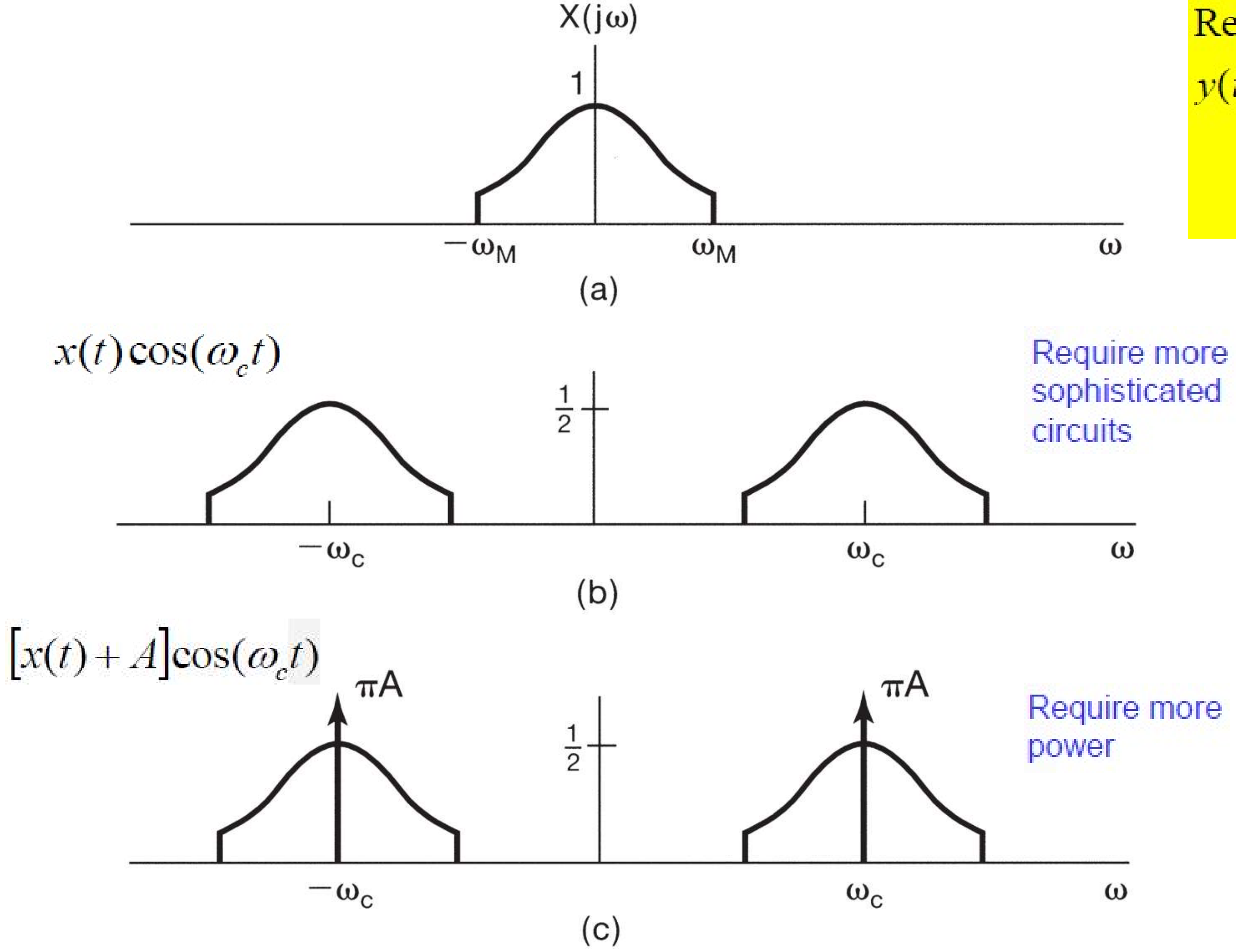


For narrowband FM, the BW of the sideband equals the BW ω_M of the modulating signal. And, the spectrum is similar to **AM**-DSB/WC.

Figure 8.36 Approximate spectrum for narrowband FM.

Recall that Demodulation for Sinusoidal AM

- Comparisons of Sync & Async Demodulation



Refer to Figs. 8.12 - 8.14, pp. 592-593.

$$
\begin{aligned}
y(t) &= (A + x(t))\cos\omega_c t \\
&= (1 + m\cos\omega_m t)\cos\omega_c t \\
&= \cos\omega_c t + m\cos\omega_m t\cos\omega_c t
\end{aligned}
$$

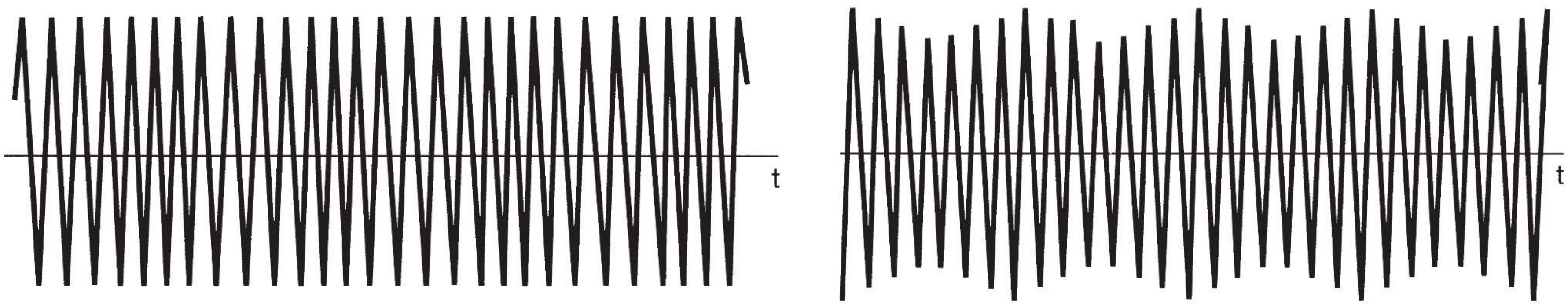
Figure 8.14 Comparison of spectra for synchronous and asynchronous sinusoidal amplitude modulation systems: (a) spectrum of modulating signal; (b) spectrum of $x(t)\cos\omega_c t$ representing modulated signal in a synchronous system; (c) spectrum of $[x(t) + A]\cos\omega_c t$ representing modulated signal in an asynchronous system.

Sect. 8.7 Sinusoidal Frequency Modulation

- Comparison of narrowband FM and AM-DSB/WC

Refer to Figs. 8.12 - 8.14, pp. 592-593.

$$\begin{aligned} y(t) &= (A + x(t))\cos\omega_c t \\ &= (1 + m\cos\omega_m t)\cos\omega_c t \\ &= \cos\omega_c t + m\cos\omega_m t\cos\omega_c t \end{aligned}$$



(a)

$$y(t) \approx \cos\omega_c t - m\sin\omega_m t\sin\omega_c t$$

(b)

$$y_2(t) = \cos\omega_c t + m\cos\omega_m t\cos\omega_c t$$

Figure 8.37 Comparison of narrowband FM and AM-DSB/WC: (a) narrowband FM; (b) AM-DSB/WC.

Sect. 8.7 Sinusoidal Frequency Modulation

- Wideband FM

$$y(t) = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right].$$

$$y(t) = \cos(\omega_c t + m\sin\omega_m t)$$ m: modulation index

$$y(t) = \cos\omega_c t\,\underbrace{\cos(m\sin\omega_m t)} - \sin\omega_c t\,\underbrace{\sin(m\sin\omega_m t)}.$$

- Both are periodic signals with fundamental frequency ω_m.
- Thus the FT of each signal is an impulse train with impulses at integer multiples of ω_m and amplitudes proportional to the Fourier series coefficients.
- The first [second] term corresponds to a carrier amplitude modulated by the periodic signal $\cos(m\sin\omega_m t)$ [$\sin(m\sin\omega_m t)$].

If the period of $\cos(m\sin\omega_m t)$ is T, then

$\cos(m\sin\omega_m t) = \cos(m\sin\omega_m (t+T))$

$= \cos(m\sin\omega_m t\cos\omega_m T + m\cos\omega_m t\sin\omega_m T)$

$\Rightarrow \cos\omega_m T = \pm 1$ and $\sin\omega_m T = 0 \Rightarrow T = \pi/\omega_m$

$\Rightarrow$ Fundamental frequency of $\cos(m\sin\omega_m t)$ is $2\omega_m$.

If the period of $\sin(m\sin\omega_m t)$ is T, then

$\sin(m\sin\omega_m t) = \sin(m\sin\omega_m (t+T))$

$= \sin(m\sin\omega_m t\cos\omega_m T + m\cos\omega_m t\sin\omega_m T)$

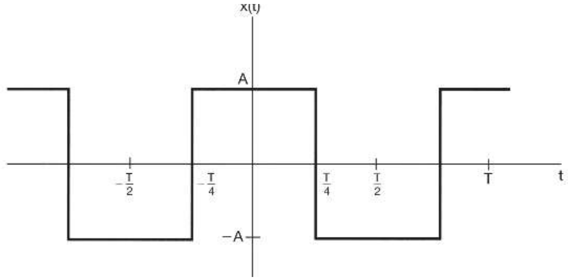
$\Rightarrow \cos\omega_m T = 1$ and $\sin\omega_m T = 0 \Rightarrow T = 2\pi/\omega_m$

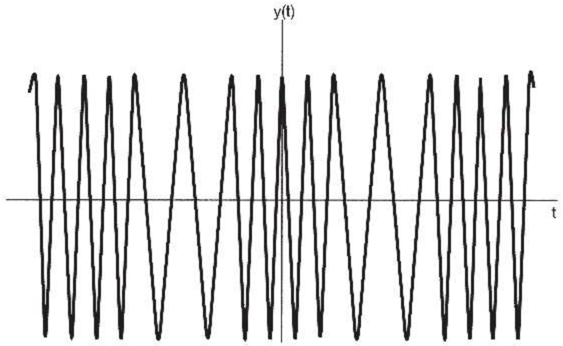
$\Rightarrow$ Fundamental frequency of $\sin(m\sin\omega_m t)$ is ω_m.

Sect. 8.7 Sinusoidal Frequency Modulation

$$\omega_i(t) = \omega_c + k_f x(t) \qquad k_f = 1 \Rightarrow \Delta\omega = A$$

- When $x(t) > 0$, $\omega_i(t) = \omega_c + \Delta\omega$
- When $x(t) < 0$, $\omega_i(t) = \omega_c - \Delta\omega$





- Periodic Square-Wave Modulating Signal

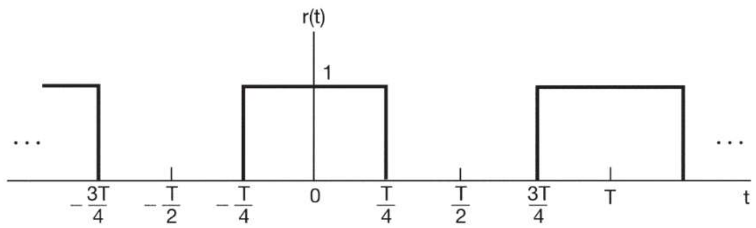
$$y(t) = r(t)\cos[(\omega_c + \Delta\omega)t] + r\left(t - \frac{T}{2}\right)\cos[(\omega_c - \Delta\omega)t],$$

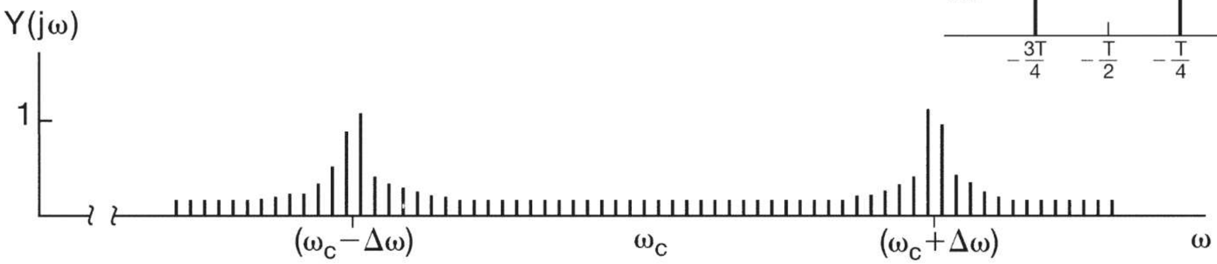
$$Y(j\omega) = \frac{1}{2}[R(j\omega + j\omega_c + j\Delta\omega) + R(j\omega - j\omega_c - j\Delta\omega)]$$
$$+ \frac{1}{2}[R_T(j\omega + j\omega_c - j\Delta\omega) + R_T(j\omega - j\omega_c + j\Delta\omega)],$$

where $R(j\omega)$ is the Fourier transform of the periodic square wave $r(t)$ and $R_T(j\omega)$ is the Fourier transform of $r(t - T/2)$.
From Example 4.6, with $T = 4T_1$,

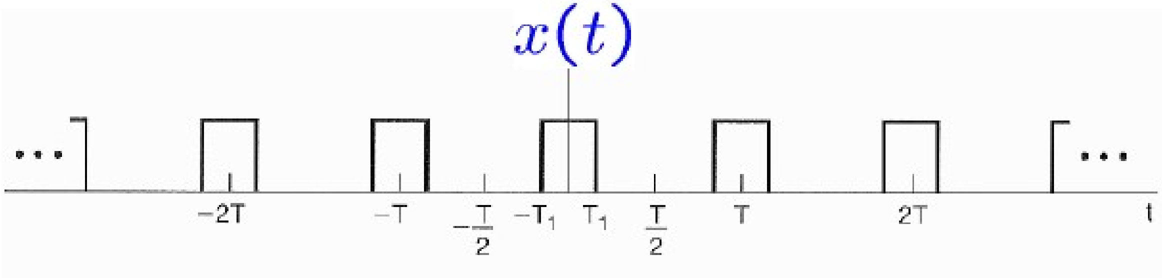
$$R(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2}{2k+1}(-1)^k \delta\left[\omega - \frac{2\pi(2k+1)}{T}\right] + \pi\delta(\omega)$$

$$R_T(j\omega) = R(j\omega)e^{-j\omega T/2}.$$





- Example 4.6

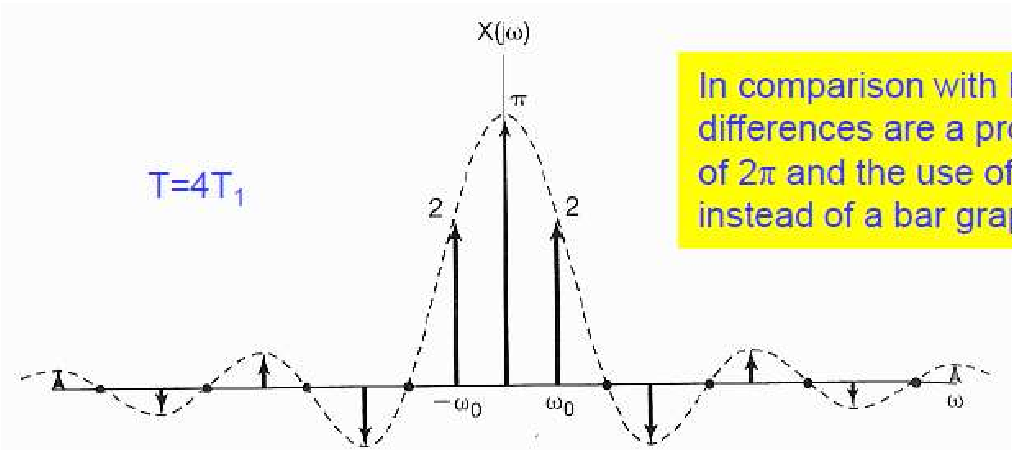


$$x(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$a_k=\frac{\sin(k\omega_0 T_1)}{\pi k}$$

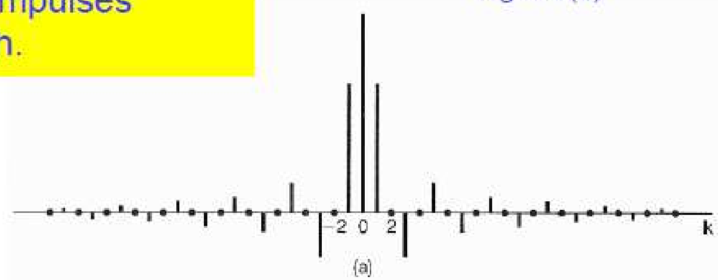
$$\Rightarrow X(j\omega)=\sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega-k\omega_0)$$

$$=\sum_{k=-\infty}^{+\infty} \frac{2\sin(k\omega_0 T_1)}{k}\delta(\omega-k\omega_0)$$



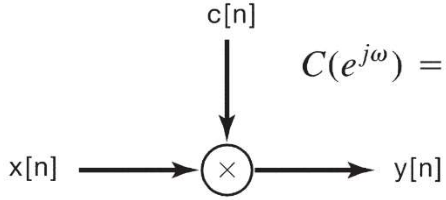
In comparison with Fig. 3.7(a), the differences are a proportional factor of 2π and the use of impulses instead of a bar graph.

Fig. 3.7(a)



Sect. 8.9 DT Modulation

- DT Sinusoidal AM & Demodulation



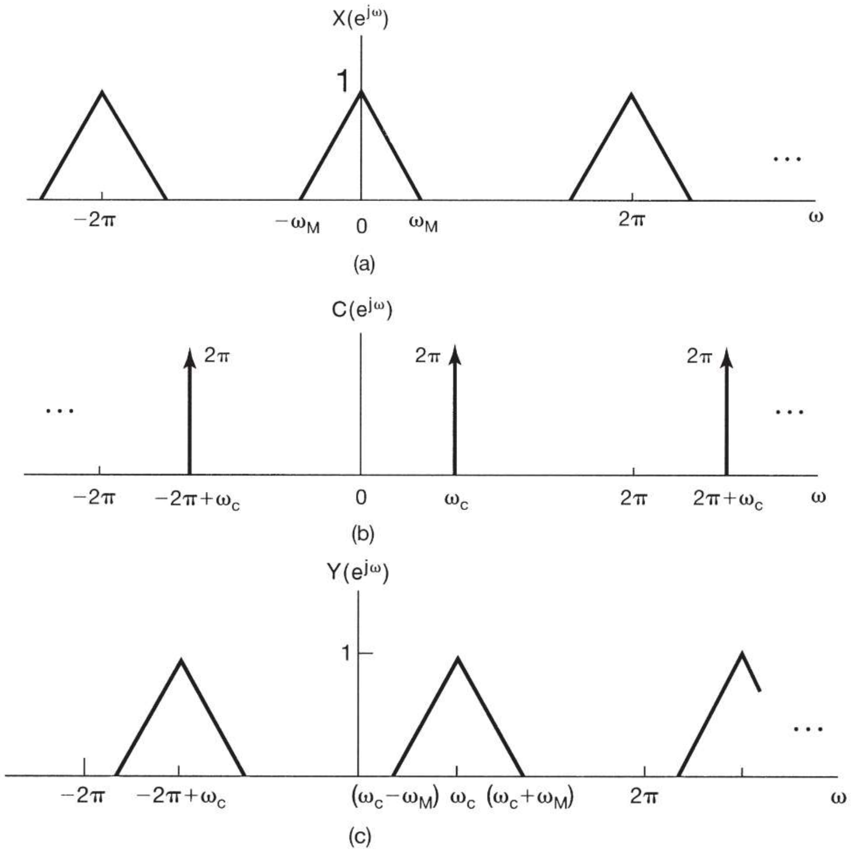
$$c[n] = e^{j\omega_c n}.$$

$$C(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_c + k2\pi),$$

$$y[n] = x[n]c[n].$$

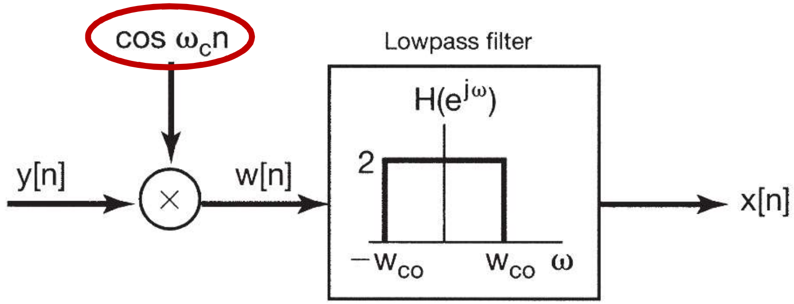
$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})C(e^{j(\omega-\theta)})d\theta.$$

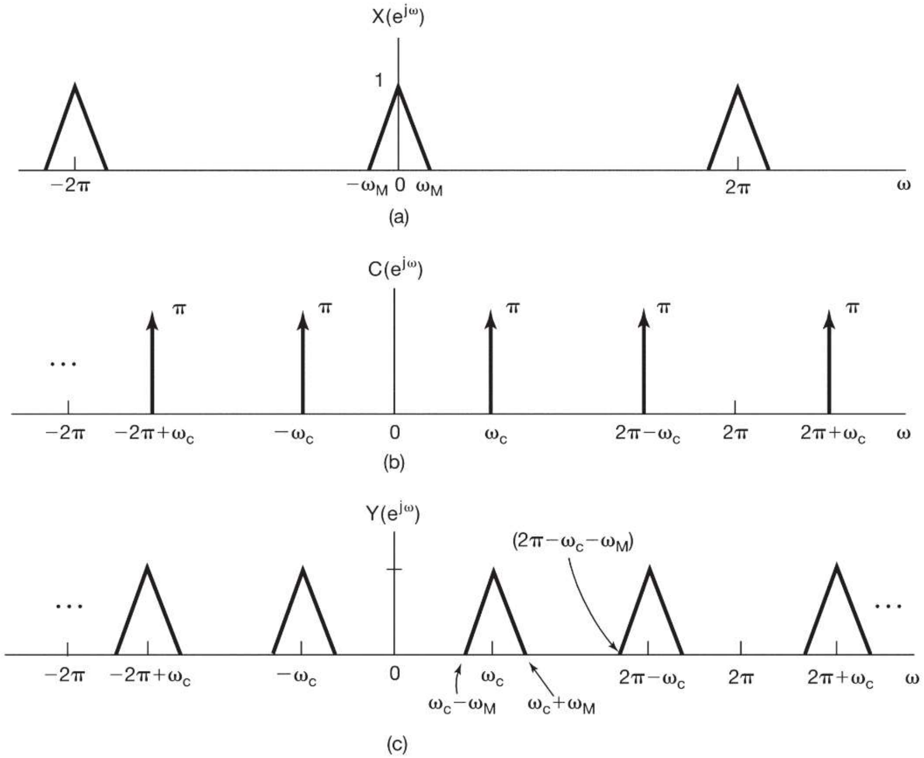
$$x[n] = y[n]e^{-j\omega_c n}.$$

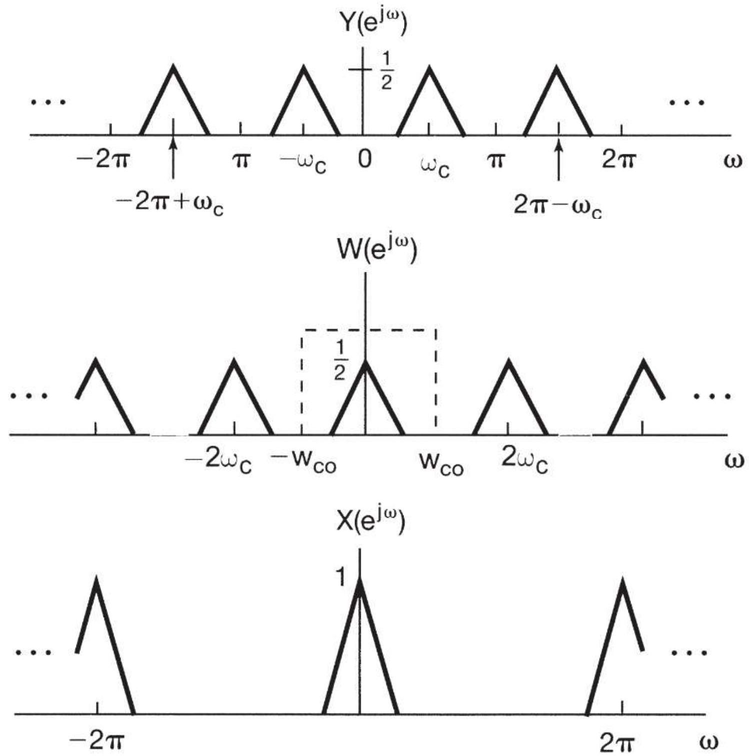


Sect. 8.9 DT Modulation

- DT Sinusoidal AM & Demodulation







$$\omega_c > \omega_M$$

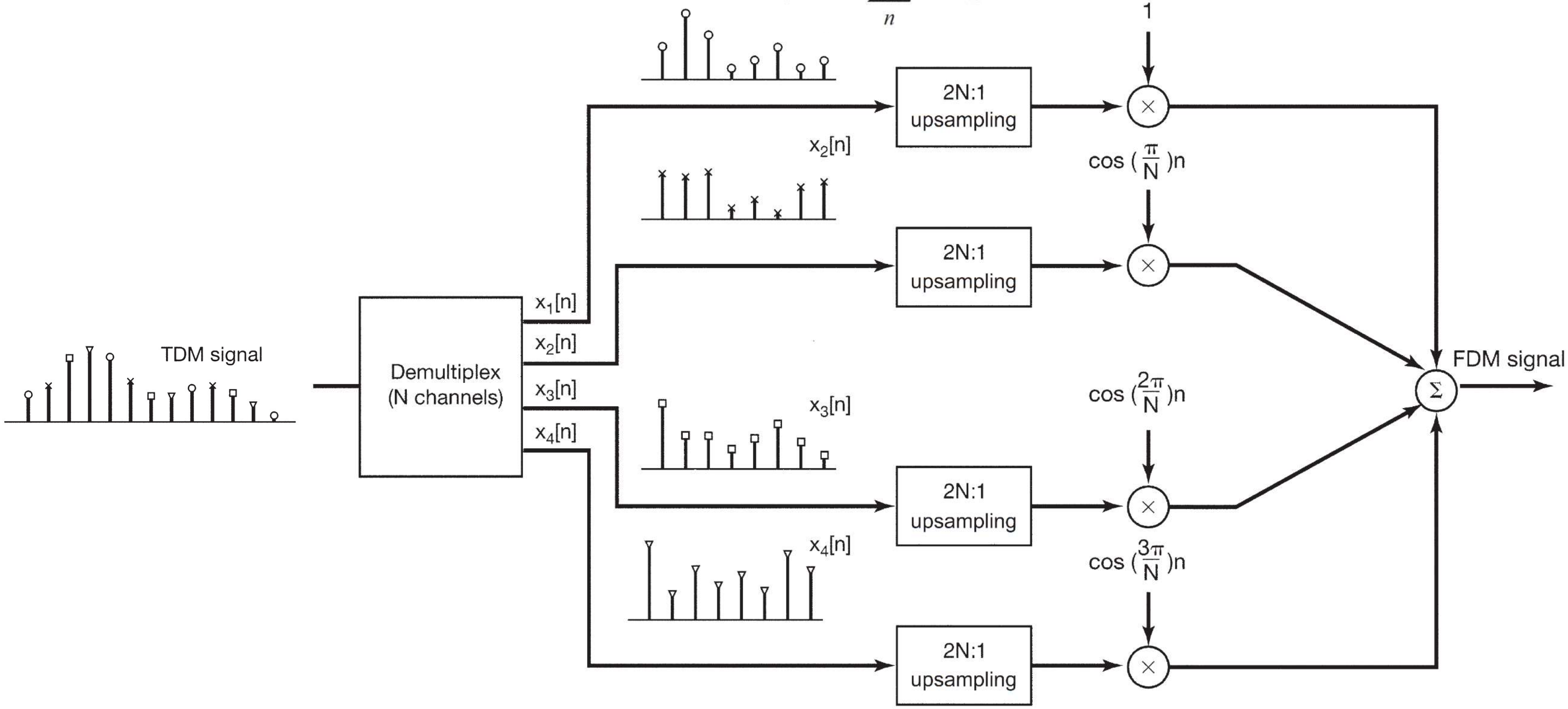
$$2\pi - \omega_c - \omega_M > \omega_c + \omega_M$$

$$\omega_M < \omega_c < \pi - \omega_M.$$

Sect. 8.9 DT Modulation

- DT Transmodulation or Transmultiplexing: TDM to FDM

$$y(t) = \sum_n a[n]p(t - nT)$$

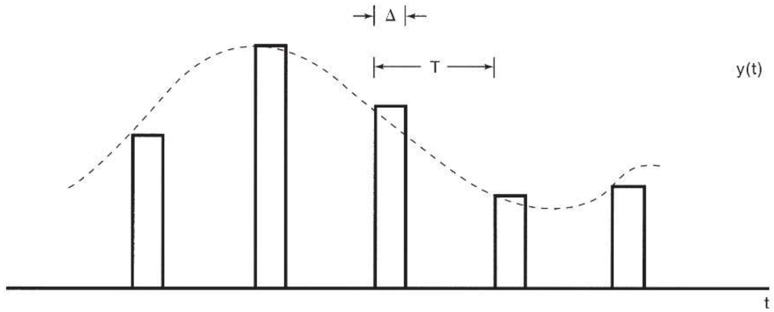


$$s(t) = \sum_n a[n]p(t - nT)\cos(\omega_c t + \theta_c)$$

Sect. 8.9 DT Modulation

$$y(t) = \sum_n a[n]p(t - nT) \quad \text{where} \quad a[n] = x(nT).$$

where $\quad p(t) = 1$ for $|t| < \Delta/2$, $p(t) = 0$ otherwise.



- Frequency-Shift Keying (FSK)

In phase-shift keying,
PAM y(t) with a sinusoidal carrier takes the form

$$s(t) = \sum_n a[n]p(t - nT)\cos(\omega_c t + \theta_n).$$

With FSK

$$s(t) = \sum_n a[n]p(t - nT)\cos((\omega_0 + \Delta_n)t + \theta_c)$$

With PSK, in each symbol interval, information can now be incorporated in both the pulse amplitude *a*[*n*] and the carrier phase ϑ_n.

$$s(t) = \sum_n ap(t - nT)\cos(\omega_c t + \theta_n).$$

For example, choosing

$$\theta_n = \frac{2\pi b_n}{M} \quad ; \quad 0 \le b_n \le M - 1$$

Sect. 8.9 DT Modulation

- Phase-Shift Keying (PSK) $s(t) = \sum_n ap(t - nT)\cos(\omega_c t + \theta_n)$. $\theta_n = \frac{2\pi b_n}{M}$; $0 \le b_n \le M-1$

Suppose that *x*[*n*] is a binary discrete-time signal.
If PSK with *M* = 4 is adopted to encode *x*[*n*], then we can set

$$
\begin{aligned}
\theta_n &= 0 && \text{if} \quad x[2n] = x[2n+1] = 0, \\
\theta_n &= \pi/2 && \text{if} \quad x[2n] = 0 \text{ and } x[2n+1] = 1, \\
\theta_n &= \pi && \text{if} \quad x[2n] = 1 \text{ and } x[2n+1] = 0, \\
\theta_n &= 3\pi/2 && \text{if} \quad x[2n] = x[2n+1] = 1,
\end{aligned}
$$

Using the identity

$$\cos(\omega_c t + \theta_n) = \cos(\theta_n)\cos(\omega_c t) - \sin(\theta_n)\sin(\omega_c t),$$

we can write with

$$s(t) = I(t)\cos(\omega_c t) - Q(t)\sin(\omega_c t),$$

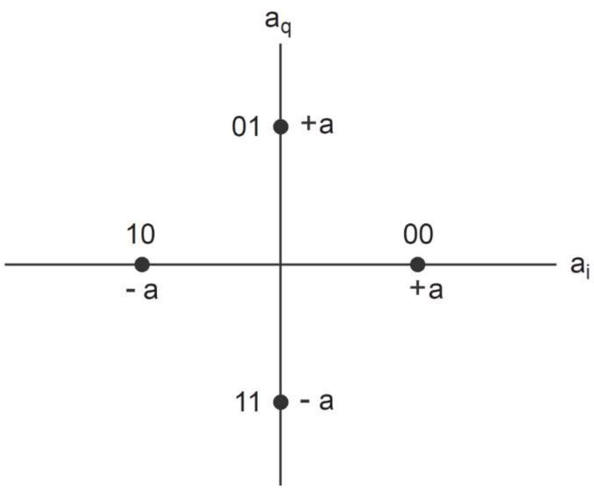
with

$$I(t) = \sum_n a_i[n]p(t - nT) \qquad Q(t) = \sum_n a_q[n]p(t - nT)$$

and

$$a_i[n] = a\cos(\theta_n)$$

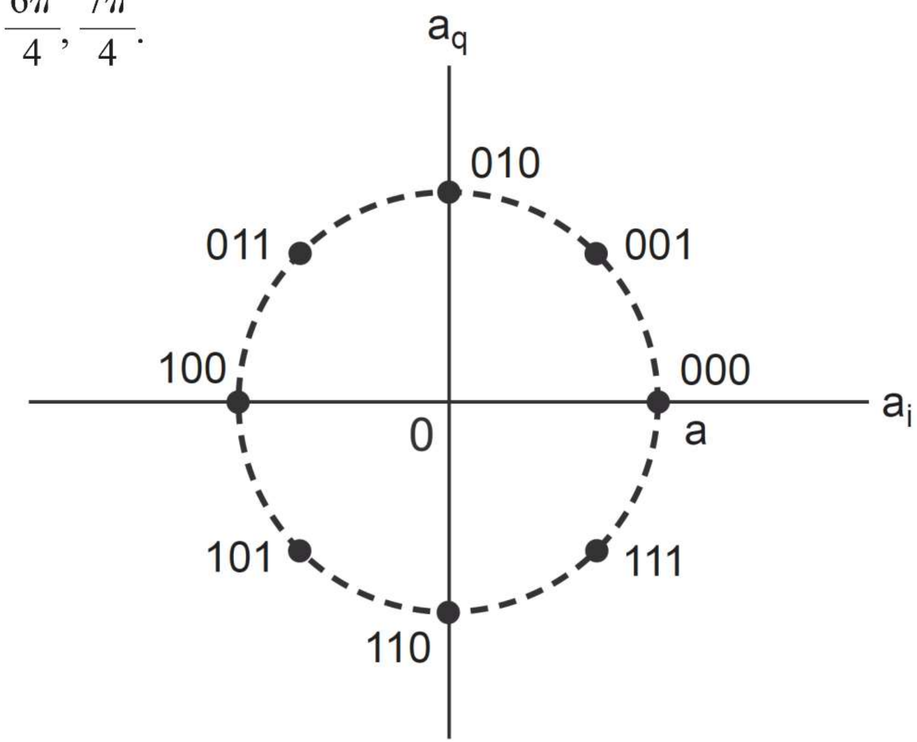
$$a_q[n] = a\sin(\theta_n).$$



Sect. 8.9 DT Modulation

- Quadrature Amplitude Modulation (QAM)

$$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{6\pi}{4}, \frac{7\pi}{4}.$$



Sect. 8.9 DT Modulation

- Demodulation

The input signal $r_i(t)$ in Figure 8.51 is

$$r_i(t) = I(t)\cos^2(\omega_c t) - Q(t)\sin(\omega_c t)\cos(\omega_c t)$$
$$= \frac{1}{2} I(t) - \frac{1}{2} I(t)\cos(2\omega_c t) - \frac{1}{2} Q(t)\sin(2\omega_c t).$$

Similarly,

$$r_q(t) = I(t)\cos(\omega_c t)\sin(\omega_c t) - Q(t)\sin^2(\omega_c t)$$
$$= \frac{1}{2} I(t)\sin(2\omega_c t) + \frac{1}{2} Q(t) - \frac{1}{2} Q(t)\cos(2\omega_c t).$$

From Figure 8.52,

$$a_i[n] = 3a \text{ and } a_q{}^2[n] = -a.$$

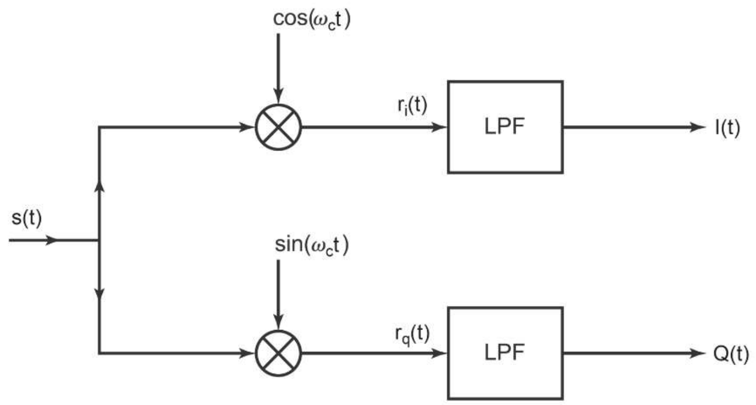


Figure 8.51 Demodulation scheme for a quadrature modulated PAM signal.

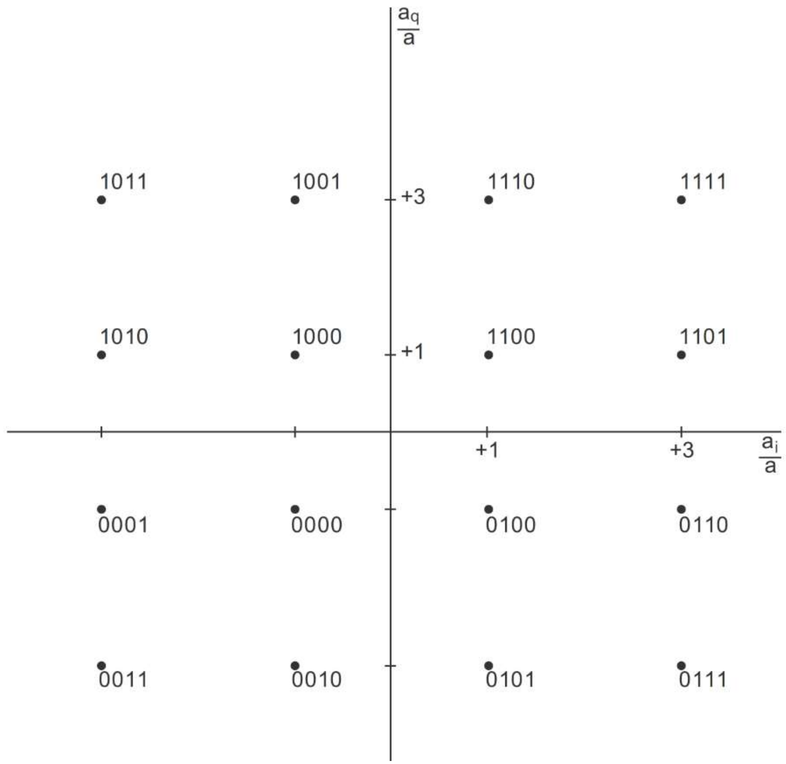


Figure 8.52 A 16-point QAM constellation.

Modulation Methods

- Continuous-Time Modulation
  - Amplitude Modulation
    - with a Complex Exponential Carrier (Sec. 8-1, demodulation: Sec. 8-2)
    - with a Sinusoid Carrier (Sec. 8-1, demodulation: Sec. 8-2)
      - Double Sided Band (Sec. 8-1, Sec. 8-2)
      - Single Sided Band (Sec. 8-4)
    - with a Pulse-Train Carrier (Sec. 8-5)
    - Pulse-Amplitude Modulation (PAM) (Sec. 8-6)
    - Quadrature Amplitude Modulation(QAM) (Sec. 8-9)
  - Angle Modulation
    - Phase Modulation (Sec. 8-7)
      Phase-Shift Keying (PSK) (Sec. 8-9)
    - Frequency Modulation (Sec. 8-7)
      Frequency-Shift Keying (FSK) (Sec. 8-9)
- Discrete-Time Modulation (Sec. 8-8, Sec. 8-11)

Multiplexing Methods

- Frequency-Division Multiplexing
  - Continuous-Time, AM with a Sinusoid Carrier (Sec. 8-3)
  - Discrete-Time (Sec. 8-8)
    - Orthogonal Frequency-Division Multiplexing (OFDM) (Sec. 8-11)
- Time-Division Multiplexing
  - Continuous-Time (Sec. 8-5, Sec. 8.6 (for PAM))
  - Discrete-Time (Sec. 8-8)
- Code Division Multiple Access (CDMA) (Sec. 8.11)