Signals and Systems MATLAB HW2
Deadline: 2019/04/26 before 23:59

Discrete Fourier Transform

The objective of this section is to learn how to use MATLAB **fft** function.

1.Background

In order to analyze the frequency domain of a finite duration and discrete-time signal $x[n]$, n=1,2,…N, its discrete Fourier transform (DFT) is defined as

$$X_k = \sum_{n=1}^{N} x[n] e^{-j\frac{2\pi}{N}(n-1)(k-1)}, k = 1, 2, ..., N$$

It is observed that DFT is the sampled Fourier transform of a finite duration signal with frequency $\omega = \frac{2\pi k}{N}$. On the other hand, the inverse DFT (IDFT) of X_k is defined as

$$x[n] = \frac{1}{N} \sum_{k=1}^{N} X_k e^{j\frac{2\pi}{N}(n-1)(k-1)}, n = 1, 2, ..., N$$

The fast Fourier transform (FFT) is equivalent to DFT with reduced computational complexity as well as inverse FFT (IFFT) to IDFT. To calculate the DFT of the signal $x[n]$ in MATLAB function, you may type:

X = fft(x);

If you want to explicitly specify the length *M*, then you can type:

X = fft(x, M);

and the **fftshift** command swaps the first and the second half of the vector **X** so that the frequency range is in $[\frac{-N}{2}, \frac{N}{2}]$ (assuming *N* is even.)

However, for signals with infinite length, we have to truncate it so that it can be computed with MATLAB. Such truncation causes *Gibbs phenomenon* (pp. 200-201 of the textbook).

2.Questions

Program a MATLAB script (save as **fftsinc.m** file) to achieve the question 1.(a)(b) and 2.(c)(d)(e).

1. Let $x(t)$ be a sinc function written as

$$x(t) = \frac{\sin(\pi t)}{\pi t}$$

Now, $x(t)$ is sampled at the rate $T_s = T / N_1$ so that $x[n] = x(nT_s)$,

$n \in \{-N_1, -N_1+1, ...0, ...N_1-1, N_1\}$ and N=2N_1+1. Let N=1001 and T=100.

(a) (20%) Use the MATLAB function **stem** to plot $x[n]$ vs n.

(b) (20%) Use the MATLAB function **fft** to compute $x[n]$, and use the MATLAB function **plot** to plot the magnitude of the **fft** output vs frequency ω. The zero frequency should be centered in your plot. Observe the *Gibbs phenomenon* in (b) and give some explanation for it in your report.

2. A way of mitigating *Gibbs phenomenon* is to multiply $x(t)$ by a finite-duration signal $w(t)$, i.e., $y(t) = x(t)w(t)$. The signal $w(t)$ is called as the window function. A famous one is *Hanning* window, which is specifically written as

$$w(t) = \begin{cases} \frac{1}{2}[1 + \cos(\frac{2\pi |t|}{T_w})], & |t| \le T_w / 2 \\ 0 & \text{, else} \end{cases}$$

where T_w denotes the duration of the window function.

Suppose $w(t)$ is also sampled at a rate $T_s = T / N_1$ so that $w[n] = w(nT_s)$,

$n \in \{-N_1, -N_1+1, ...0, ...N_1-1, N_1\}$ and N=2N_1+1. Let T_w=T/2, N=1001,

and T=100.

(c) (20%) Use the MATLAB function **stem** to plot $w[n]$ vs n.

(d) (20%) Use the MATLAB function **stem** to plot $y[n]$ vs n, where $y[n] = x[n]w[n]$, and $x[n]$ is the signal plotted in 1(a).

(e) (20%) Use the MATLAB function **fft** to compute $y[n]$ in (d), and use the MATLAB function **plot** to plot the magnitude of the **fft** output vs frequency ω. The zero frequency should be centered in your plot. Observe the *Gibbs phenomenon* here and give some explanation for comparison with 1.(b) in your report.

Note: We expect that if executing your **fftsinc.m** file, there will be total five figures come out in order. (Question (a)~(e) has one figure respectively).

3.CEIBA Submission

- Please upload a compressed file (.zip, .rar or .tar), which includes your **m-files (**save as **fftsinc.m** file**)** and a **word file (**save as **report.doc** file**)**. Please show the relevant plots mentioned above in the word file (report.doc) and some explanation.
- The compressed file name should be **ID_MATLAB2**.
  (ex: B07901xxx_MATLAB2)