Signals and Systems HW7
Deadline: 2019/05/17 before 18:30
(You should submit hand-writing paper to BL B1 EE student office.)

1. A continuous-time signal

$$x(t) = \text{sinc}(200t)$$

where $\text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$

is sampled at a rate of 150Hz using a train of pulses

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

where $T_s = 1/150$. The sampled signal is written as $y(t) = x(t)p(t)$.

(a) (14%) Compute and sketch the Fourier transform of $y(t)$.

(b) (14%) To reconstruct the signal, $y(t)$ is filtered by an ideal filter with frequency response

$$H(jw) = \frac{1}{150}\text{rect}\left(\frac{w}{300\pi}\right)$$

where $\text{rect}\left(\dfrac{t}{T}\right) = \begin{cases} 1, & |t| < \dfrac{T}{2} \\ 0, & \text{elsewhere} \end{cases}$.

The result of the filtered signal is denoted by $\hat{x}(t)$. Compute and sketch the Fourier transform of $\hat{x}(t)$.

(c) (14%) Now the signal is pre-filtered by an anti-aliasing filter. The block diagram is drawn in Figure 1. Compute and sketch Fourier transform of the sampled signal $v(t)$.

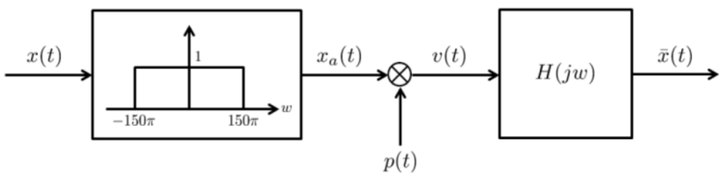

Figure 1. Sampling a signal using an anti-aliasing filter

(d) (14%) Compute and sketch the reconstructed signal $\bar{x}(t)$.

(e) (14%) To assess the quality of the reconstruction, the energy loss of the reconstructed signal is considered. Specifically, use Parseval's relation to calculate E_1 and E_2, where

$$E_1 = \int_{-\infty}^{\infty} |x(t) - \hat{x}(t)|^2 dt \quad \text{and} \quad E_2 = \int_{-\infty}^{\infty} |x(t) - \bar{x}(t)|^2 dt$$

Which one is smaller, indicating a better reconstruction?

2. A continuous-time signal

$$x(t) = \sin(200\pi t)$$

is sampled by a periodic rectangular function defined by

$$p(t) = \sum_{k=-\infty}^{\infty} \text{rect}(\frac{t-0.004k}{0.0008}).$$

The sampled signal is written as $y(t) = x(t)p(t)$.

(a) (15%) Please find the Fourier transform of $y(t)$.

(b) (5%) Explain whether $x(t)$ can be recovered from $y(t)$ or not.

(c) (10%) If $y(t)$ is passed through an ideal lowpass filter of bandwidth 100Hz and unit gain, determine the output signal.