Signals & Systems

Spring 2019

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2019/02/25

Tutorial on Matlab

- Time & Location:
  - 2/25 (一) 15:30~17:00 EE2-143
  - 2/27 (三) 16:30~18:00 EE2-143
- What would be covered?
  - MATLAB安裝說明
  - 介紹MATLAB基本操作
  - MATLAB程式語法
  - 簡單訊號處理範例程式

TA Office Hours

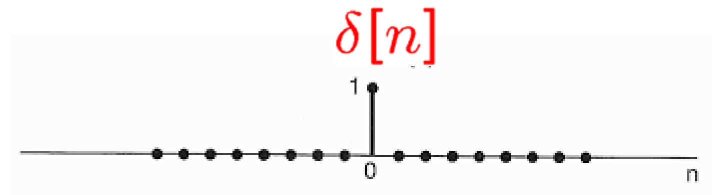
- 星期一 11:20~13:10 辜炳叡小助教 博理B1電機系K
- 星期二 13:00~14:00 林志皓小助教 博理B1電機系K
- 星期三 15:00~17:00 蘇峯廣小助教 博理B1電機系K
- 星期四 10:00~12:00 陳泓廷小助教 博理B1電機系K
- 星期五 16:40~18:00 許秉倫小助教 博理B1電機系K

蘇峯廣

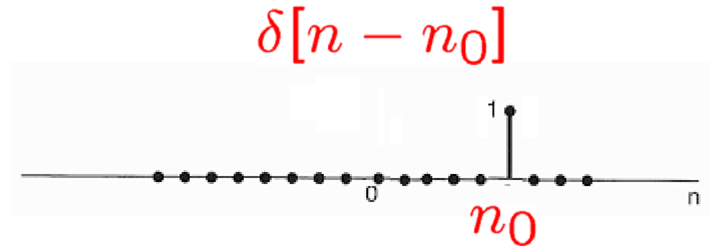
林志皓

1.4 The Unit Impulse and Unit Step Functions

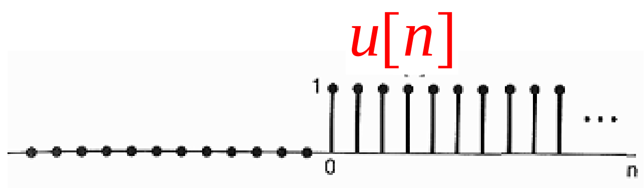
- 1.4.1 The DT Unit Impulse and Unit Step Sequences
- Definitions
    - Unit impulse (or unit sample)



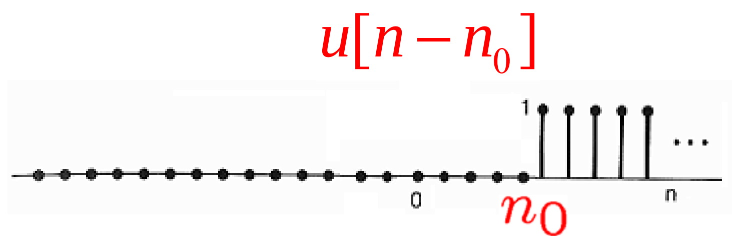
$$\delta[n] = \begin{cases} 1 & , n = 0 \\ 0 & , \text{otherwise} \end{cases}$$



    - Unit step

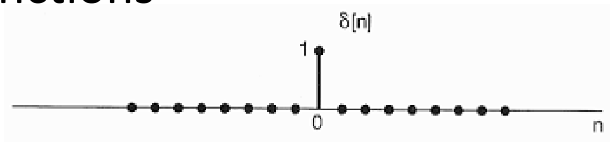


$$u[n] = \begin{cases} 1 & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$



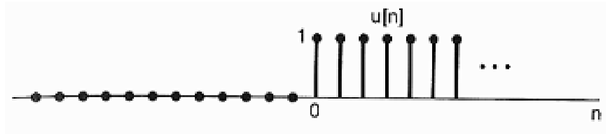
1.4 The Unit Impulse and Unit Step Functions (cont'd)

- Relations between Impulse and Step Functions
- First difference

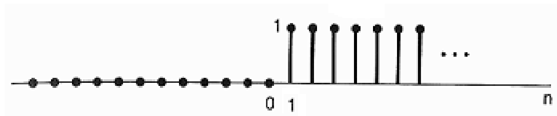


$\delta[n]$

$\delta[n] = u[n-1] - u[n]$



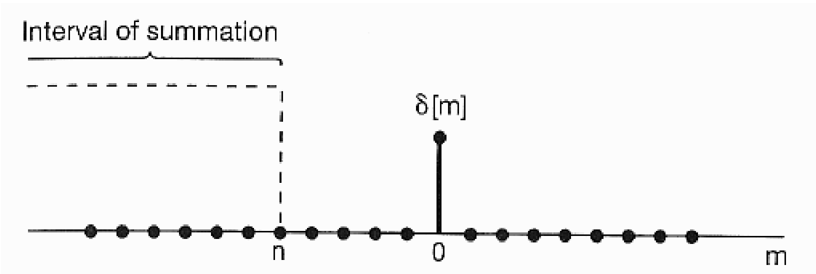
$u[n]$



$u[n-1]$

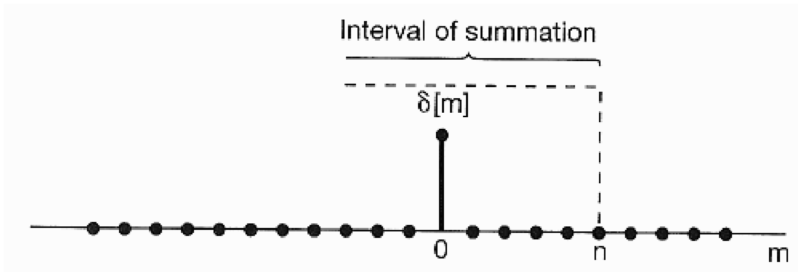
- Running sum

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \begin{cases} 1, & m \geq 0 \\ 0, & m < 0 \end{cases}$$



(a)

$n < 0$



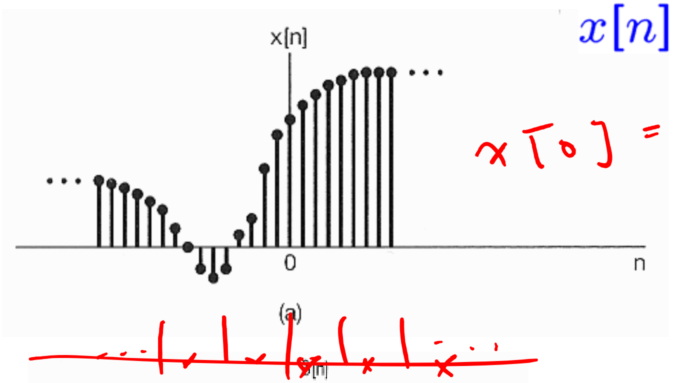
(b)

$n \geq 0$

1.4 The Unit Impulse and Unit Step Functions (cont'd)

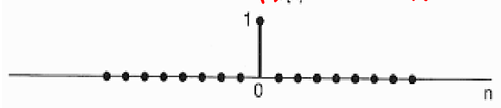
- Sampling (sifting) property
- For $x[n]$

$$x[n]\delta[n] = x[0]\delta[n]$$

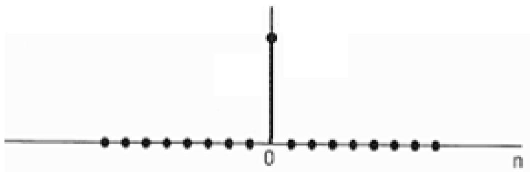


$x[n]$

$x[0] = ?$

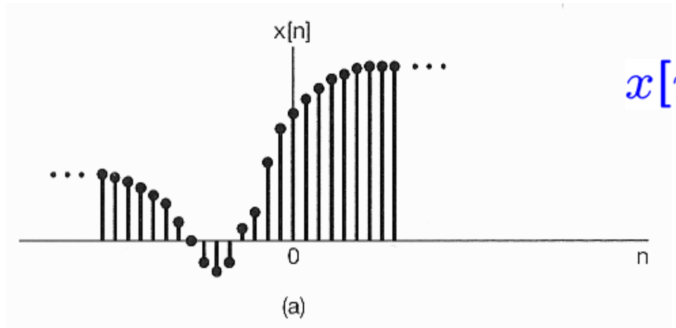


$\delta[n]$

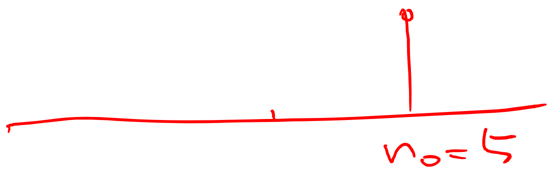


$x[n]\delta[n]$

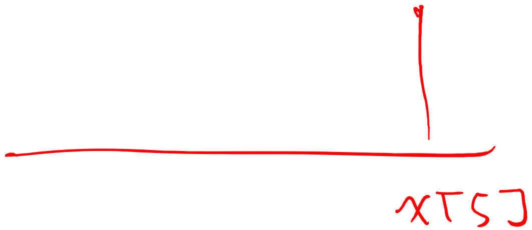
- More generally,



$x[n]$



$\delta[n-5]$

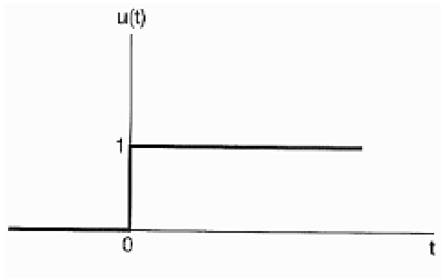


$x[5]\delta[n-5]$

1.4 The Unit Impulse and Unit Step Functions

- 1.4.2 The CT Unit Impulse and Unit Step Sequences
- Definitions
  - Unit step function

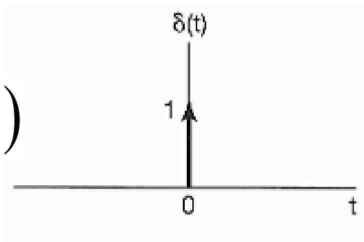
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



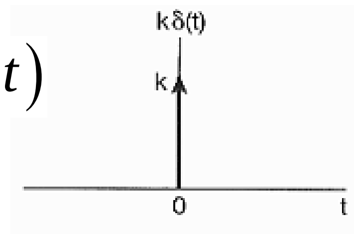
  - Unit impulse function

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$$

$\delta(t)$

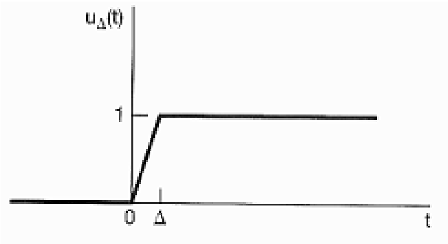


$k\delta(t)$

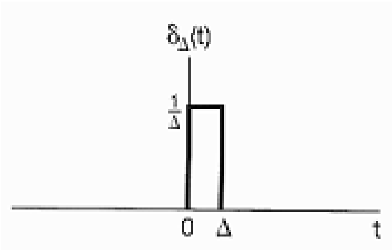


1.4 The Unit Impulse and Unit Step Functions

- 1.4.2 The CT Unit Impulse and Unit Step Sequences
- Approximation

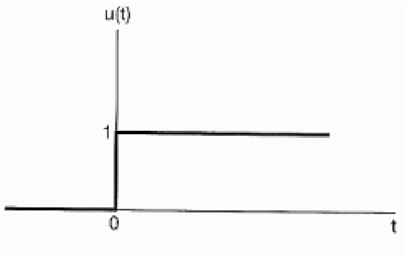


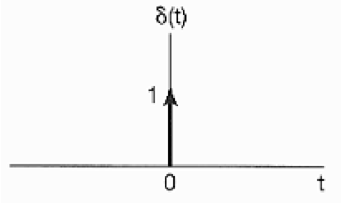
$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$



$$u(t) = \lim_{\Delta \to 0} u_\Delta(t)$$

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$





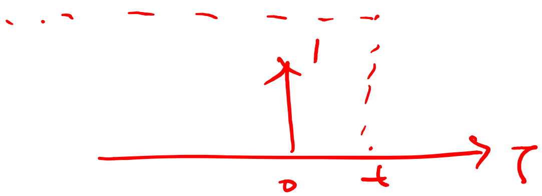
1.4.2 The CT Unit Impulse and Unit Step Sequences (cont'd)

- Relations between Impulse and Step Functions
- First derivative

$$\delta(t) = \frac{du(t)}{dt}$$

- Running integral

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau \quad = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



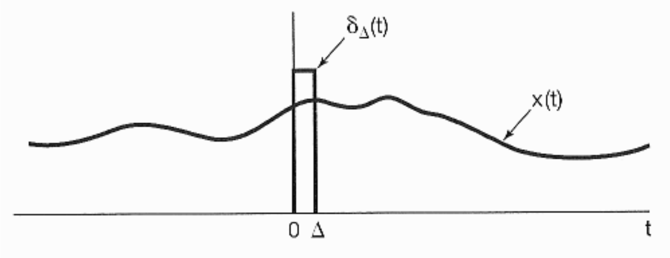
1.4.2 The CT Unit Impulse and Unit Step Sequences (cont'd)

- Sampling (sifting) property
- For *x*(t)

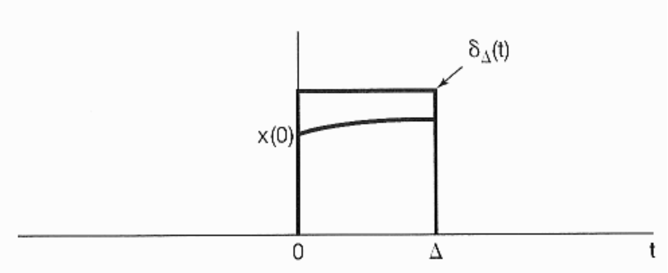
$$x(t)\delta(t) = x(0)\delta(t)$$

- More generally,

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$



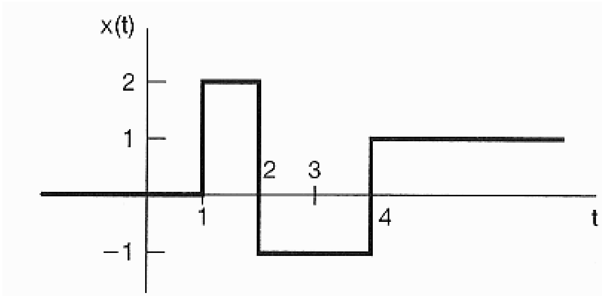
(a)



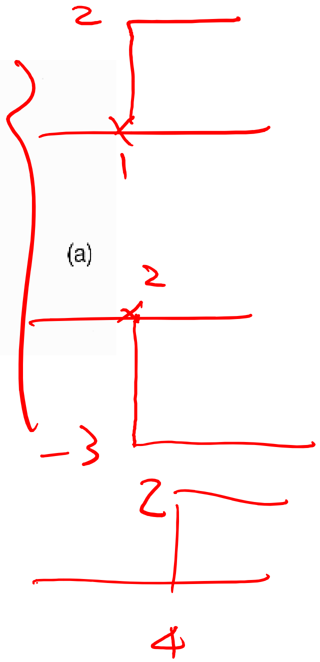
(b)

1.4.2 The CT Unit Impulse and Unit Step Sequences (cont'd)

- Example 1.7
  Express x(t) and x'(t) in terms of CT unit impulse/step functions.



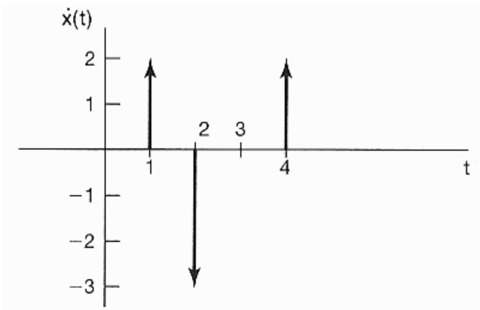
(a)



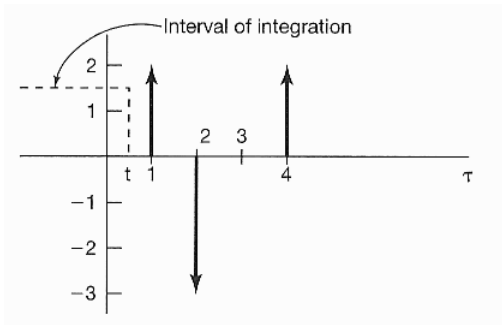
$$x(t) = 2u(t-1) - 3u(t-2) + 2u(t-4)$$

$$\dot{x}(t) = 2\delta(t-1) - 3\delta(t-2) + 2\delta(t-4)$$

$$x(t) = \int_0^t \dot{x}(\tau)d\tau$$

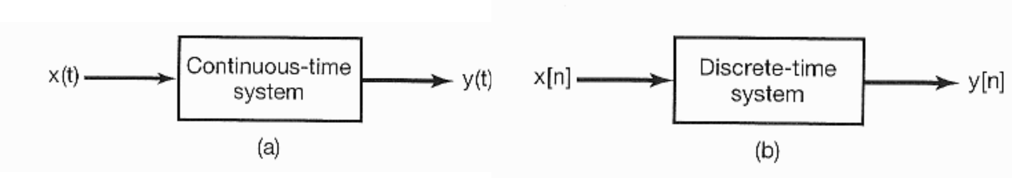


(b)



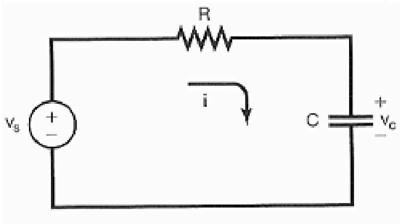
(c)

1.5 CT and DT Systems



- A system can be viewed as a process in which input signals are transformed into other signals (outputs).
- Example 1.8 RC circuit (a CT system)

Input signal: $v_s(t)$



Output signal: $v_c(t)$

$$i(t) = \frac{v_s(t) - v_c(t)}{R} \qquad i(t) = C\frac{dv_c(t)}{dt}$$

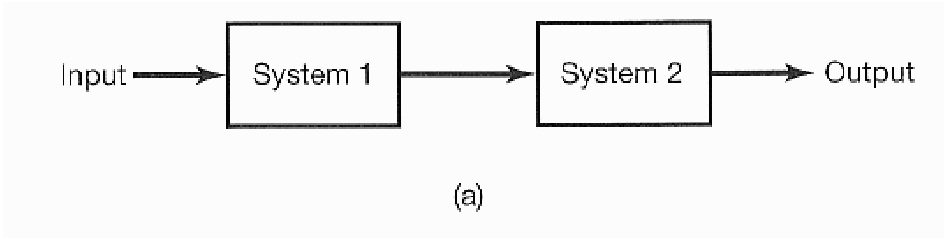
$$\Rightarrow \frac{v_s(t) - v_c(t)}{R} = C\frac{d}{dt}v_c(t)$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

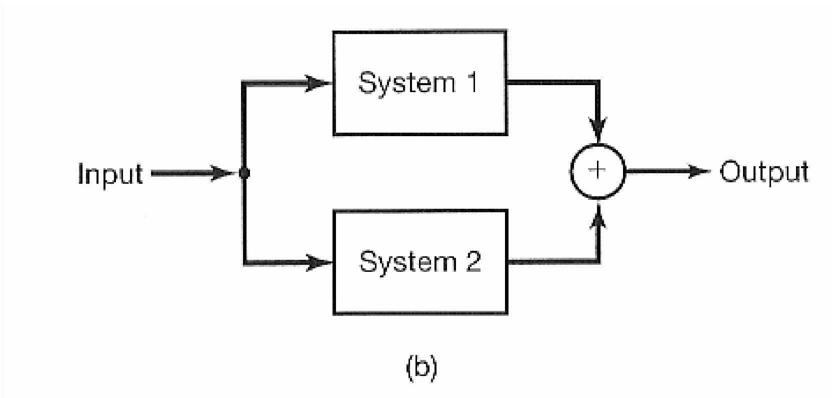
$$\Rightarrow \frac{dy(t)}{dt} + ay(t) = bx(t) \qquad a = b = \frac{1}{RC}$$

1.5.2 Interconnections of Systems

- Series or cascade interconnection of 2 systems (e.g., receiver/amplifier)

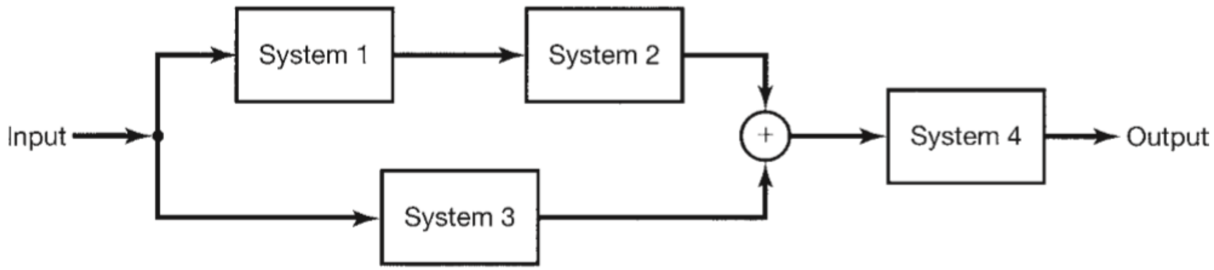


- Parallel interconnection of 2 systems
  (e.g., audio systems with multi-microphones/speakers)

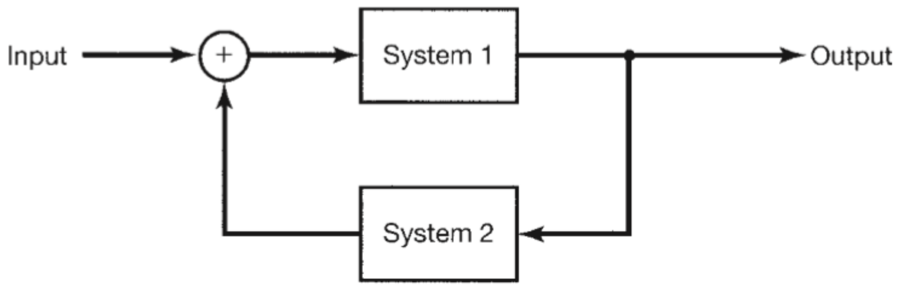


1.5.2 Interconnections of Systems

- Hybrid of series and cascade interconnections

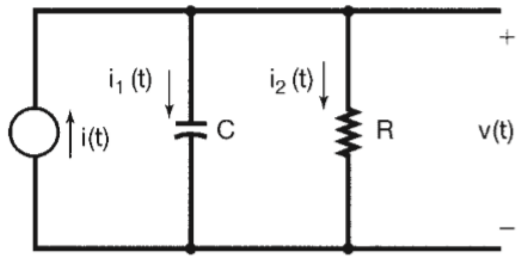


- Feedback interconnections (e.g., control systems, circuits, etc.)

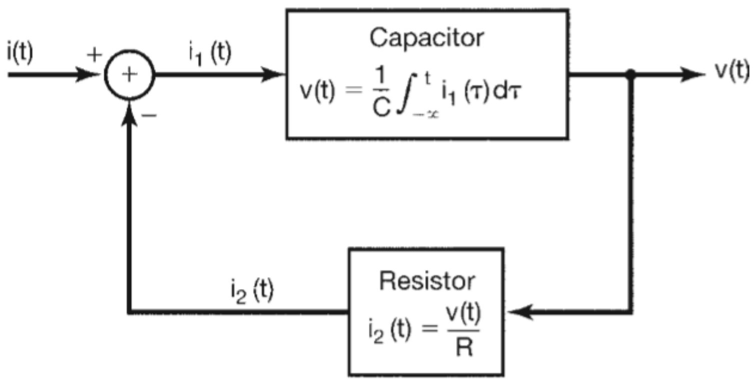


1.5.2 Interconnections of Systems

- An example of a feedback system in circuits
  (a) Simple electrical circuit



  (b) Block diagram in which the circuit is depicted as the feedback interconnection of two circuit elements



1.6 Basic System Properties

- Key Concepts
  - Memory and memoryless (1.6.1)
  - Invertibility (1.6.2)
  - Causaility (1.6.3)
  - Stability and BIBO stable (1.6.4)
  - Time invariance (1.6.5)
  - Linearity (additivity property for a linear system) (1.6.6)
  - Superposition property for a linear system
  - Incrementally linear system and zero-input response

1.6.1 Systems with and without Memory

- CT
  - If $y(t)$ is independent of $x(t+\tau)$ where $\tau \neq 0$, the systems is memoryless.
- DT
  - If $y[n]$ is independent of $x[n+k]$ where $k \neq 0$, the systems is memoryless.
- Example
  - Memoryless systems $y[n] = (2x[n] - x[n]^2)^2$

    $y[n] = x[n]$ (identity)

    $y(t) = x(t)$ (identity)

  - Systems with memory

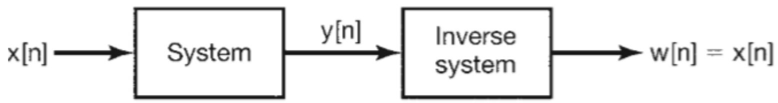
    $$y[n] = \sum_{k=-\infty}^{n} x[k] \quad \text{(accumulator)}$$

    $$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \quad \text{(integral)}$$

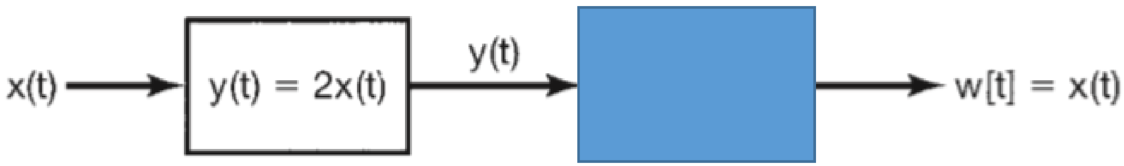
    $$y[n] = x[n-1] \quad \text{(delay)}$$

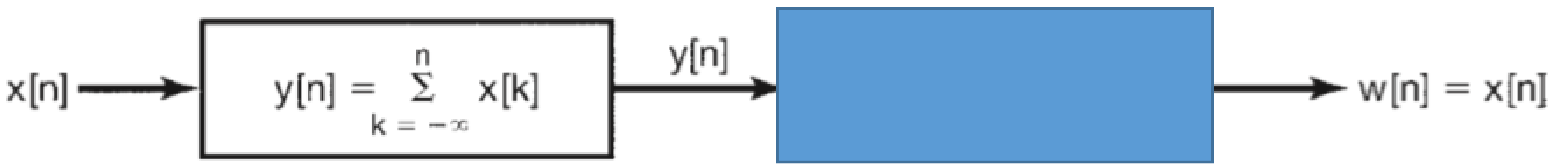
1.6.2 Invertibility and Inverse Systems

- For a system, if there exits another system that can retrieve the input from the output, then the system is invertible.



- Examples
  - Is $y(t) = 2x(t)$ invertible?
  - Is the summation operation reversible?
  - Is $y(t) = x(t)^2$ invertible?





1.6.3 Causality

- Causal systems
  - The present output ($y(t)$ or $y[n]$) depends only on the input at the present time ($x(t)$ or $x[n]$) & those in the past.
  - Future inputs do NOT affect the present output.

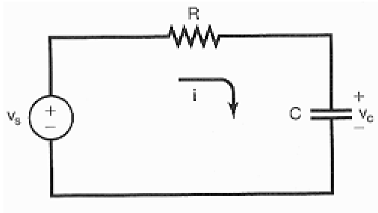
  - In a causal CT system
    $y(t)$ is independent of $x(t+\tau)$ where $\tau > 0$.

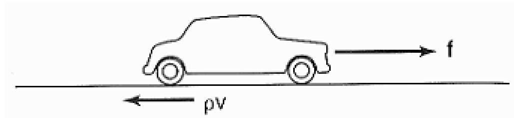
  - In a causal DT system
    $y[n]$ is independent of $x[n+k]$ where $k > 0$.

1.6.3 Causality (cont'd)

- Causal systems $y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau$ $\quad y[n]=x[n]-x[n-1]$

  Circuit System and motion system are also causal systems,
  since the future input is impossible to affect the present output.



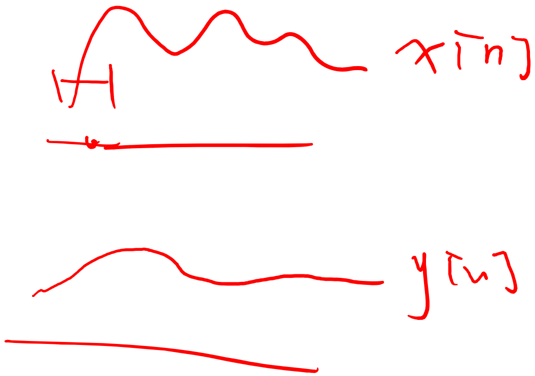


- Non-causal systems

$$y[n] = x[n] - x[n+1]$$

$$y(t) = x(t+1)$$

$$y[n] = \frac{1}{2M+1}\sum_{k=-M}^{+M} x[n-k].$$



→ moving average

1.6.3 Causality (cont'd)

- Example 1.12
  Are the following two systems causal and why?
  (i) $y[n] = x[-n]$ ✗
  (ii) $y(t) = x(t)\cos(t+1)$ ✓

(i) $y[0] = x[0]$

$y[1] = x[-1]$

$y[-1] = x[1]$

(ii) $y(t) = x(t) \cdot \underline{g(t)}$

1.6.4 Stability

- Stable systems do not diverge for bounded inputs.
  - Input: $x(t)\ or\ x[n]$
  - Output: $y(t)\ or\ y[n]$
- Bounded-input bounded-output stable (BIBO stable)
  - Continuous case: $|y(t)| < \infty$ for all t, if $|x(t)| < \infty$ for all t
  - Discrete case: $|y[n]| < \infty$ for all t, if $|x[n]| < \infty$ for all t
  - Example: Are the following systems BIBO and why?

$$y[n] = x[n]^2$$

$$y[n] = 1.01y[n-1] + x[n] \text{ (think about your bank account)}$$

1.6.4 Stability (cont'd)

- Example: Are the following systems BIBO and why?

  (i) $y(t) = tx(t)$ X

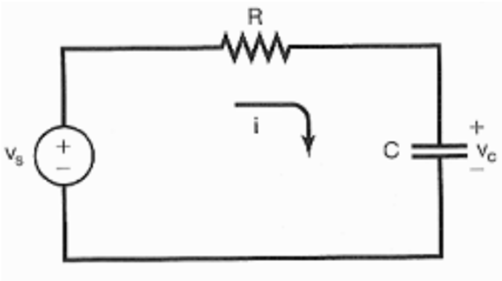
  (ii) $y(t) = e^{x(t)}$ V

$$-B \leq x(t) \leq B$$

$$e^{-B} \leq y(t) \leq e^{B}$$

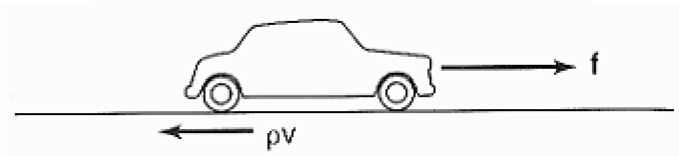
1.6.5 Time Invariance

- Time-invariant systems
  - Behavior & characteristics of a system are fixed over time.



Input signal: $v_s(t)$

Output signal: $v_c(t)$



Input signal: force $f(t)$

Output signal: velocity $v(t)$

- If R, C, m (mass of the vehicle), and ρ are fixed, then the above two systems are all time-invariant.

1.6.5 Time Invariance

- Time-invariant systems
  - Behavior & characteristics of a system are fixed over time.
  - For a time-invariant system, a time shift in the input signal results in an identical time shift in the output signal.

$$x[n] \to y[n] \quad \Longleftrightarrow \quad x[n-n_0] \to y[n-n_0]$$

$$x(t) \to y(t) \Leftrightarrow x(t-t_0) \to y(t-t_0)$$

1.6.5 Time Invariance (cont'd)

- Examples 1.14 & 1.15
  - Is the following system time-invariant?

$$y(t) = \sin[x(t)]$$

$x_1(t)$ $\qquad y_1(t) = \sin[x_1(t)]$

$x_2(t) = x(t - t_0)$

$x_2(t) = x_1(t - t_0) \qquad y_3(t) = \sin[x_2(t)] = \sin[x_1(t - t_0)]$

$y_2(t) = \sin(x_2(t))$

$\cancel{y_2(t) = y_1}(t - t_0)$

$= \sin(x(t - t_0))$

$= y(t - t_0)$ *

- Is the following system time-invariant? And counter examples?

$$y[n] = nx[n]$$

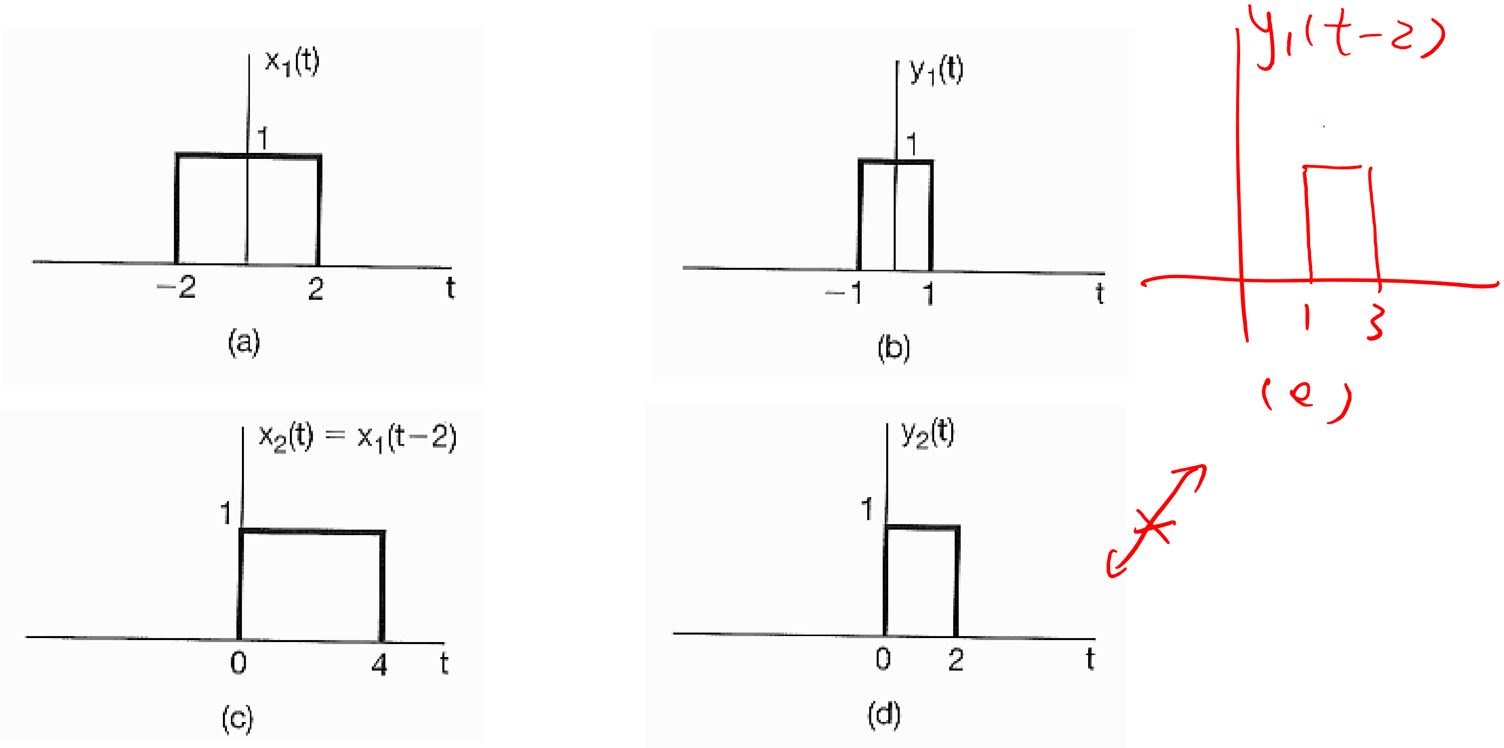
$\downarrow \quad \downarrow$

$n \, x[n - n_0]$

" $y[n - n_0] = (n - n_0) \, x[n - n_0]$ "

1.6.5 Time Invariance (cont'd)

- Examples 1.16
  - Is the system $y(t) = x(2t)$ time-invariant? (X)
    Can you give a quick guess to the answer?



1.6.6 Linearity

- Linear Systems
  - Take DT systems for an example. Suppose that $y_1[n]$ and $y_2[n]$ are the outputs of a system when the inputs are $x_1[n]$ and $x_2[n]$, respectively.
  - If we have
    (1) $x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$ (additivity)
    (2) $\mathrm{a}x_1[n] \rightarrow ay_1[n]$ (scaling or homogeneity)

    a : any complex #

    Then, the system is linear.

1.6.6 Linearity (cont'd)

- Superposition Property of Linear Systems
  - For a DT liner system:

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + \ldots$$

$$\longrightarrow \; y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + \ldots$$

if $\quad x_k[n] \to y_k[n]$

  - For a CT linear system:

$$x(t) = \sum_k a_k x_k(t) = a_1 x_1(t) + a_2 x_2(t) + \cdots\cdots$$

$$\to y(t) = \sum_k a_k y_k(t) = a_1 y_1(t) + a_2 y_2(t) + \cdots\cdots$$

if $\quad x_k(t) \to y_k(t)$

1.6.6 Linearity (cont'd)

- Examples 1.18 & 1.19
  ✗ - Is the system $y(t) = (x(t))^2$ linear? ?

$x(t) = a x_1(t) + b x_2(t) \Leftrightarrow y(t) = a y_1(t) + b y_2(t)$

  - Is the system $y[n] = \mathcal{Re}\{x[n]\}$ linear? ✗

$x[n] = x_r[n] + j x_i[n]$

$j x[n] = j x_r[n] - x_i[n]$

1.6.6 Linearity (cont'd)

- Example 1.20
  - Is the system $y[n] = 2x[n] + 3$ linear?
    Does it satisfy the "zero-in/zero-out" property of linear systems?

$$x[n] = a x_1[n] + b x[n]$$

$$y[n] = 2a x_1[n] + 2b x[n] + 3$$

$$\not\parallel$$

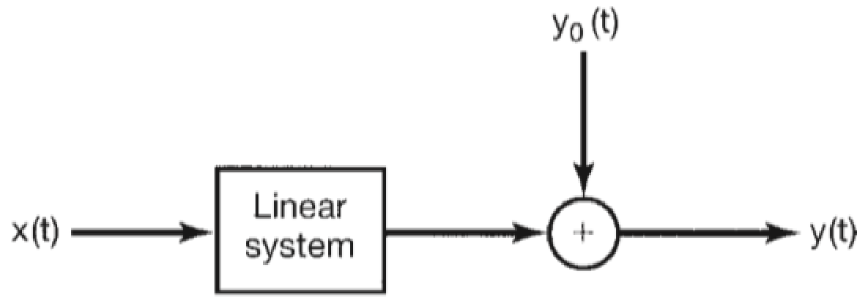
$$a y_1[n] = 2a x_1[n] + 3a$$

$$b y_2[n] = 2b x_2[n] + 3a$$

- Actually, the above system is incrementally linear…Why?

1.6.6 Linearity (cont'd)

- Structure of an Incrementally Linear System
  - Incrementally linear system = linear system + zero-input response



$y_0(t)$: zero-input response (the response when the input is $x(t) = 0$)

- Example 1.20 $y[n] = 2x[n] + 3$ (linear system + zero-input response)

$\Rightarrow y[n-1] = 2x[n-1] + 3$

$\boxed{y[n] - y[n-1] = 2(x[n] - x[n-1])}$

Chapter 1 Signals & Systems

- Sec. 1.1 Continuous-Time & Discrete-Time Signals
- Sec. 1.2 Transformations of the Independent Variable
- Sec. 1.3 Exponential & Sinusoidal Signals
- Sec. 1.4 The Unit Impulse & Unit Step Functions
- Sec. 1.5 Continuous-Time & Discrete-Time Systems
- Sec. 1.6 Basic System Properties
- HW #1 will be out soon!

Signals

Systems

Chapter 2 Linear Time Invariant Systems

- Sec. 2.1 Discrete-time LTI Systems: The Convolution Sum
- Sec. 2.2 Continuous-time LTI Systems: The Convolution Integral
- Sec. 2.3 Properties of Linear Time-invariant Systems
- Sec. 2.4 Causal LTI Systems Described by Differential and Difference Equations
- Sec. 2.5 Singularity Functions
- Sec. 2.6 LTI Systems in the Multiple Dimensional Case
- Sec. 2.7 Several Well-known LTI Systems
- Sec. 2.8 Summary

2.1 DT LTI Systems: The Convolution Sum

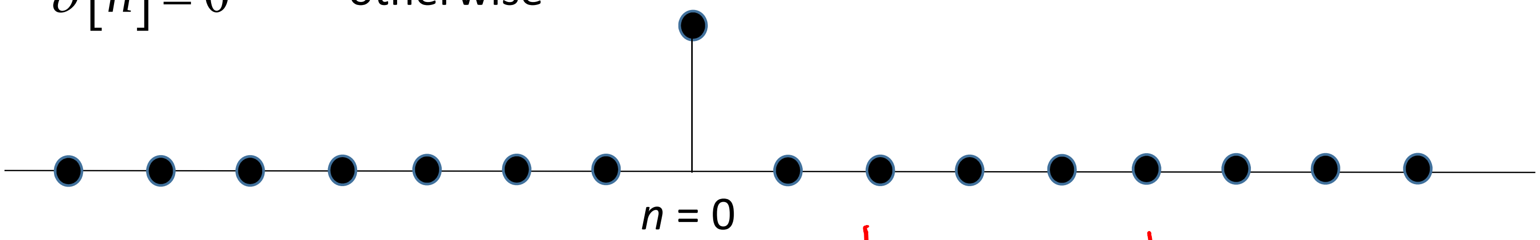
- Highlights
  - Definition of DT convolution
  - Unit impulse response
  - Any DT LTI system can be modeled by a DT convolution operation.
  - Any DT signals can be represented by a sum of impulses.

2.1.1 Representation of DT Signals in term of Impulses

- Recall that, unit impulses are…

$\delta[n] = 1$ when n = 0,

$\delta[n] = 0$ otherwise



linear combination

- Any DT signals can be represented by a sum of impulses.

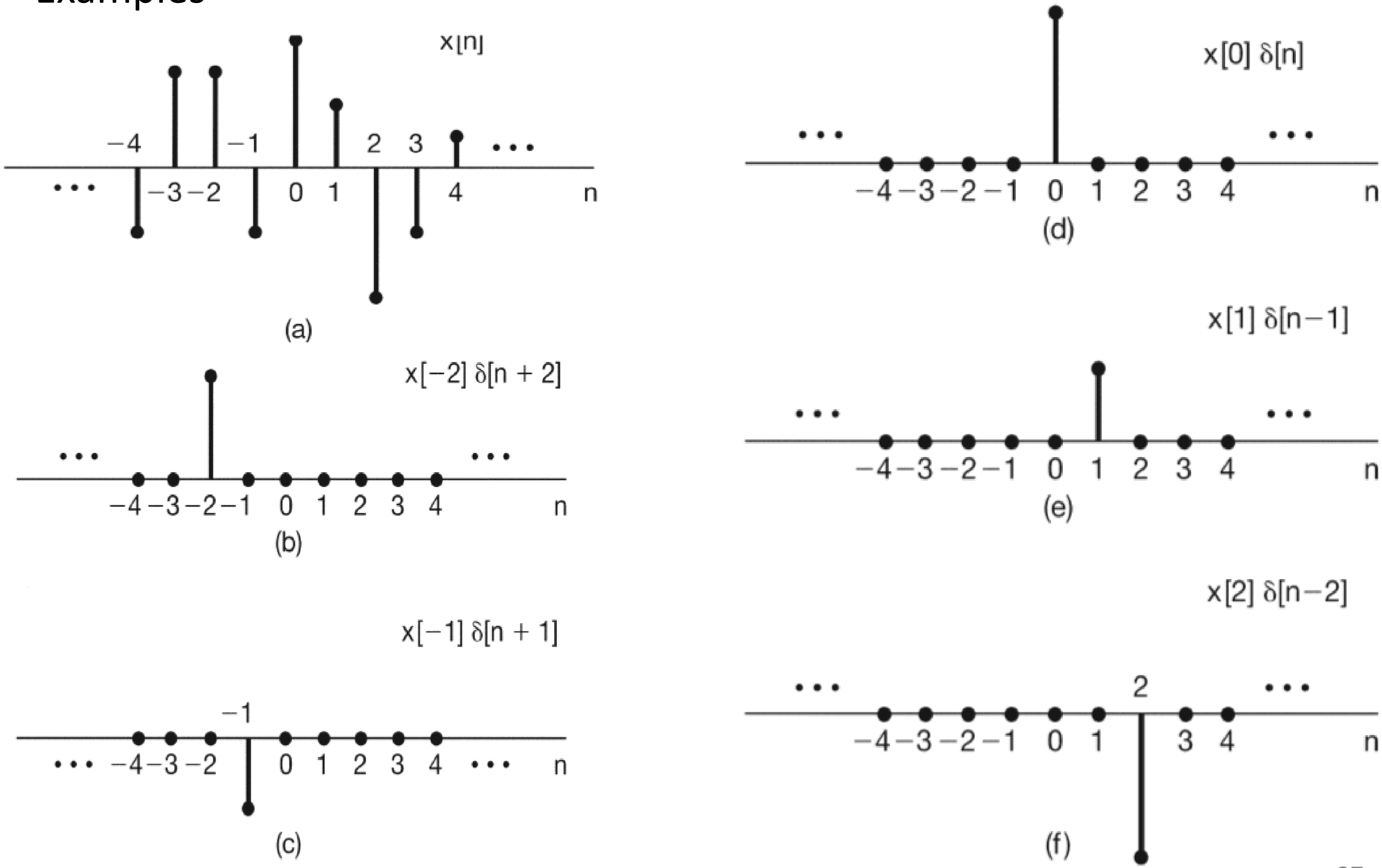
coeff

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + x[3]\delta[n-3] + \ldots$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\boxed{\delta[n-k]}.$$

2.1.1 Representation of DT Signals in term of Impulses

- Examples



2.1.2 The DT Unit Impulse Response and the Convolution Sum Representation of LTI Systems

- For a DT signal, we can represent it as:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

- If a system is linear, then its output corresponding to $x[n]$ can be expressed as:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

where $\boxed{h_k[n]}$ is the system output with $\delta[n-k]$ as input.

- Why?

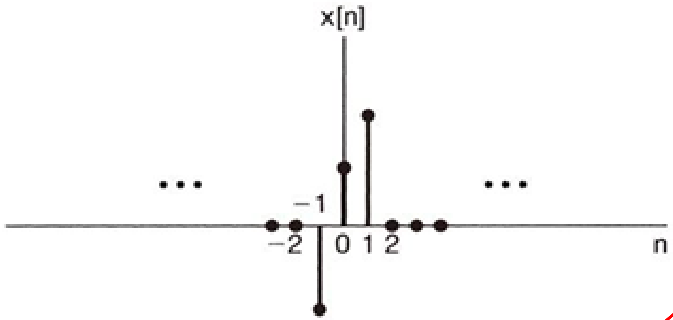
Input

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Output

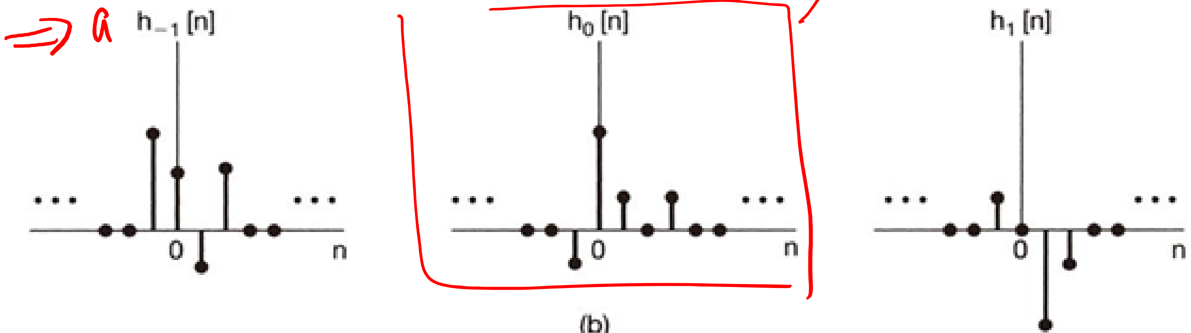
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\boxed{h_k[n]} + \boxed{}$$

$\delta[n] \rightarrow \boxed{} \rightarrow h_0[n]$



(a)

$a\,\delta[n+1] \Rightarrow a$



(b)

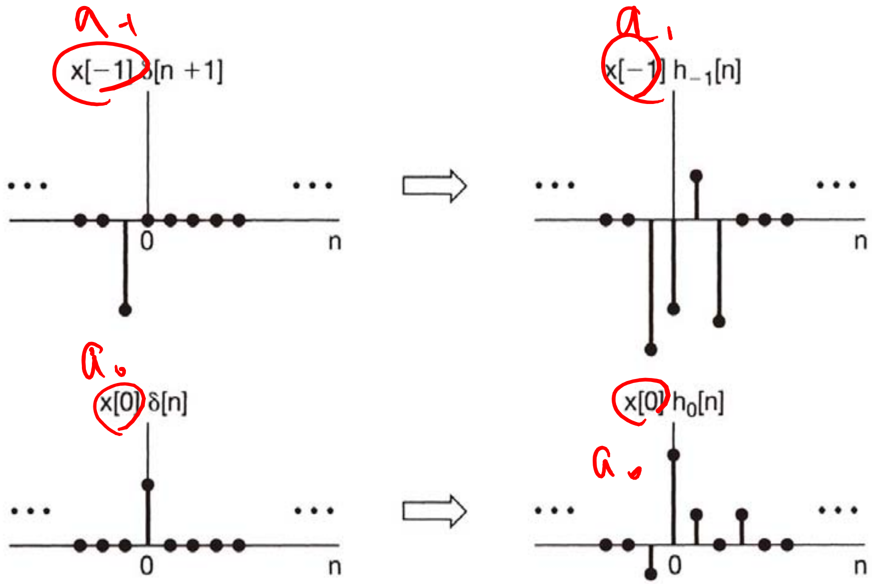
For a system that is linear but not time-invariant

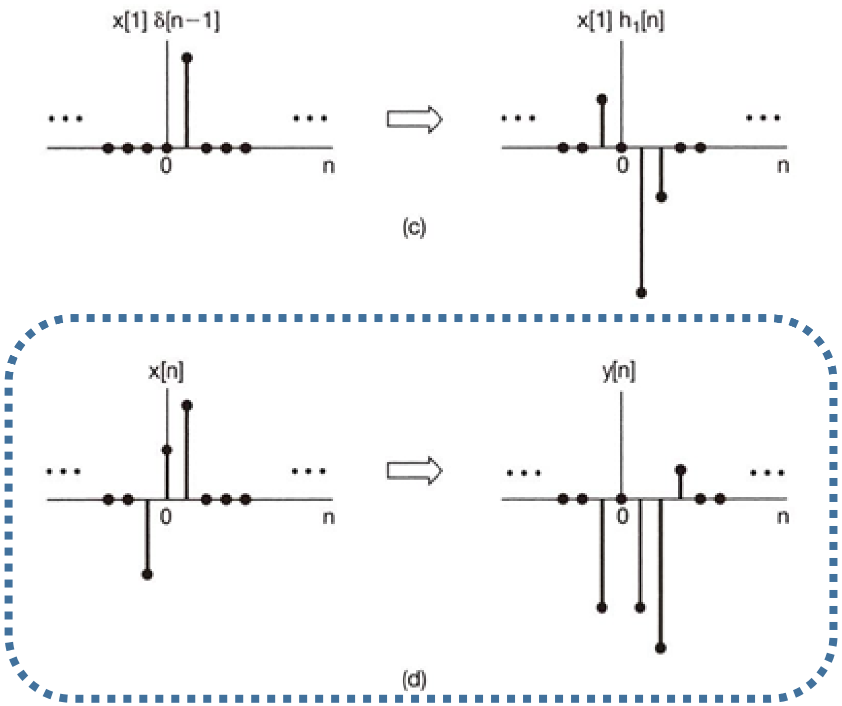
Input

$$x[n]=\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Output

$$y[n]=\sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$





For a system that is linear but not time-invariant

Input

$$x[n]=\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Output

$$y[n]=\sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

- If the system is time-invariant, then

$$h_k[n] = h_0[n-k] \checkmark$$

- Denote $h_0[n]$ by $h[n]$, we have

$$y[n]=\sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

This is the Discrete-Time Convolution.

- The convolution operator is typically denoted by *.

$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

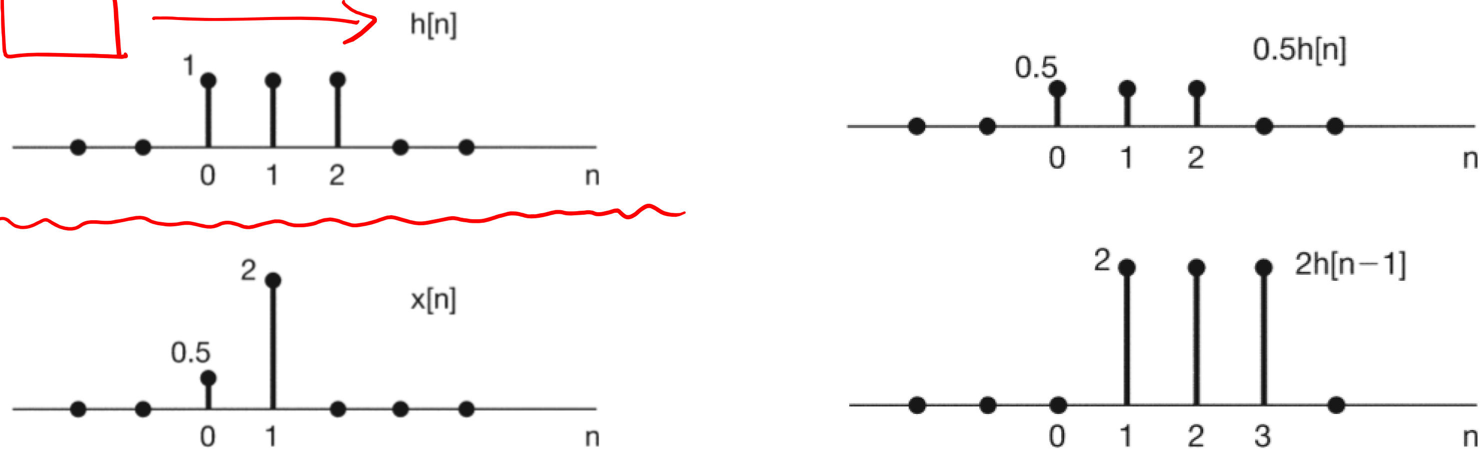
2.1 DT LTI Systems: The Convolution Sum

- Highlights
  - Definition of DT convolution
  - Unit impulse response
  - Any DT LTI system can be modeled by a DT convolution operation.
  - Any DT signals can be represented by a sum of impulses.

Example 2.1
Consider an LTI system with impulse response $h[n]$ and input $x[n]$.
We have

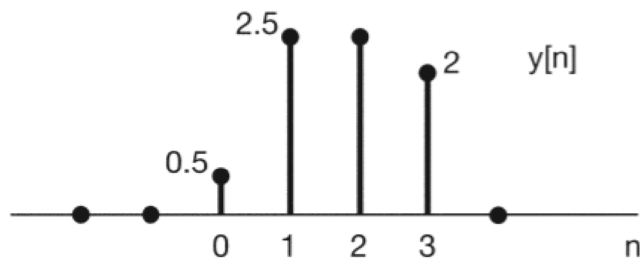
$$y[n] = x[n] * h[n] = x[0]h[n-0] + x[1]h[n-1] = 0.5h[n] + 2h[n-1].$$

$\delta[n] \rightarrow$



(a) (b)

$$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$



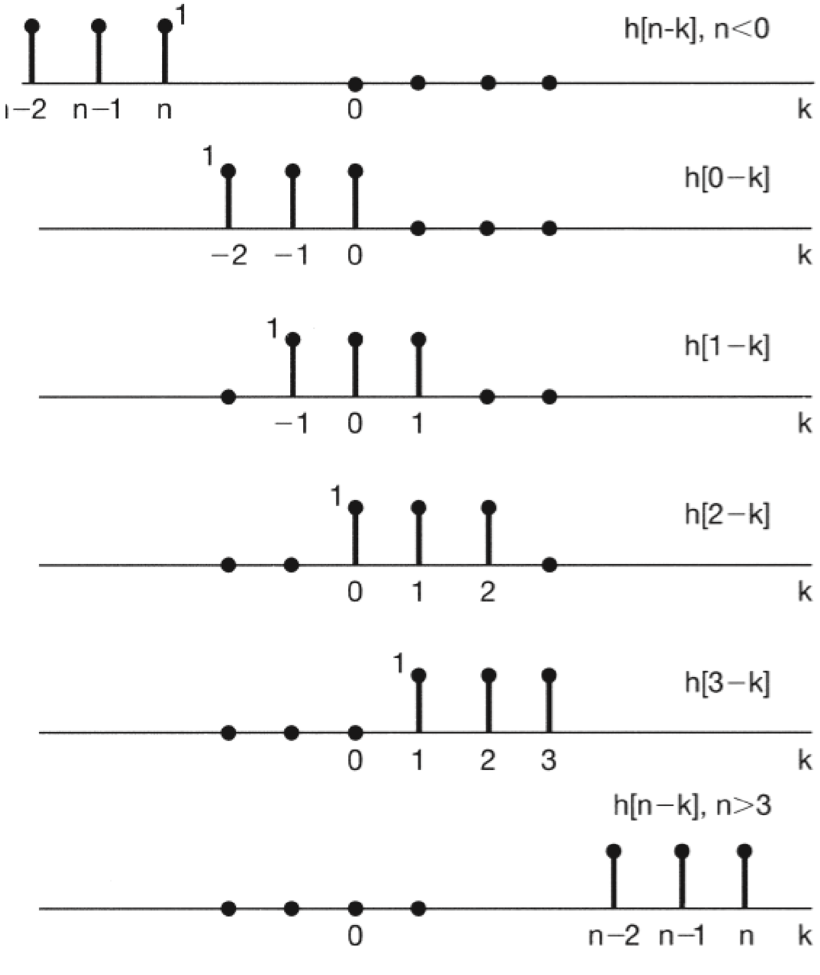
(c)

Example 2.2
Same as Example 2.1 but from a different point of view…

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

That is, $y[n]$ is the sum of the products of $x[k]$ and $h[n-k]$ over k.

Example 2.2 $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$



(b)

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = 0.5.$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = 0.5 + 2.0 = 2.5.$$

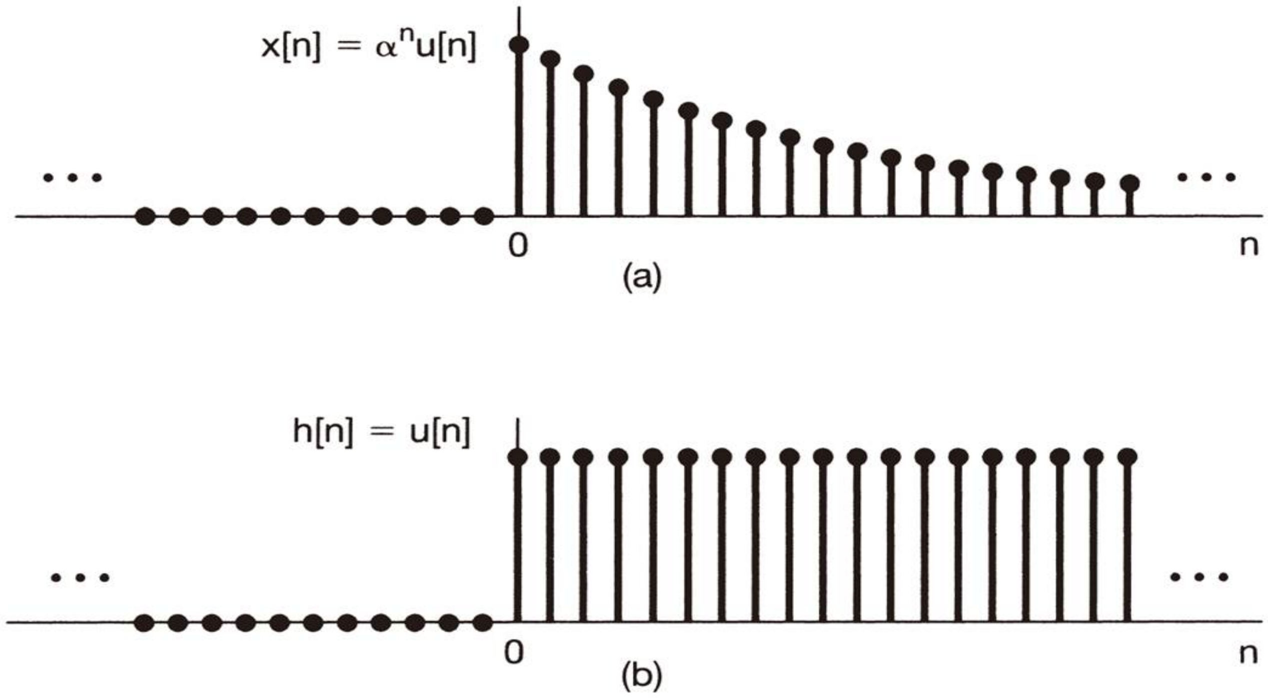
$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = 0.5 + 2.0 = 2.5.$$

$$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k] = 2.0.$$

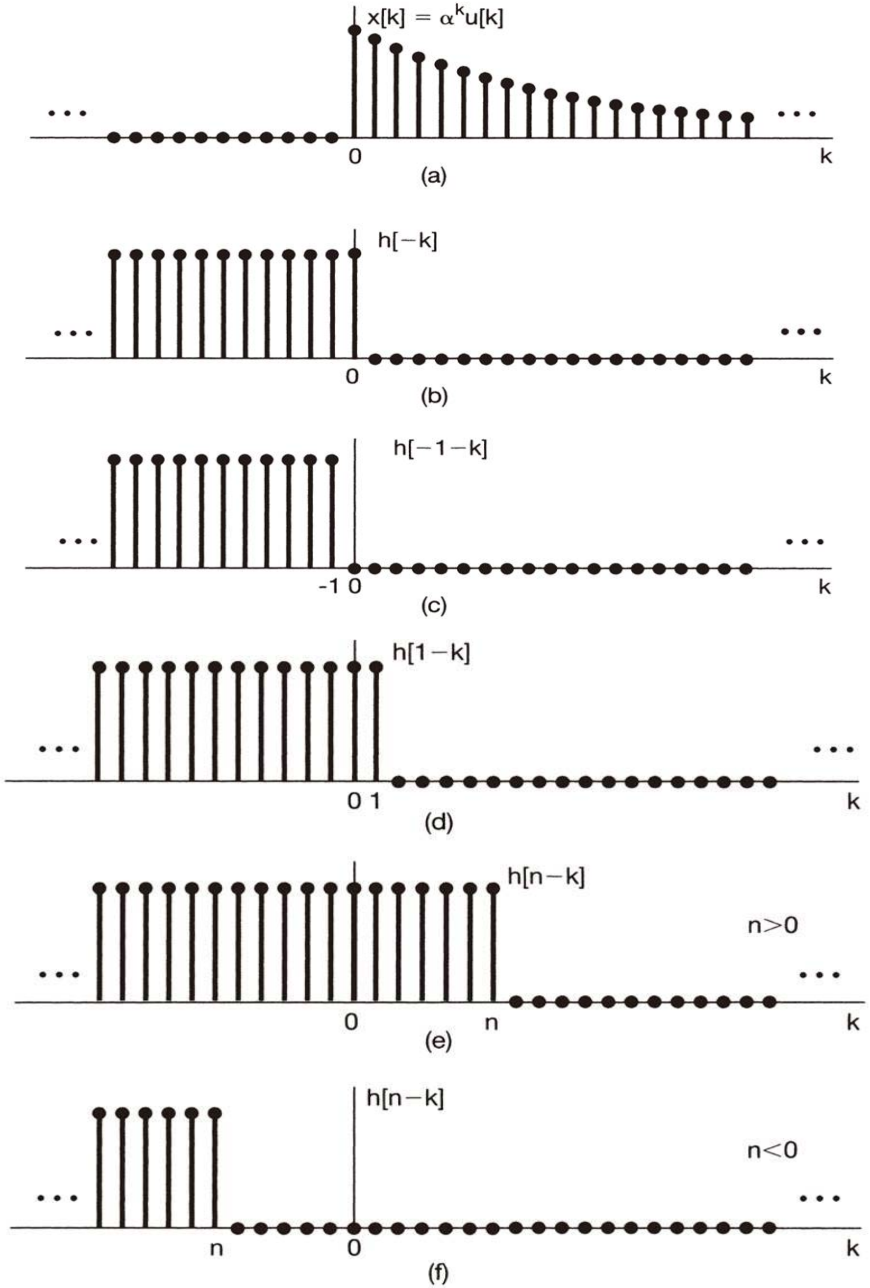
$y[n] = 0$ otherwise

Example 2.3 $x[n] = \alpha^n u[n],$

$h[n] = u[n],$



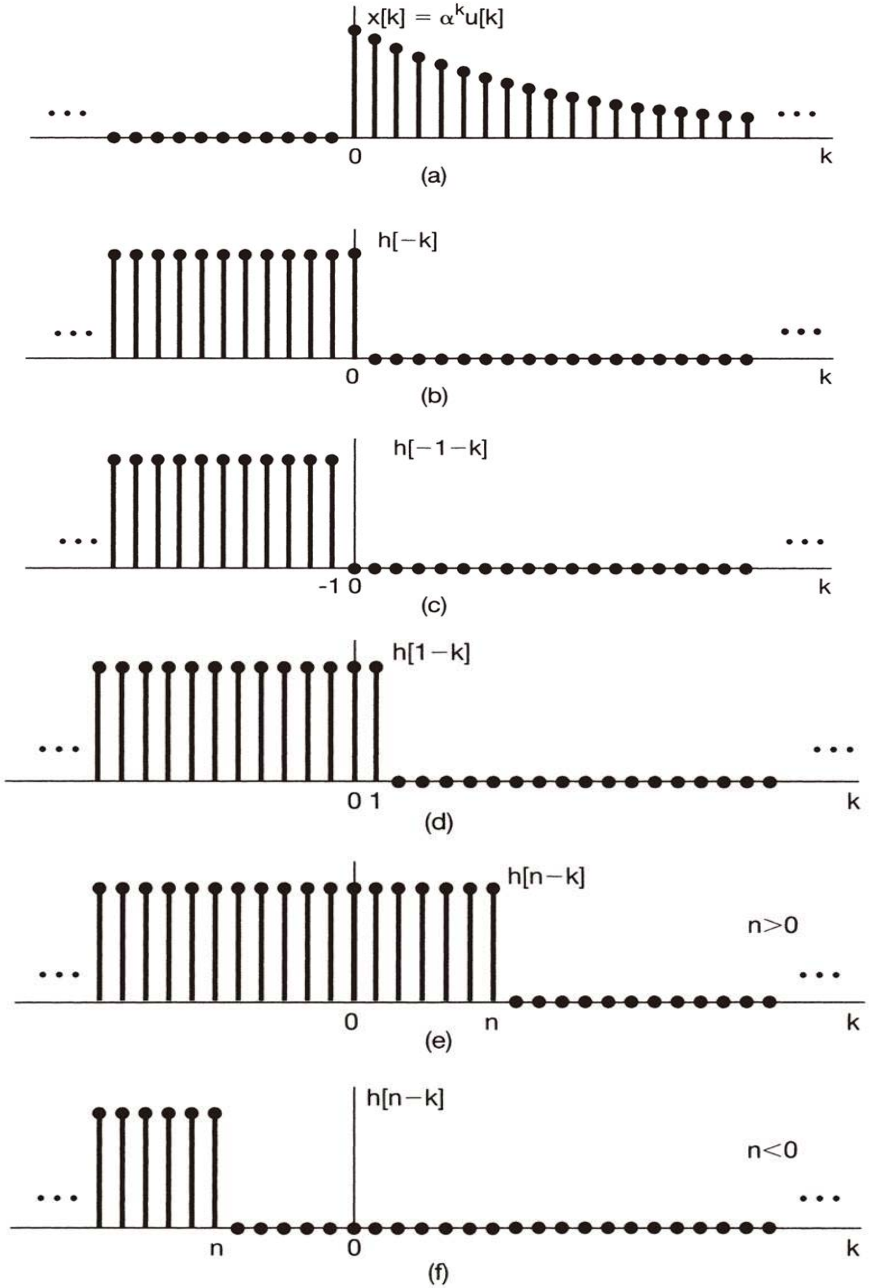
Example 2.3 (cont'd) $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$



$$x[n] = \alpha^n u[n],$$

$$h[n] = u[n],$$

Example 2.3 (cont'd) $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$



$$x[n] = \alpha^n u[n],$$

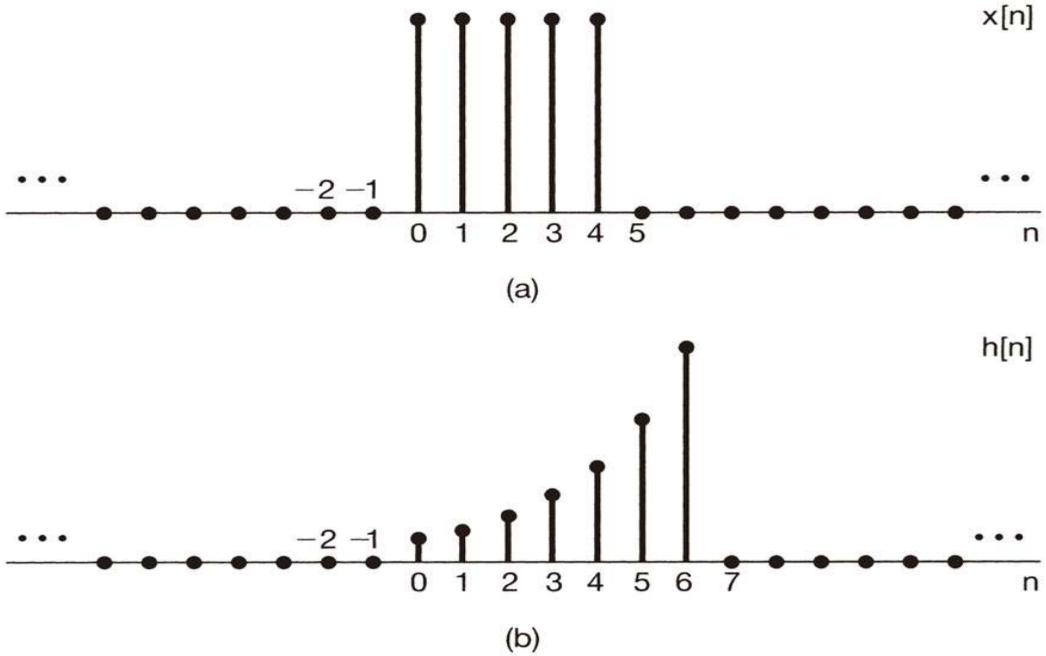
$$h[n] = u[n],$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{n} \alpha^k,$$

$$\boxed{y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)u[n].}$$

Example 2.4 $x[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & otherwise \end{cases}$ $h[n] = \begin{cases} \alpha^n, & 0 \le n \le 6 \\ 0, & otherwise \end{cases}$.

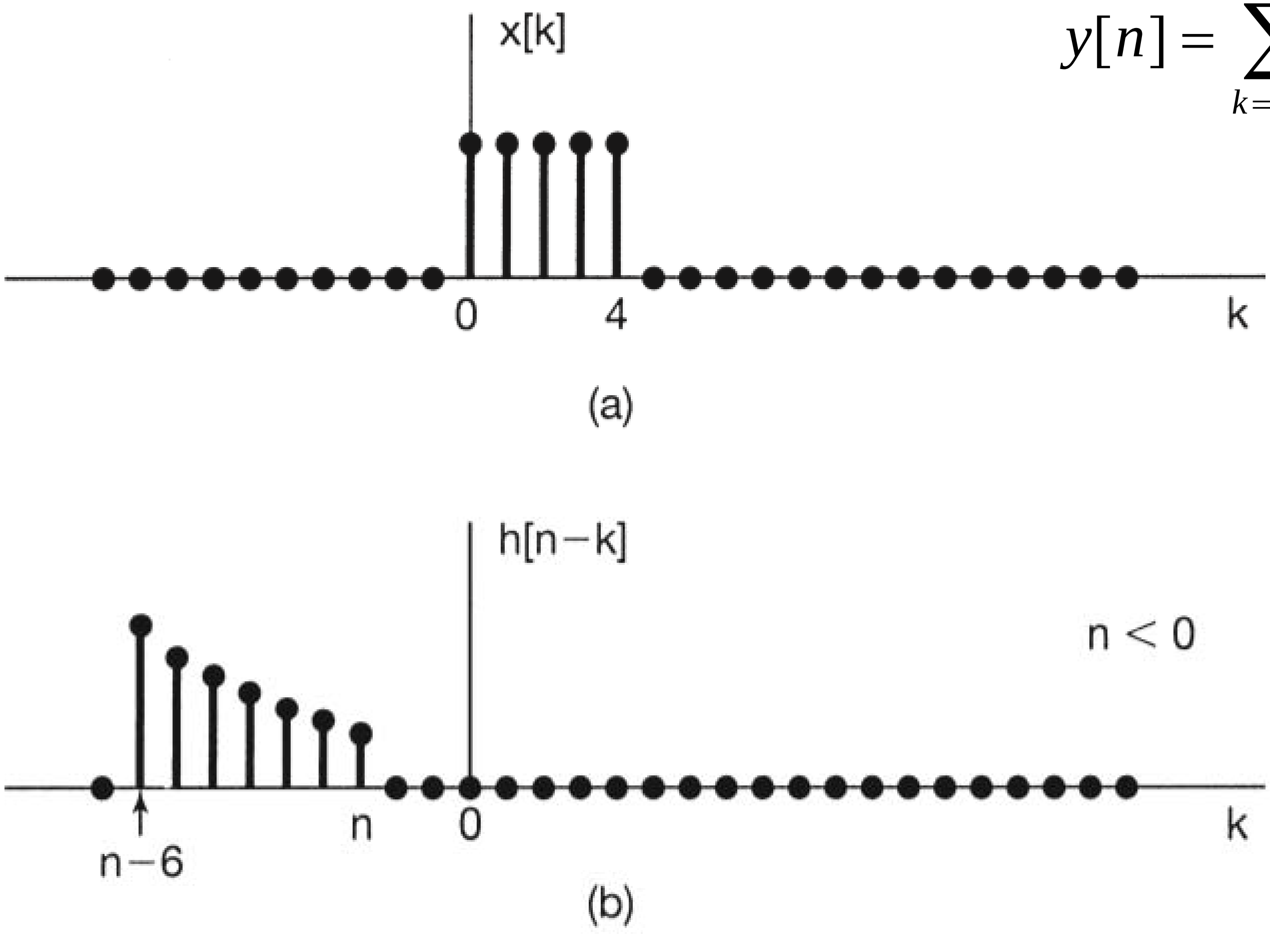
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



Example 2.4 (cont'd)

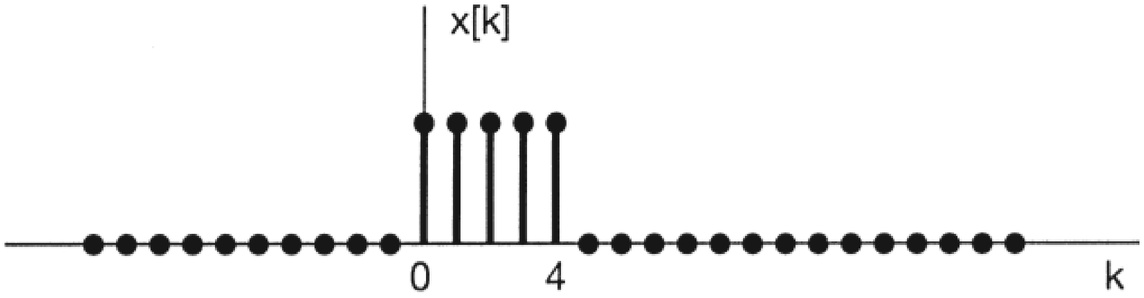
Interval 1: $n < 0$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = 0$$

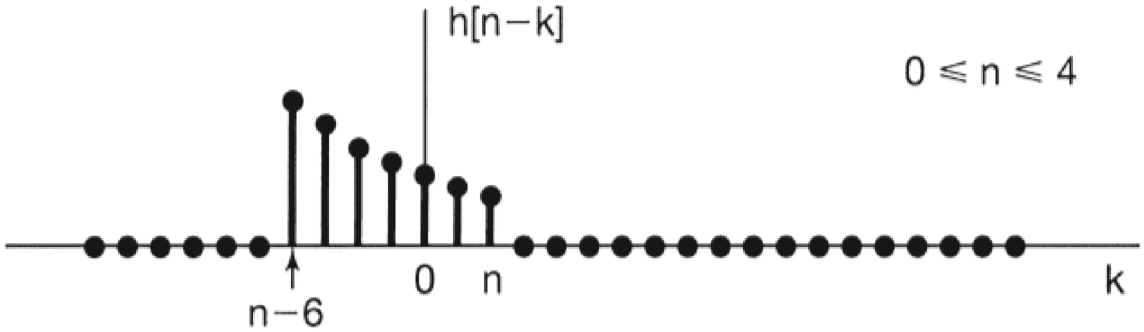


Example 2.4 (cont'd)

Interval 2: $0 \leq n \leq 4$ $$y[n] = \sum_{r=0}^{n} \alpha^r = \frac{1-\alpha^{n+1}}{1-\alpha}.$$



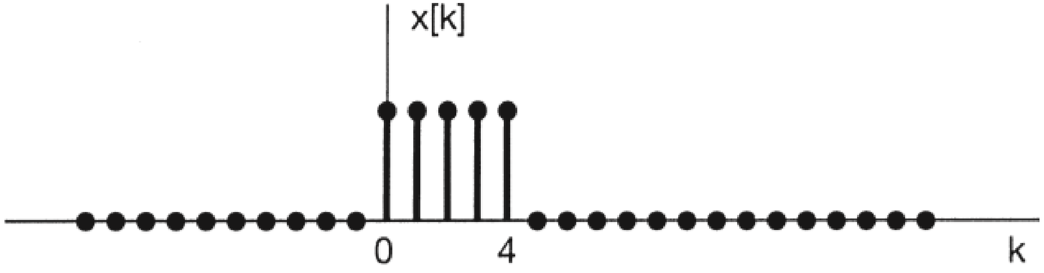
(a)



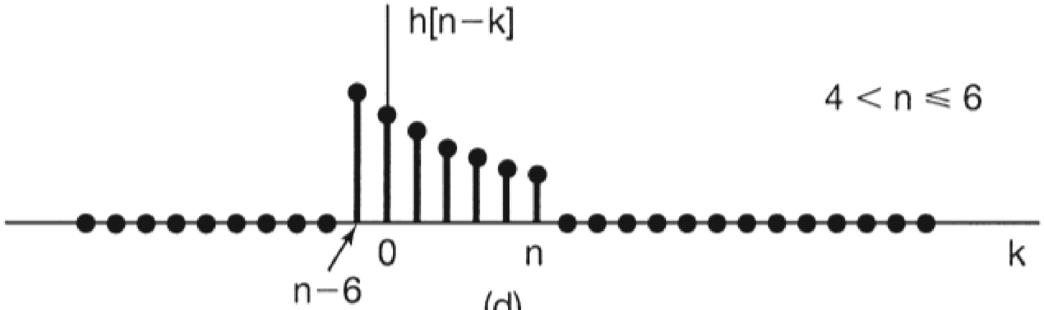
(c)

Example 2.4 (cont'd)

Interval 3: $4 < n \leq 6$ $$y[n] = \alpha^n \sum_{k=0}^{4} (\alpha^{-1})^k = \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}.$$



(a)

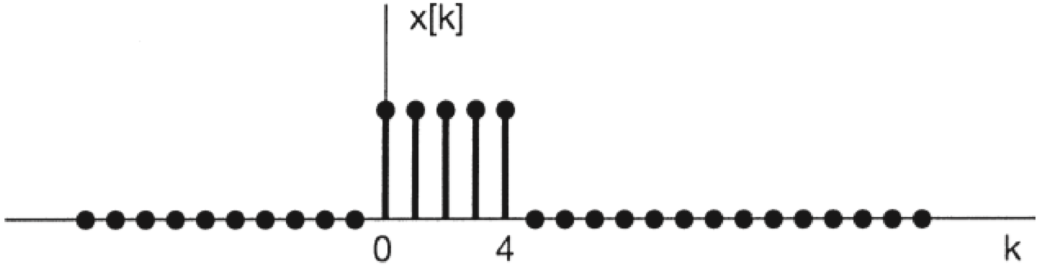


(d)

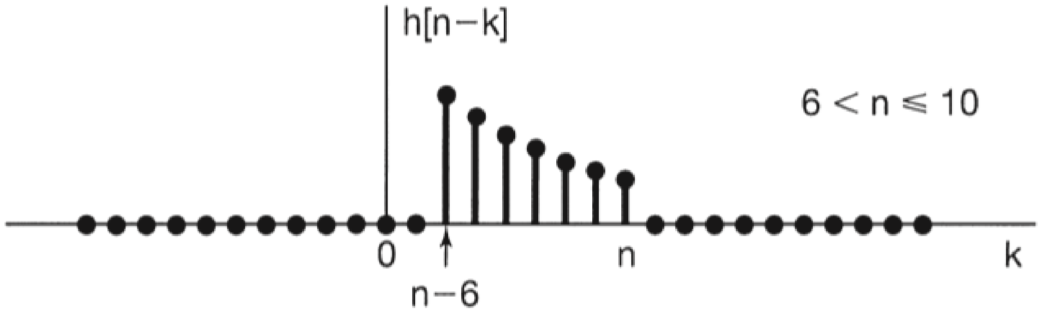
Example 2.4 (cont'd)

Interval 4: $6 < n \le 10$

$$y[n] = \sum_{r=0}^{10-n} \alpha^{6-r} = \frac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}.$$



(a)

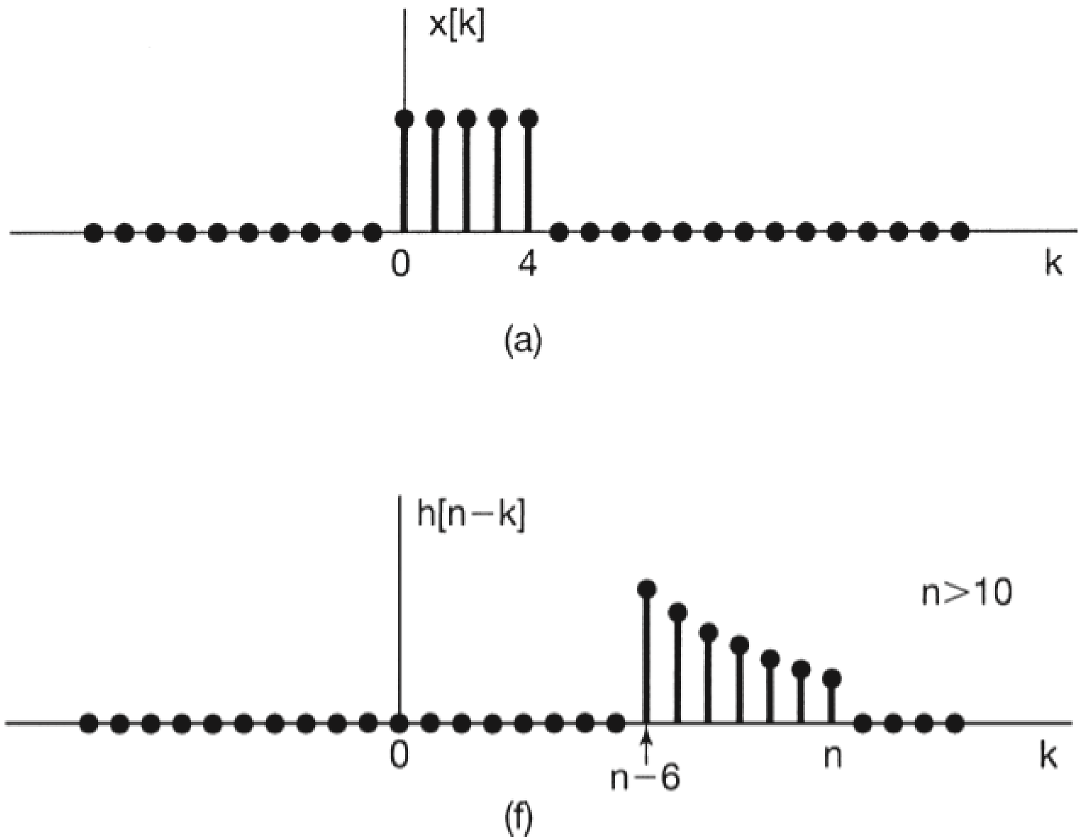


(e)

Example 2.4 (cont'd)

Interval 5: $n > 10$

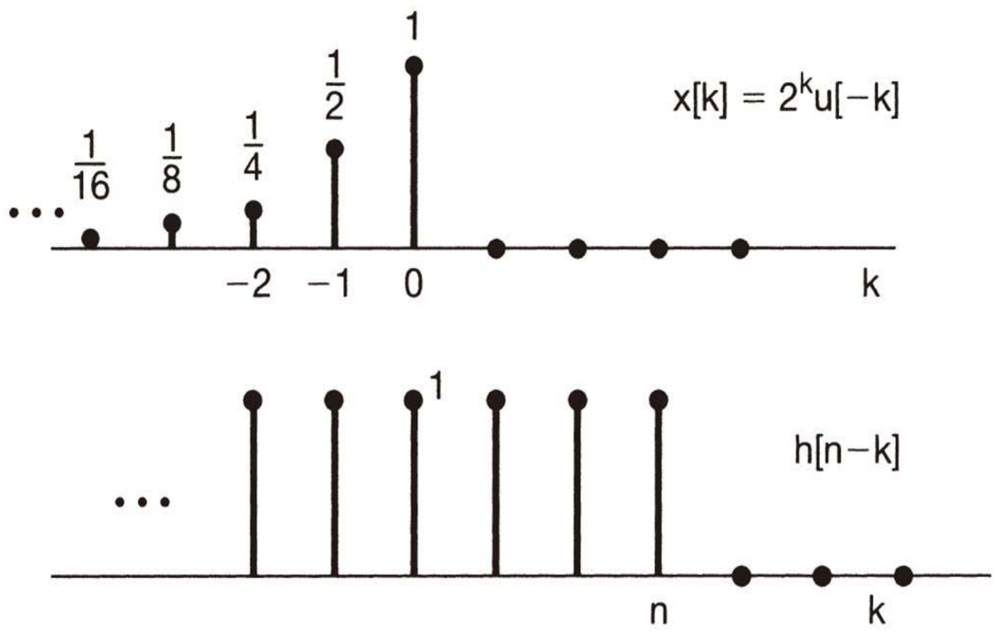
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = 0.$$



Example 2.5

$$x[n] = 2^n u[-n], \qquad h[n] = u[n].$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



(a)

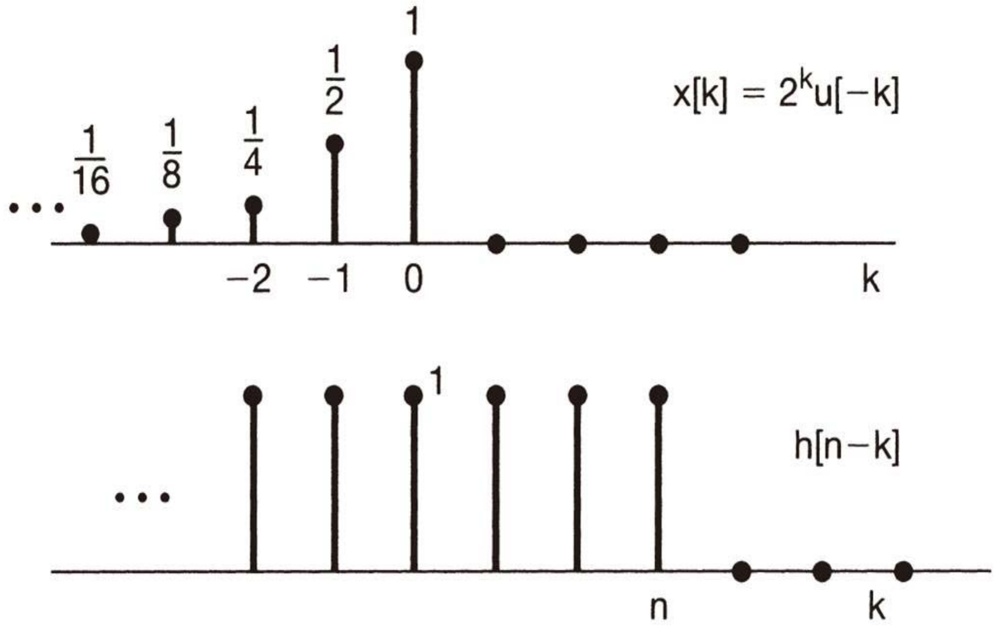
Interval 1: $n \geq 0$

$$y[n] = \sum_{k=-\infty}^{0} x[k]h[n-k] = \sum_{k=-\infty}^{0} 2^k = 2$$

Example 2.5

$$x[n] = 2^n u[-n], \qquad h[n] = u[n].$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



(a)

Interval 2: $n < 0$

$$y[n] = \sum_{k=-\infty}^{n} 2^k = \sum_{l=-n}^{\infty} (\frac{1}{2})^l = \sum_{m=0}^{\infty} (\frac{1}{2})^{m-n}$$

$$= \left(\frac{1}{2}\right)^{-n} \sum_{m=0}^{\infty} (\frac{1}{2})^m = 2^n \cdot 2 = 2^{n+1}.$$

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