Signals and Systems HW9

Deadline: 2019/06/10 before 18:30

(You should submit hand-writing paper to BL B1 EE student office.)

1. Consider a discrete-time right-sided LTI system with transfer function

$$H(z)=\frac{1-a^{*}z}{z-a}, |a|<1$$

where a^{*} is the conjugate of a.

(a) (10%) Sketch the pole-zero plot for this system in the z-plane.

(b) (10%) Use the graphical method to show the magnitude of the system's frequency response is unity for all frequencies.

(c) (20%) Use the graphical method to sketch the phase of the system's response for $a=1/2$.

(d) (10%) Use the result from (b) to prove that any system with a transfer function of the form

$$H(z)=\prod_{k=1}^{K}\frac{1-a_k^{*}z}{z-a_k}, |a_k|<1, \forall k=1,\ldots,K$$

corresponds to a stable and causal all-pass system.

2. The final-value theorem states that for a signal $x[n]$ with unilateral z-transform $X(z)$ and for which $x[n]=0$ for n<0, the final value of $x[n]$ (i.e. $\lim_{n\to\infty} x[n]$) can be obtained from $X(z)$ through the relation

$$\lim_{n\to\infty} x[n]=\lim_{z\to 1}(z-1)X(z)$$

(a) (15%) Show that $Z\{x[n+1]-x[n]\}=(z-1)X(z)-zx[0]$, where $Z\{f\}$ denotes the z-transform of f.

(b) (15%) Show that

$$Z\{x[n+1]-x[n]\}=-x[0]+\lim_{n\to\infty}\{\sum_{k=1}^{n-1}x[k]z^{-k+1}(1-z^{-1})+x[n]z^{-n+1}\}$$

(c) (20%) Use the results in (a) and (b) to prove that $\lim_{n\to\infty} x[n]=\lim_{z\to 1}(z-1)X(z)$