Lecture 11

Antoine Abi Chakra

Overview

- Animated Circular Object and Stationary Circular Object
- Collision Response (Reflection)

Modeling Pinball Animation as Ray

- Located at center point B_s at top of frame
- Moving in direction given by normalized vector v and speed k units
- In other words, k is the magnitude of the vector v

$$B(t) = B_s + \vec{v}(t)$$

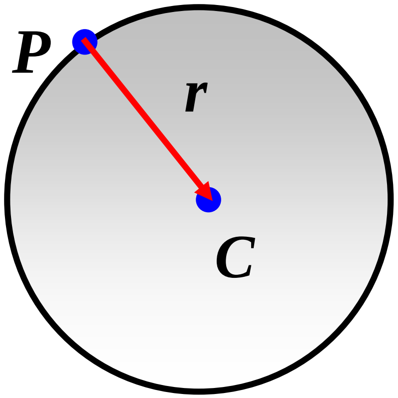
$$\Rightarrow B(t) = B_s + k\hat{v}(t) \qquad t \in [0,1]$$

Circular Pillars

- Circular pillars are stationary and defined by center point C and radius r
- Boundary of circle defined as all points P whose distance from center C is equal to radius r

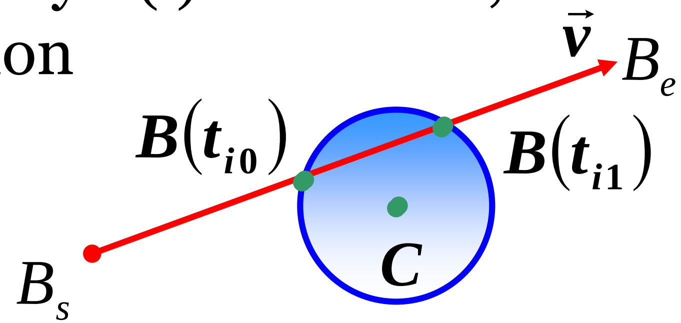
$$\|\boldsymbol{C}-\boldsymbol{P}\| = \boldsymbol{r} \quad \Rightarrow \|\boldsymbol{C}-\boldsymbol{P}\|^2 = \boldsymbol{r}^2$$

$$\Rightarrow (C-P)^2 = r^2$$



Ray-Circle Intersection (1/6)

- To solve for intersection between ray $B(t)$ and circle, replace P with $B(t_i)$ in circle equation



$$(C - B(t))^2 = r^2$$

$$\Rightarrow (C - B_s - t\vec{v})^2 = r^2$$

- Note: $C - B_s = \overrightarrow{B_sC}$

$$\Rightarrow t^2(\vec{v} \bullet \vec{v}) - 2t\left(\overrightarrow{B_sC}\right) \bullet \vec{v} + \left(\overrightarrow{B_sC}\right) \bullet \left(\overrightarrow{B_sC}\right) - r^2 = 0$$

Ray-Circle Intersection (2/6)

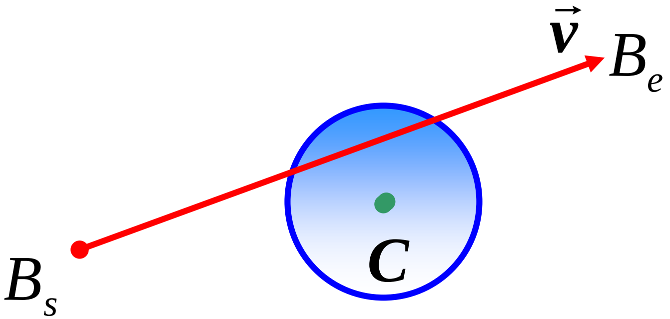
$$\Rightarrow at^2 + bt + c = 0$$

where

$$a = \vec{v} \bullet \vec{v}$$

$$b = -2\left(\overrightarrow{B_s C}\right) \bullet \vec{v}$$

$$c = \left(\overrightarrow{B_s C}\right) \bullet \left(\overrightarrow{B_s C}\right) - r^2$$



Ray–Circle Intersection (3/6)

Solve for t,

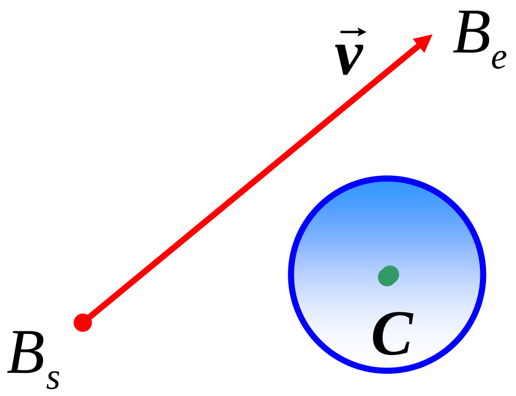
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ray-Circle Intersection (4/6)

$$\text{Given}: \mathrm{a} = \vec{v} \bullet \vec{v}, \ \mathrm{b} = -2\left(C - B_s\right) \bullet \vec{v}, \text{ and}$$

$$\mathrm{c} = \left(C - B_s\right) \bullet \left(C - B_s\right) - r^2$$

$$\text{If } \mathrm{b}^2 - 4\mathrm{ac} < 0 \Rightarrow \text{ray misses circle}$$

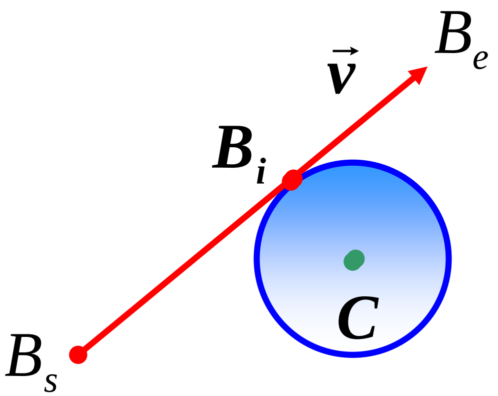


Ray-Circle Intersection (5/6)

$$\text{If } \mathrm{b}^2 - 4\mathrm{ac} \equiv 0 \Rightarrow \text{ray grazes circle}$$

$$t_i = \frac{-\mathrm{b}}{2\mathrm{a}} \in [0,1]$$

$$\boldsymbol{B_i} = \boldsymbol{B}(\boldsymbol{t_i}) = \boldsymbol{B_s} + \vec{\boldsymbol{v}}\boldsymbol{t_i}$$



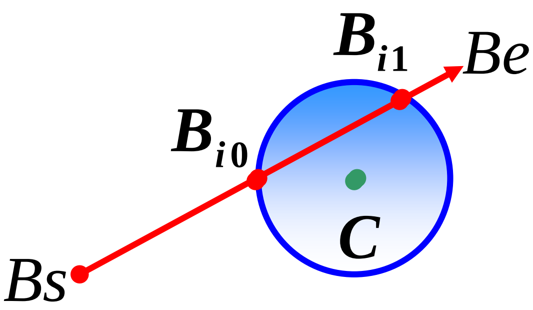
Ray-Circle Intersection (6/6)

$$\text{If } \mathrm{b}^2 - 4\mathrm{ac} > 0 \Rightarrow \text{ray intersects circle at } B_{i0} \text{ and } B_{i1}$$

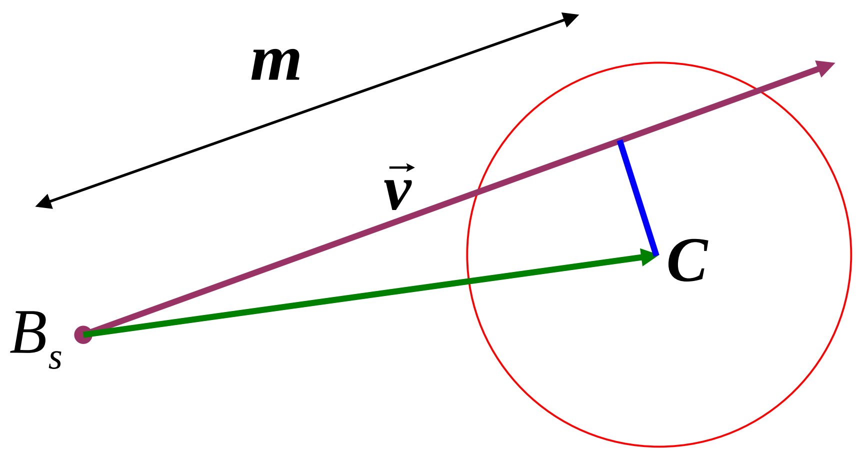
$$t_{i0} = \frac{-\mathrm{b} - \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}} \qquad t_{i1} = \frac{-\mathrm{b} + \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$$

$$t_i = \min(t_{i0}, t_{i1}) \text{ and } t_i \in [0,1]$$

$$\boldsymbol{B}(\boldsymbol{t_i}) = \boldsymbol{B_i} = \boldsymbol{B_s} + \vec{\boldsymbol{v}}\boldsymbol{t_i}$$



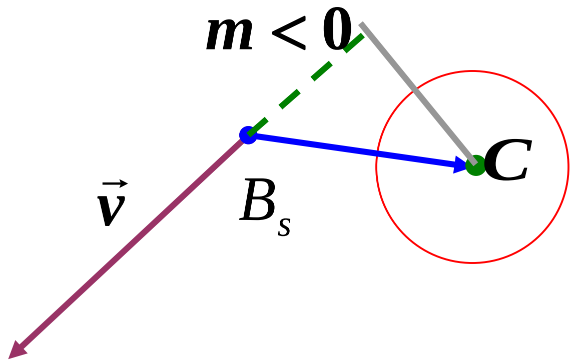
Test for Non-Collision (1/3)



Compute projection of $\overrightarrow{B_s C}$ onto $\hat{v}$

Test for Non-Collision (2/3)

$$m = \overrightarrow{B_s C} \bullet \frac{\vec{v}}{\|\vec{v}\|}$$



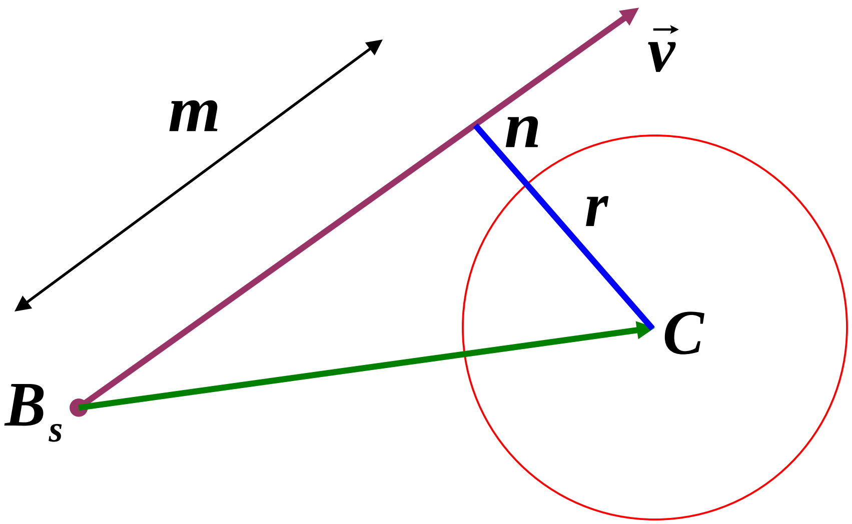
If $m < 0$ && B_s outside circle

$\Rightarrow$ circle behind ray origin

Test for Non-Collision (3/3)

$$\text{Compute}: \; n^2 = \left\| B_s C \right\|^2 - m^2$$

If $n^2 > r^2$ ray will miss the circle



… Otherwise: Compute t_i (1/2)

- There are two ways to compute the time of intersection:
  - Using the quadratic equation

$$t_{i0} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad t_{i1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$t_i = \min(t_{i0}, t_{i1}) \text{ and } t_i \in [0,1]$$

(Or make sure that the intersection point is between B_s and B_e)

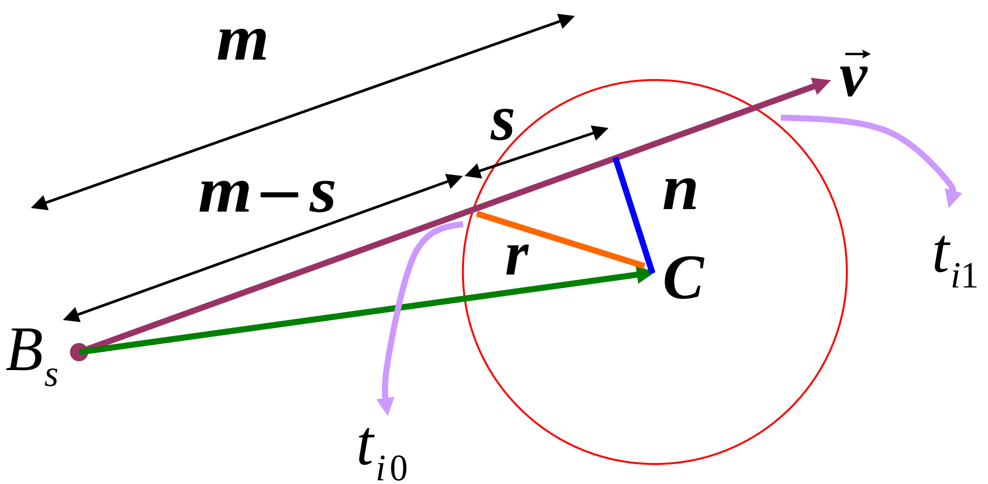
Compute t_i (2nd method) (2/2)

Compute: $s^2 = r^2 - n^2$

Since $n^2 \le r^2 \Rightarrow s^2 \ge 0 \Rightarrow s \ge 0$

$$t_{i0} = \frac{m - s}{\|\vec{v}\|}$$

$$t_{i1} = \frac{m + s}{\|\vec{v}\|}$$



(Make sure that the intersection point is between B_s and B_e)

Overview

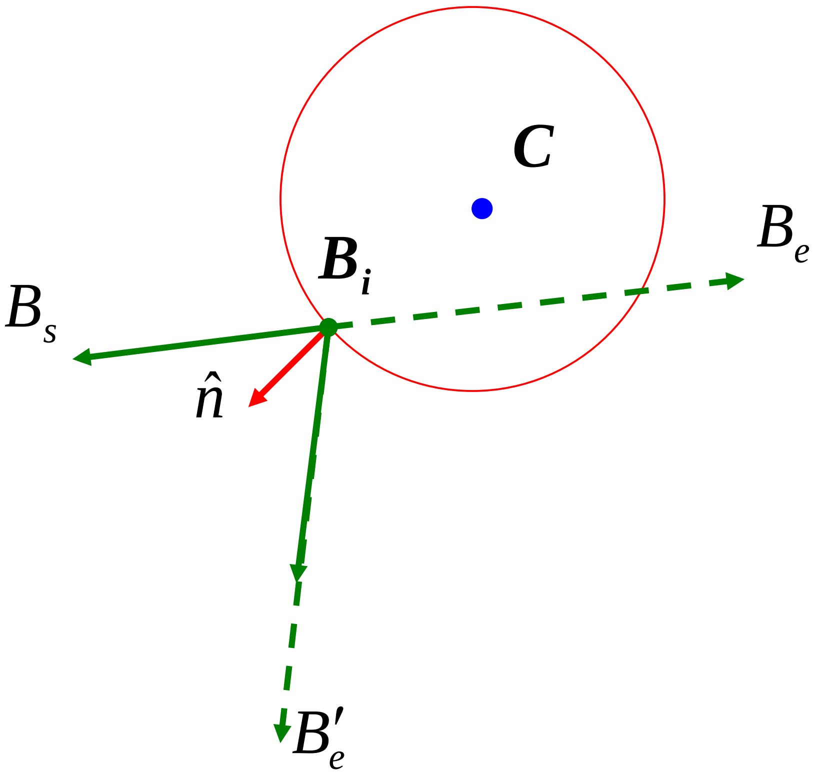
- Animated Circular Object and Stationary Circular Object
- Collision Response (Reflection)

Reflection (1/4)

$$\text{Compute}: \ \boldsymbol{B_i} = \boldsymbol{B_s} + \vec{\boldsymbol{v}}\boldsymbol{t_i}$$

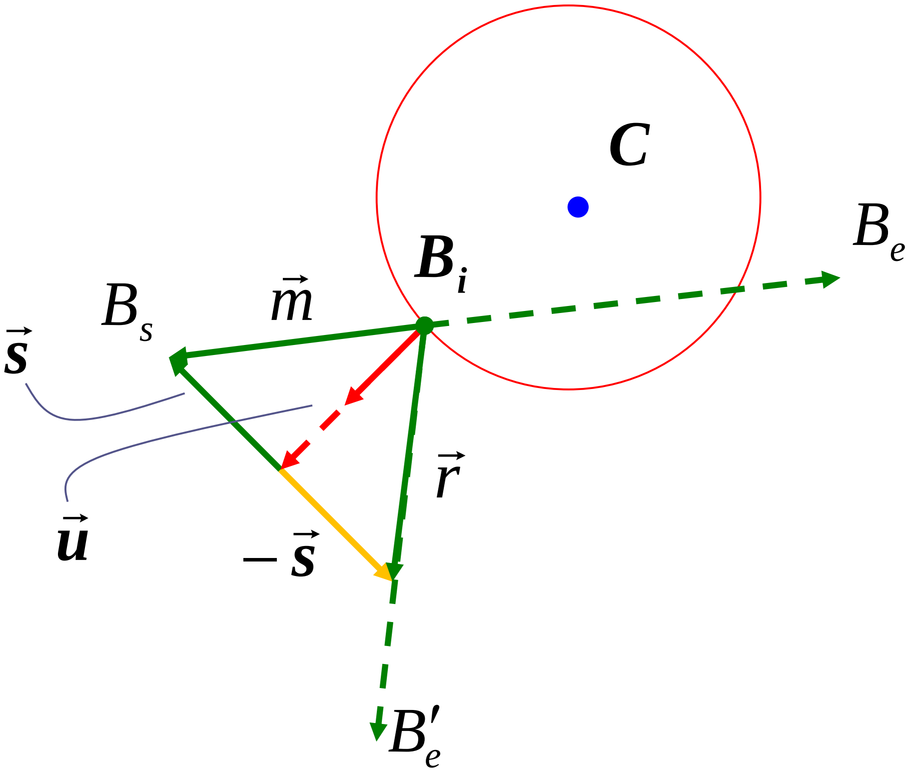
$$\text{Compute}: \ \vec{n} = \overrightarrow{CB_i}$$

$$\hat{n} = \frac{\overrightarrow{CB_i}}{\left\| \overrightarrow{CB_i} \right\|}$$



Reflection (2/4)

$$\vec{m} = \overrightarrow{B_i B_s} = B_s - B_i$$



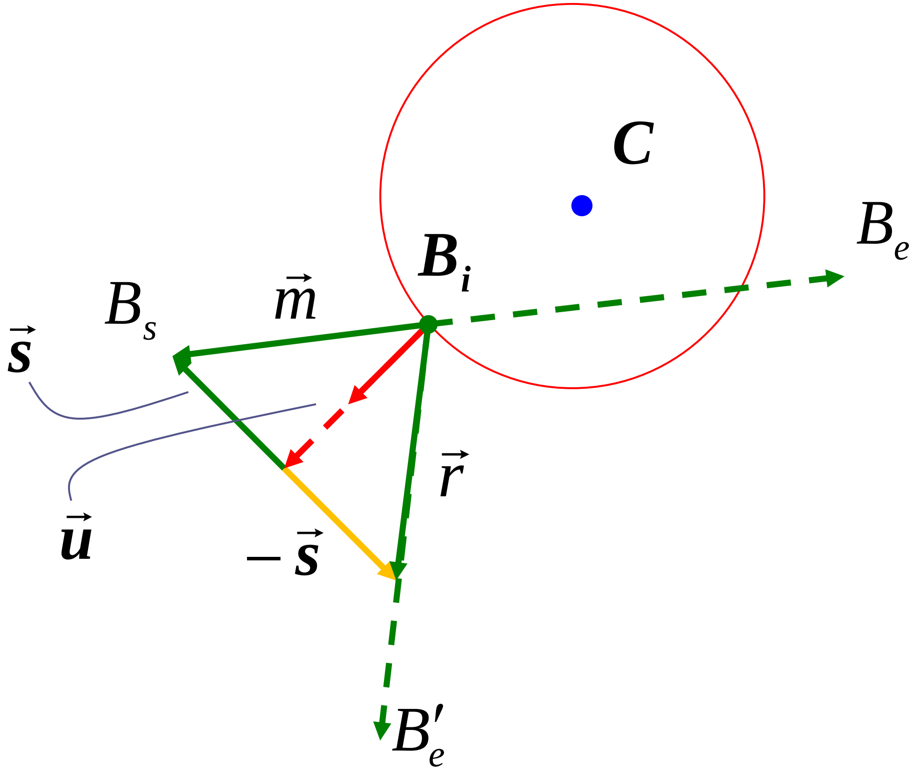
Reflection (3/4)

$$\vec{u} + \vec{s} = \vec{m}$$

$$\vec{u} - \vec{s} = \vec{r}$$

$$2\vec{u} = \vec{m} + \vec{r} \Rightarrow \vec{r} = 2\vec{u} - \vec{m}$$

$$\vec{u} = (\vec{m} \bullet \hat{n})\hat{n}$$



$$\text{Reflection}: \vec{r} = 2(\vec{m} \bullet \hat{n})\hat{n} - \vec{m}$$

Reflection (4/4)

$$\text{Given}: \boldsymbol{B_s}, \boldsymbol{B_i}, \boldsymbol{t_i} \text{ and } \vec{\boldsymbol{n}}$$

$$\vec{m} = \overrightarrow{B_i B_s} = B_s - B_i$$

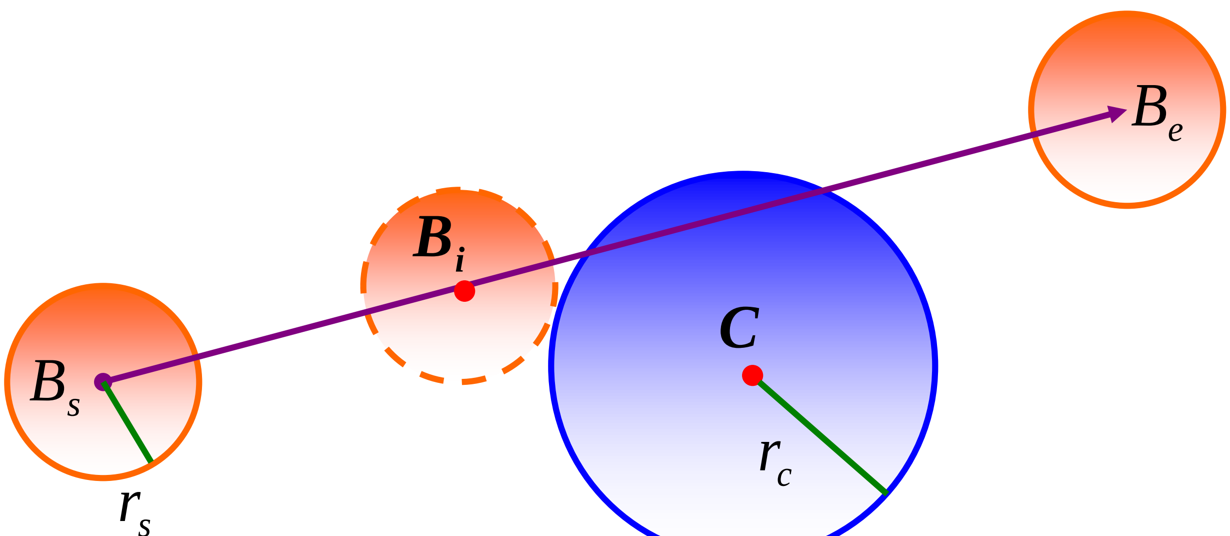
$$\vec{\boldsymbol{r}} = 2(\vec{\boldsymbol{m}} \bullet \hat{\boldsymbol{n}})\hat{\boldsymbol{n}} - \vec{\boldsymbol{m}} \qquad \hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{\|\vec{\boldsymbol{r}}\|}$$

$$\Rightarrow B_e' = B_i + k\hat{r}(1 - t_i)$$

(k is the length of the original vector v. Refer to slide 4)

Pinball-Circular Pillar Collision (1/2)

- Animated pinball modeled by a circle with center B_s and radius r_s
- Stationary circular pillar defined by center point C and radius r_c



Pinball-Circular Pillar Collision (2/2)

- Similar to intersection tests between ray from B_s to B_e and circle of radius (r_s+r_c)