1 Horizontal Composition

1.1 Setup

$$\mathcal{C} \xrightarrow{F} \mathcal{D}\ ,\mathcal{C} \xrightarrow{F'} \mathcal{D}\ ,\mathcal{D} \xrightarrow{G} \mathcal{E}\ ,\mathcal{D} \xrightarrow{G'} \mathcal{E}\ A \underset{\bullet}{\xrightarrow{\alpha}} B$$

$$\begin{tikzcd}
 & F \arrow[dr] \arrow[dd, Rightarrow, "\alpha"] & & G \arrow[dr] \arrow[dd, Rightarrow, "\beta"] & \\
\mathcal{C} \arrow[ur, dash] \arrow[dr, dash] & & \mathcal{D} \arrow[ur, dash] \arrow[dr, dash] & & \mathcal{E} \\
 & F' \arrow[ur] & & G' \arrow[ur] &
\end{tikzcd}$$

1.2 Definition of $(\beta \circ \alpha)_a$

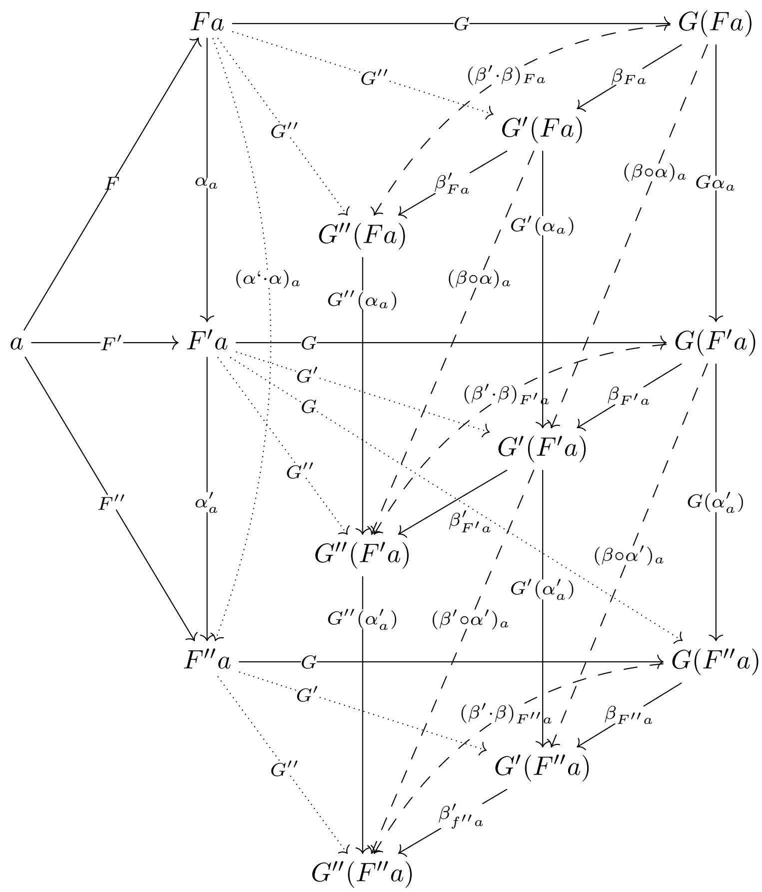
$(\beta \circ \alpha)_a = G'\alpha_a \circ \beta_{Fa} = \beta_{F'a} \circ G\alpha_a$

$$\begin{tikzcd}
 & Fa \arrow[rr, "G"] \arrow[dd, "\alpha_a"] \arrow[dr, dotted, "G'"] & & G(Fa) \arrow[dl, "\beta_{Fa}"'] \arrow[dd, "G(\alpha_a)"] \arrow[ddl, dashed, "(\beta\circ\alpha)_a"] \\
a \arrow[ur, "F"] \arrow[dr, "F'"'] & & G'(Fa) \arrow[dd, "G'(\alpha_a)"] & \\
 & F'a \arrow[dr, dotted, "G'"'] & & G(F'a) \arrow[dl, "\beta_{F'a}"] \\
 & & G'(Fa) &
\end{tikzcd}$$

2 Interchange Law

Proposition 1 (Interchange Law).

$$(\beta' \circ \alpha') \cdot (\beta \circ \alpha) = (\beta' \cdot \beta) \circ (\alpha' \cdot \alpha)$$



2.1 Definition of $(\beta' \circ \alpha')_a$

$(\beta' \circ \alpha')_a = G''(\alpha_a) \circ \beta'_{F'a} = \beta'_{F''a} \circ G'(\alpha'_a)$