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Amundi
Investment Solutions

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Automation and Enhancement of ETF and Index Valuation Processes

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Amundi Investment Solutions

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Academic Year: 2023/2024

Confidentiality Notice

Some information in this report has been blurred out for confidentiality reasons. Thank you for your understanding.

Dedication

“

To my mother and my father

It is very difficult to find enough phrases or expressions that can express my gratitude and appreciation. You have instilled in me a sense of responsibility, optimism, and above all self-confidence to face the difficulties of life. May Almighty God preserve you, grant you health, happiness, peace of mind, and protect you from all harm.

To my brothers and my grandparents
With all the love I have for you. May God bless you and keep you.

To all my family and my friends.

Thank you.

”

- Ahmed

Acknowledgments

At the end of this work, it is a pleasant duty to express in a few lines the gratitude I owe to all those who contributed, directly or indirectly, to the realization of this work and who made my internship fruitful and profitable. First of all, I wish to express my deep gratitude to my supervising professor, Mr. **Philippe Bich**, for his availability, patience, responsiveness, and support throughout my project, as well as his assistance in writing the report. He made himself available whenever needed to guide me, direct me, and advise me throughout the development of this project.

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The advice and substantive remarks he continually provided allowed me to overcome my difficulties and improve my skills. I express my sincere thanks to Mr. **Bou-Ly WU**, Deputy Head of the ETF Data Digital Team, for his advice and support during the internship period.

I will not forget to thank the team, in particular **Mohamed Amine Hssaini**, **Yousaf Fajer**, **Zélia Cazalet**, and **Laarissi Soukaina**, who welcomed me into the team from the very first day. Their kindness and strong encouragement were a source of motivation to overcome the difficulties encountered and the completion of this project.

I also wish to thank the teaching and administrative staff of the University Panthéon-Sorbonne for their efforts to offer us an excellent education.

Abstract

My final year internship at Amundi, a renowned asset management company, focused on the practical application of my skills in finance and software development.

As a **Data Analyst**, a versatile position, I joined the Data and Digital team of the ETFs department. I contributed to the automation of manual tasks by developing tools with **VBA** and **Python**, and by using **SQL** to extract data intended for external partners such as Bloomberg, FundInfo, and MorningStar. The interoperability between SQL and Python allowed me to analyze our data and provide crucial information on their quality.

I also participated in the project of migrating VBA processes to a centralized Python solution (**Process Launcher**), offering increased flexibility for sending reports to our internal clients and facilitating the configuration for the integration and dispatch of new reports.

Keywords: Data Analyst, Programming, Python, VBA, SQL, Automation, ETFs, Data Analysis, index strategy

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Contents

Confidentiality Notice . . . . **ii**

Dedication . . . . **iii**

Acknowledgments . . . . **iv**

Abstract . . . . **v**

List of Figures . . . . **viii**

List of Tables . . . . **ix**

List of Acronyms . . . . **x**

General Introduction **1**

1 Presentation of the company and its activities **2**

1.1 Amundi : a recent company that has become a world leader in asset management 3

1.1.1 Amundi: Chronology of key events and key figures at June 30, 2024 4

1.1.2 Amundi's core activities 5

1.1.3 Company culture 5

2 Exchange-Traded Fund Valuation **7**

2.1 ETFs with Domestic Constituents 8

2.1.1 Calculating the net asset value 8

2.2 ETFs with International Constituents 9

2.2.1 Calculating the Net Asset Value 10

2.2.2 Currency Hedged ETF 11

2.2.3 Intraday Indicative Value and the Estimated NAV 12

2.2.4 Providing Liquidity 13

3 Proprietary indices **17**

3.1 The procedure for creating and implementing index strategies. 18

3.2 Presentation of the VolSpot Index Strategy 18

3.2.1 Swap Pricing 18

3.2.2 Martingale Representation Theorem 21

3.2.3 Pricing the Swaption 23

3.2.4 Generalized Black-Scholes and Black-76 Formulas 24

3.2.5 Conclusion . . . 25
3.2.6 Types of Index Returns . . . 26
3.3 Implementation of the Swaption Roll Strategy . . . 26
3.3.1 Calculation, and Expiration Dates . . . 26
3.3.2 Interest Rate Swaption Valuation Method . . . 27
3.3.3 The Single-Curve Stripping Method . . . 27
3.3.4 The Dual-Curve Stripping Method . . . 27
3.3.5 Pricing Methodology . . . 28
3.3.6 Selection of Swaptions Entering the Portfolio . . . 30
3.4 Conclusion . . . 33

4 Missions and Add-On Value . . . 34
4.1 Analysis of missions and results achieved . . . 35
4.1.1 Internal and external requests . . . 35
4.1.2 Control missions . . . 35
4.1.3 Projects and Achievements . . . 36
4.2 Assessment of the internship . . . 41
4.2.1 Developed skills . . . 41
4.2.2 Challenges Faced and Solutions . . . 42

Conclusion . . . 43

Bibliography . . . 44

Appendices . . . 46

List of Figures

1.1 Chronology of the company's main events since its creation 4

2.1 ETF with Domestic Constituents—Hedged Position 13
2.2 International ETF—Correlating Hedged Position 13
2.3 International ETF—After the Creation 14

3.1 Example of a Bloomberg screen allowing the selection of a set of instruments to construct the USD 23 IR curve. 29
3.2 Example of a volatility surface of a swaption product. 30
3.3 Method for entering straddle groups 31

4.1 Process Launcher Tool 37
4.2 Process Launcher Tool 37
4.3 Screenshot of Morning Check template 39
4.4 Screenshot of Primary Daily Report 40

List of Tables

List of Acronyms

ADV *Average Daily Volume*

AP *Authorized Participant*

ATM *At The Money*

AUM *Assets Under Management*

CCL *Cash Components Level*

ETF *Exchange-traded fund*

FRA *Forward Rate Agreement*

HPPL *Hedging Portfolios Profit & Loss*

IL *Index Level*

Intrady *Intraday Trading (Buying and selling within the same day)*

IRS *Interest Rate Swap*

ITM *In The Money*

LIBOR *London Inter Bank Offer Rate*

MtM *Market-to-Market*

MVHP *Market Value of Hedging Portfolios*

NAV *Net Asset Value*

OTC *Over the Counter*

OTM *Out of The Money*

General Introduction

As part of my Master 2 program in **Modeling and Mathematical Methods in Economics and Finance** at the University Paris 1 Panthéon-Sorbonne, I had the opportunity to complete my final-year internship at Amundi ETF, a division of Amundi, the leading European asset management company and a key global player in the Exchange-Traded Fund (ETF) market. This internship allowed me to apply the skills developed throughout my academic training, which included finance, risk management, and data analysis—critical components in the rapidly evolving financial sector. The increasing digitalization of financial markets, along with the rise of Big Data and automation, has significantly transformed the finance industry. Data analytics now plays a central role in optimizing investment strategies and enhancing decision-making. This transformation is particularly important in asset management firms like Amundi ETF, where the effective use of data is essential for optimizing ETF and index valuation processes and maintaining a competitive advantage.

During my six-month internship within Amundi ETF's Data Digital team, I primarily worked as a Data Analyst. My main responsibilities included automating ETF processes and improving the quality and dissemination of fund data across external platforms. I also contributed to the migration of manual tasks into centralized systems using programming languages such as VBA and Python, which facilitated smoother reporting and better data management.

The purpose of this report is to present a detailed overview of my internship experience, focusing on the company's ETF and index valuation processes. The report is structured into four key chapters. In the first chapter, a presentation of Amundi's activities and its significant role in the global asset management market will be provided. The second chapter discusses the technical aspects of ETF valuation, explaining the procedures involved in valuing both domestic and international ETFs. The third chapter covers the proprietary index strategies used in finance, including the implementation of swaption roll strategies and pricing methodologies. Finally, the fourth chapter analyzes my specific tasks during the internship, detailing the projects I worked on, the results achieved, the skills developed, and the challenges faced.

This internship has allowed me to gain valuable insights into the financial industry's ongoing digital transformation, especially in ETF management and data processing. The experience has not only reinforced my technical and analytical skills but has also provided a solid foundation for my future career in finance.

Chapter 1

Presentation of the company and its activities

1.1 Amundi : a recent company that has become a world leader in asset management

Created on January 1, 2010, the company resulted from the merger of the asset management activities of Crédit Agricole (Crédit Agricole Asset Management, CAAM) and Société Générale (Société Générale Asset Management, SGAM). This merger positioned Amundi as a major player in the asset management market, both in Europe and internationally.

Amundi quickly became the leading asset manager in Europe and one of the top ten globally. The company manages a wide range of investment solutions for institutional, corporate, and individual clients, with over €2 trillion in assets under management across more than 30 countries. Their offerings span a full spectrum of asset classes, including equities, bonds, real estate, alternative investments, and passive management solutions.

Primarily owned by the Crédit Agricole group, its principal shareholder, Amundi was listed on the stock exchange on November 4, 2015, when the company was introduced on the Euronext Paris market. This listing marked a significant milestone for the company, enabling it to strengthen its position in the European and global financial markets.

Since its inception, Amundi has solidified its standing through a continuous strategy of expansion and innovation.

Firstly, geographic expansion has been central to consolidating its position. Amundi has actively sought opportunities to extend its operations beyond national borders, establishing itself in strategic regions such as Asia, North America, and Eastern Europe. By entering these key markets, the company not only diversified its revenue streams but also reinforced its international presence. This approach allowed Amundi to capture a broader client base and better meet the diverse needs of clients across different regions of the world.

Simultaneously, acquiring local asset management companies has been another cornerstone of its expansion strategy. These acquisitions provided Amundi with opportunities to integrate new expertise and enhance its product portfolio.

They also facilitated entry into new markets and strengthened its position in specific segments of the financial sector. Each acquisition was carefully chosen to align with Amundi's strategic goals, enabling the company to enrich its capabilities and broaden its offerings.

Innovation has also been a key factor in Amundi's success. The company has consistently invested in developing innovative financial products to meet the evolving needs of investors. One of the highlights of these innovations is the focus on sustainable and responsible investment strategies, showcasing Amundi's ability to anticipate and adapt to growing concerns around environmental, social, and governance (ESG) issues. These initiatives have not only strengthened Amundi's reputation as a forward-thinking player but also attracted clients who care about the social and environmental impact of their investments.

Additionally, digitalization has played a crucial role in driving this innovation. By implementing advanced technological tools, Amundi has enhanced its operational processes and improved the customer experience. The use of digital technologies allows for more refined financial data analysis, better risk management, and more personalized services, meeting the demands of an increasingly discerning and diverse clientele.

1.1.1 Amundi: Chronology of key events and key figures at June 30, 2024

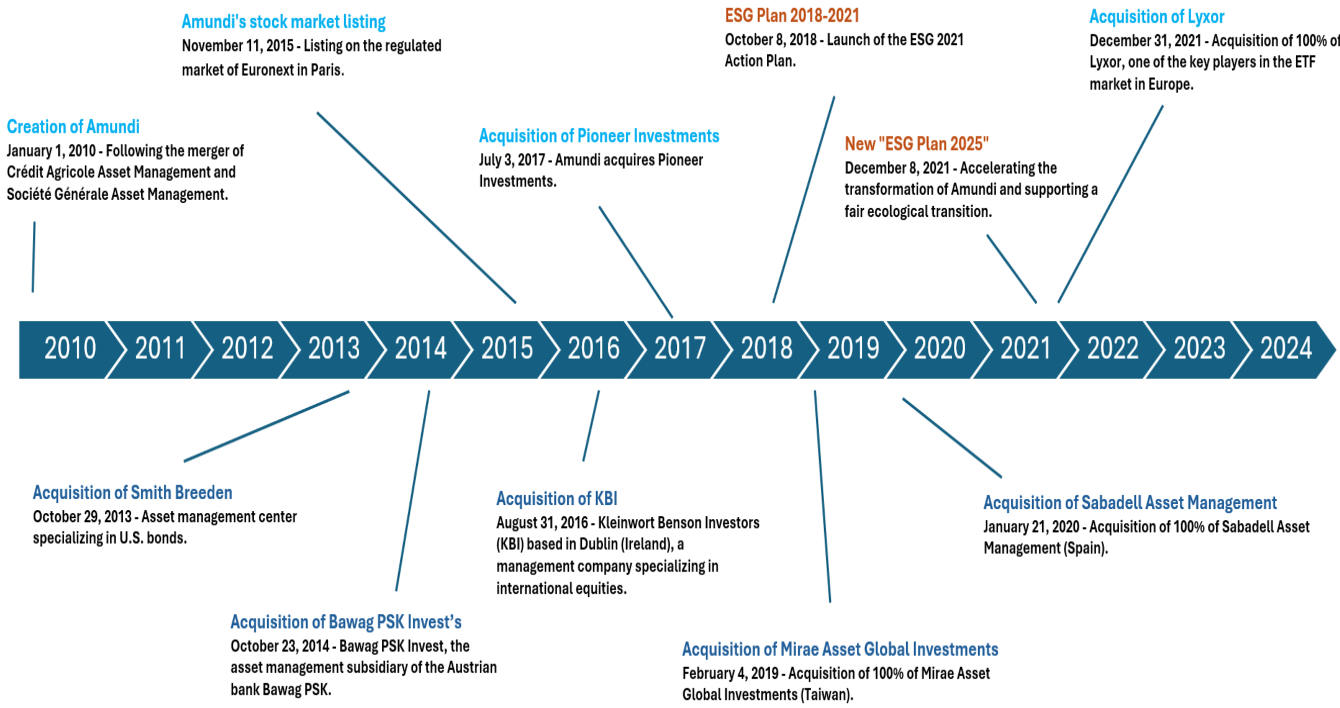

Figure 1.1: Chronology of the company's main events since its creation

- **1st European asset manager** ranked in the global top 10
- More than **€2,000 billion** in assets under management
- Over **100 million** individual clients through approximately **600** partner networks and private banks
- **1,000** institutional clients and companies
- **5,500** employees, including **900** investment professionals
- More than **60,000** users of technology platforms
- **2nd largest player in Europe** in the ETF market with over **€200 billion** in assets under management
- **1st** among the top 10 global asset managers for its voting policy on environmental and social issues

1.1.2 Amundi's core activities

In its asset management activities, Amundi offers both active and passive investment strategies. Active management stands out through careful stock selection and strategic asset allocation, while passive management, a field where Amundi excels, allows investors to track the performance of specific market indices through index funds, and particularly ETFs (Exchange Traded Funds).

As the second-largest player in Europe's ETF market, with over €200 billion in assets under management, Amundi launched its first ETF as early as 2001, ahead of all other European providers. As an ETF pioneer, its expertise has helped shape the market into what it is today, offering a broad range of diversified products that allow investors access to various markets with competitive fees and high liquidity.

Amundi also stands out for its strong commitment to responsible investing. The company consistently incorporates ESG (Environmental, Social, and Governance) criteria into its investment decisions, offering a range of funds, including ETFs, specifically designed to promote sustainable development.

Additionally, Amundi provides tailored services for institutions, assisting them in managing their portfolios with customized solutions and investment advice that align with their goals and constraints. Amundi's cash management solutions are also essential for institutions, providing optimized options for managing liquidity.

For individual investors, Amundi offers a variety of investment products, from funds to structured products, including discretionary management services, where clients can entrust their portfolios to the company's experts. These solutions are designed to cater to different investment profiles, from conservative to more daring approaches.

Innovation and research are also at the heart of Amundi's activities. The company has dedicated financial research teams that provide in-depth analysis to support investment decisions. Furthermore, Amundi invests in the development of new products and services, particularly in digital investment, ETFs, and new sustainable investment approaches.

Lastly, Amundi provides advanced technological platforms to facilitate asset management, incorporating Big Data tools and artificial intelligence to enhance investment performance and manage risks more efficiently.

1.1.3 Company culture

As mentioned earlier in this section, Amundi stands out particularly for its ability to offer tailored solutions, adapted to the specific needs of its clients, as well as its strong commitment to responsible investment.

"Our priority is to create investment solutions that meet your needs and the market context. We support you in your investment decisions through financial services and innovative technological solutions. As pioneers in responsible investment, we can play a major role in the energy transition of our economy." – *Valérie Baudson, CEO of Amundi*

The company focuses on 4 founding pillars:

1. Proximity to clients
2. Quality of products and services
3. Organizational efficiency
4. Social commitment

Proximity to clients: Amundi places great importance on maintaining a close relationship with its clients. This involves a deep understanding of their needs, expectations, and financial goals. This proximity allows Amundi to offer personalized and tailored solutions while staying attuned to market developments and investor preferences.

Quality of product and service offerings: Amundi strives to provide a wide range of high-quality products and services, covering a broad spectrum of assets and investment strategies. This includes the creation of innovative products and rigorous risk management.

Organizational efficiency: Organizational efficiency at Amundi is reflected in an agile and optimized operational structure, capable of responding quickly to market challenges while controlling costs. This includes continuous improvement of internal processes, the use of technologies to automate and simplify tasks, and efficient management of human resources.

Social commitment: Amundi is deeply committed to sustainable development and corporate social responsibility (CSR). This commitment is reflected in the promotion of responsible investments, the integration of ESG criteria in investment decisions, and concrete actions to support the environment, human rights, and ethical business practices.

Finally, the Amundi brand, in addition to its founding pillars, is also based on the company's strategy and brand platform. the company's strategy and a brand platform to develop a territory and a narrative a unique identity.

Chapter 2

Exchange-Traded Fund Valuation

2.1 ETFs with Domestic Constituents

The purest form of an exchange-traded fund (ETF) is one that trades alongside a basket of domestic securities. We'll start by discussing how to value these funds. This involves calculating the net asset value (NAV) of the fund, examining any discounts or premiums, and looking at cash amounts. We'll end with figuring out the intraday indicative value (IIV).

ETFs with domestic assets are the largest group, making up about 45% of all U.S.-listed ETF assets.

The key to ETF transparency is making sure all the numbers needed to calculate the ETF's fair value are published daily. Six key pieces of information are released every day:

1. Net asset value
2. Intraday indicative value
3. Total cash
4. Estimated cash
5. Shares outstanding
6. Accrued dividends (for some funds)

The NAV represents the most recent official value of the ETF based on the latest market close. The IIV shows the current value of the fund based on the market prices of its underlying securities. The total and estimated cash amounts reflect the excess cash in the fund, which helps determine the amount of balancing cash needed for creating or redeeming shares. The shares outstanding indicate how many shares have been issued and can vary daily with new creations and redemptions.

2.1.1 Calculating the net asset value

The NAV (Net Asset Value) of an ETF is calculated daily based on the latest closing prices of the fund's assets and the total cash in the fund. This provides a consistent value for the fund, which can be compared with other funds for performance and accounting purposes. In the world of funds, being able to compare statistics is crucial, so standardized reporting was essential for ETFs.

To find the NAV of an ETF, you add up the value of all the assets in the fund, including securities and cash, subtract any liabilities, and then divide by the number of shares outstanding.

$$\text{NAV} = \frac{\text{Assets} - \text{Liabilities}}{\text{Shares Outstanding}} \tag{2.1}$$

Where :

- **Assets:** This represents the total value of all the securities and holdings in the ETF's portfolio (stocks, bonds, cash, etc.).
- **Liabilities:** These are any obligations or debts that the ETF owes. This could include fees (Management fees, Administrative Fees etc.), dividends payable, or any other financial commitments.
- **Shares Outstanding:** This refers to the total number of ETF shares that are currently owned by investors and are being traded in the market.

Every day, ETFs share detailed information, including what exactly the fund is holding. This level of transparency is often highlighted as a big advantage of ETFs. In contrast, mutual and closed-end funds don't provide daily updates on their holdings. Mutual funds show their daily net asset value (NAV), but their holdings are only updated quarterly. Closed-end funds also provide NAV either daily or weekly, with holdings updated quarterly as well. With ETFs, we can always see the precise assets and liabilities of the fund. This transparency helps prevent any unexpected changes in the fund's investment style. To find an ETF's NAV, you can simply use the creation unit (CU) and the total cash data, which are updated daily.

$$\text{NAV} = \sum \left(\frac{\text{Shares per each component stock} * \text{Last Price}}{\text{CU Shares}} \right) + \frac{\text{Total Cash}}{\text{CU Shares}} \tag{2.2}$$

Where :

- **Shares per each component stock:** The number of shares held by the ETF for each stock in its portfolio.
- **Last Price:** The market price of each individual stock at the most recent close or last trade.
- **CU Shares (Creation Unit Shares):** The total number of shares that make up a creation unit (CU), which is the set of ETF shares that can be created or redeemed at one time. Typically, ETFs are traded in large blocks known as "creation units" rather than in single shares.
- **Total Cash:** The total cash that the ETF holds, which could include dividends or interest earnings that are not yet reinvested.

The NAV of an ETF is shown in terms of its share price. That's why we divide the total assets by the number of shares outstanding, and we do the same for the creation unit calculations.

2.2 ETFs with International Constituents

The valuation principles for exchange-traded funds (ETFs) with domestic stocks apply similarly to those with international stocks. The main differences lie in timing and currency. When the stocks in an ETF trade in a different time zone, the intraday indicative value (IIV) stays the same for the equity part but changes based on the relative foreign exchange (FX) rate.

International ETFs and make up about 27% of the ETF market in the U.S. as of September 10, 2009. This chapter will cover how to calculate NAV, IIV, and estimated NAV (eNAV) for ETFs with international holdings. For U.S. investors looking to diversify by investing overseas, these funds are crucial. They provide access to various regions and countries, like India or Japan. However, their diverse components add complexity to valuation, including timing differences and currency risks.

2.2.1 Calculating the Net Asset Value

The way we calculate the NAV for a U.S.-listed ETF with international holdings is quite similar to how we calculate it for domestic ETFs. We use the creation unit, total cash, and the number of ETF shares in the creation unit. Plus, we convert currencies to U.S. dollars (USD) since the ETF is listed and quoted in dollars.

$$\text{NAV} = \left(\sum \frac{\text{Shares per each component stock} * \text{Last Price}}{\text{Currency Rate}}\right) /\text{CU Shares} + \frac{\text{Total Cash}}{\text{CU Shares}} \tag{2.3}$$

Note: This applies to currency rates in a divisible form (e.g., if 1 U.S. dollar was equal to 95.30 yen. So, to convert the total value of shares multiplied by their price in yen into USD, we need to divide that number by the currency rate).

International ETF Execution Example

Let's look at what happens when a U.S. customer wants to buy an NAV execution in a U.S.-listed ETF with Japanese stocks. Here are the steps:

1. The customer tells an Authorized Participant (AP) to buy $2 million worth of **Amundi Japan Topix UCITS ETF - DAILY HEDGED USD (C)** (a U.S.-listed ETF with Japanese stocks) at the official NAV price.

2. The AP then buys a basket of Japanese stocks as close to the end of trading the next day in Japan as possible.

3. To buy these Japanese stocks, the AP needs to borrow yen to pay for them.

4. The AP gives the basket of stocks to the issuer's agent and gets the ETF in return.

5. The AP then gives the ETF to the customer and gets U.S. dollars as payment.

6. The AP is now settled with the ETF but has a fluctuating position in currencies. It holds U.S. dollars from the customer and owes yen, which it borrowed to buy the Japanese stocks. The AP will use the U.S. dollars to buy yen and settle the currency position.

The FX transaction of the AP is crucial because it sets the price of the ETF for the customer. The customer will get the USD ETF based on the Japanese underlying assets, USD/YEN exchange rate, and cash per creation unit. The official NAV of the ETF uses a standard accounting method to set the FX rate, while the customer's value is based on the actual FX transaction. The AP aims to execute its currency transaction at the same time as the fund to keep the price difference between the customer and the ETF as small as possible.

2.2.2 Currency Hedged ETF

Currency-hedged ETFs are a useful tool for investors because they make it easier to invest in international markets without taking on currency risk. When investing, it's important to understand what we are aiming to achieve, before we buy stocks in foreign countries, it's crucial to know how different currencies can impact our returns. Before currency-hedged ETFs existed, investors had to use more complicated methods, like shorting another ETF or the currency itself, to protect their investments from currency fluctuations. These strategies were typically only used by professionals and institutional investors.

With currency-hedged ETFs, the fund still needs to convert to the base currency to buy stocks, but they use forward contracts to lock in the value of the currency at the time the assets enter the fund. These contracts are created when the assets are added to the fund, removed during redemptions, and often rolled over every month. This process helps investors gain exposure to foreign stocks without unintentionally betting on the movement of a foreign currency.

Currency-hedged ETFs operate much like other international ETFs, with one key difference. To eliminate currency exposure, the portfolio manager sells forward contracts that match the value of the assets coming into the fund. On a monthly basis, they adjust these contracts to align with the fund's value. Throughout the month, as assets enter or leave the fund, the manager adjusts their forward contract positions to avoid tracking errors. However, the value of the hedge may not always perfectly match the value of the stocks, since both can fluctuate differently over time. This means the fund could be slightly over- or under-hedged during the month.

Calculating the NAV for a U.S.-listed currency-hedged ETF is mostly the same as the method described earlier in this chapter, with one key difference. We also need to add the profit and loss (PL) from the forward contracts as a separate part of the NAV. Keeping in mind that since the hedge and the value of the equity in USD don't always match perfectly, we have to calculate the forward PL separately.

$$\begin{aligned} \text{NAV} = & \left(\frac{\sum(\text{Shares per each component stockLast Price})}{\text{Currency Rate}} \right) / \text{CU Shares} \\ & + \text{Total Cash} + \frac{\text{Forward Position P\&L}}{\text{CU Shares}} \end{aligned} \tag{2.4}$$

2.2.3 Intraday Indicative Value and the Estimated NAV

Understanding the intraday value of a U.S.-listed ETF with international constituents is more complex than for U.S.-based ETFs. This is due to the timing differences between when the ETF and its underlying basket of international stocks are trading. The U.S.-listed ETF acts as a price discovery tool, estimating where the underlying basket will trade when its local market opens. Unlike U.S.-based ETFs, which trade closely with their underlying basket, international ETFs often trade at a premium or discount to their indicative value (IIV) because there is no arbitrage mechanism when the underlying markets are closed.

The published NAV of an international fund in the U.S. reflects a one-day lag. For example, a Japanese ETF listed in the U.S. will have its NAV based on Japan's previous trading day, even though the U.S. trading day has already started. The ETF's IIV will reflect the most recent market activity, such as Japan's close, but significant events occurring after that can influence the ETF's price. Since there is no real-time arbitrage between the ETF and its underlying basket during U.S. trading hours, the prices move independently, often reflecting different time zones.

To properly value a U.S.-traded international ETF, traders often use a method called estimated net asset value (eNAV). This involves estimating how various factors will affect the underlying basket's value and building those into a new price. This method was originally used for closed-end funds (CEFs) where NAVs were published infrequently. While eNAV values are available for CEFs, they are not commonly available for ETFs, limiting access to those with the resources to calculate them.

Currently, there's a growing competition to calculate eNAV because it helps traders price large ETF blocks more accurately, increasing order flow, commissions, and spreads. The eNAV process is subjective, with different opinions on valuation, making international ETFs valuable as a way to estimate future pricing.

Abasic formula for calculating an eNAV:

$$\mathrm{eNAV} = \sum \left(\frac{\text{Shares per each component stock} * \text{Last Local Price}}{\text{FX rate}} \right) \frac{(1+x)}{\text{Creation Unit Shares}} + \frac{\text{Estimated Cash}}{\text{Creation Unit Shares}} \tag{2.5}$$

where x = the expected percentage change in the underlying constituents.

In this formula, the variable x is created based on a set of correlated proxy assets. It uses estimated cash to provide a real-time estimate of the ETF's value. Typically, we don't need to estimate the actual NAV (Net Asset Value) because it's published after the fact using the closing prices of the assets and total cash. When calculating an estimated NAV (eNAV) to approximate the fund's value, we use the estimated cash in the fund. This gives us an idea of what would be needed for a creation or redemption at the next available opportunity.

2.2.4 Providing Liquidity

Making markets or providing liquidity for an ETF where the underlying asset isn't trading at the same time as the ETF is quite different from doing so for a domestic version. For example, if a customer is a big buyer of a Japanese ETF, the liquidity provider will sell the ETF to that customer. The liquidity provider (LP) then has two main goals: either buying back shares of the ETF at a lower price or buying a pre-planned hedge for their position.

When the ETF and the underlying basket trade at the same time, the LP can buy the basket to stay perfectly hedged. This situation is shown in 2.1. Later, the LP might create ETF shares to balance their position and clear their balance sheet.

However, if the basket isn't trading at the same time as the ETF, the LP will depend on the eNAV and a related hedge to manage their risk. Even with a good hedge, unwinding the position can still be complex.

Figure 2.1: ETF with Domestic Constituents—Hedged Position

The ETF and basket position can be collapsed via the creation/redemption mechanism.

Figure 2.2: International ETF—Correlating Hedged Position

The ETF and correlated hedge cannot be collapsed.

The process isn't as straightforward as just creating an ETF when the opportunity arises. In this example, the LP has sold the ETF to a customer and has taken a position that matches the ETF and trades during the same hours. You can see this in 2.2.

Unlike domestic positions, this one can't be easily closed out using the creation and redemption method. If the LP tried to create more of the ETF to offset its short position, it would end up with a short position in the creation unit, as shown in 2.3.

The creation and redemption methods involve swapping one asset for a basket of others, so they represent the same exposures. Because the hedge doesn't match the creation basket, the position isn't closed out this way. However, this method can help in switching between the ETF and its underlying basket if it's more beneficial.

There are some exceptions, like cash creation and redemption for certain ETFs. Sometimes the AP can exchange cash for the ETF or vice versa, especially if the ETF's underlying assets are restricted in their home country. In such cases, the LP might sell the ETF and buy a matching hedge during the day. At the end of the trading day, it could unwind the hedge and take on the risk between market closes. Then, the LP would create more ETF shares by delivering cash and receiving ETF shares to cover its short position. This process varies based on trading conditions in different markets and is no longer just simple arbitrage between the ETF and its underlying basket.

Figure 2.3: International ETF—After the Creation

After doing a creation of the ETF, you are left short the basket versus the hedge.

The process is more complex than just creating an ETF whenever possible. In this case, the LP has sold the ETF to the customer and taken a position. The variability in eNAV calculations, the diverse user base, and the arbitrage mechanisms in ETFs have made some international ETFs very popular among traders and investors. Unlike domestic ETFs, which focus on speed and financing, international ETFs need global systems and a keen trading sense. A firm will only rely on its automated hedge for so long before it stops taking risks. During the market disruptions of late 2008–2009, there were significant opportunities in ETFs that traders capitalized on, providing liquidity and aligning discounts and premiums when they could.

Measuring potential available ETF liquidity

The liquidity of Exchange-Traded Funds (ETFs) is often misunderstood. This is largely because many investors do not realise that ETF liquidity does not only derive from a fund's trading volume on a market, but also from the liquidity of the basket shares it's comprised of in turn. An ETF's true liquidity is not purely a reflection of its trading activity, but is fundamentally linked to the cash creation and redemption mechanism. This process allows authorized participants (APs) to swap ETF shares for the stocks underlying the ETF and vice versa. The final result is that an AP can almost eat their ETF whenever they need—it can be converted into its underlying basket of stocks and from there back to the ETF. ETF Liquidity and the Creation/Redemption Process The liquidity of an ETF is often discussed, mainly because many market participants fail to figure out that ETF is refill capable. That is, shares can be issued or withdrawn as per market demand just as if they were cash. The process is mediated by authorized participants (AP), who act as intermediates between the market and ETF's underlying basket of stocks. If AP wants to create part of an ETF, they purchase the constituent stocks in its basket, wrap those to mirror the ETF's composition precisely, and then give them an exchange traded fund provider for new shares of ETF. This process revolves around the creation unit, which is a specific bundle of ETF shares–usually between 50,000 and 200,000 shares. The number of shares required for each stock in the basket depends on its weight in the index that the ETF tracks. Therefore, the liquidity of an ETF is not just a function of trading volume but also how easy it is to trade stocks in its underlying basket.

One feasible way to assess the implied liquidity of an ETF is consider daily average volume (ADV) of each stock in the underlying basket. By analyzing how much of any given stock's ADV can be traded before heading prices slant dramatically upwards (typically assumed around 50%), we can judge whether there is ample potential liquidity available for forming or redeeming ETF shares. To find out the liquidity of the underlying stocks, let us calculate how many creation units of the ETF can be created or redeemed. The New Stock Market Dictionary, 1990 For instance, let's take an ETF designed to mirror the Dow Jones Industrial Average (DJIA), which is comprised of 30 large-cap stocks. Let's say we have three of the stocks in troubles: Apple (AAPL); Microsoft (MSFT) and Boeing (BA). In this example, each creation unit comprises 50,000 ETF shares, and how this liquidity is to be made available will be determined by how much Apple, Microsoft and Boeing can be traded without affecting their price in any significant way. Suppose that the ADV and number of small lots are required for each stock to form a creation unit are as follows:

1. Apple (AAPL): ADV = 40,000,000 shares, Small Lots Needed Per Creation Unit = 100
2. Microsoft (MSFT): ADV = 30,000,000 shares, Small Lots Needed Per Creation Unit = 150
3. Boeing (BA): ADV = 10,000,000 shares, Small Lots Needed Per Creation Unit = 50 We suppose that each stock's trade can be done half if ADV.

Calculation of Implied Liquidity

Calculate the number of shares that can be traded without impacting price:

1. Apple: 50% of 40,000,000 = 20,000,000 shares
2. Microsoft: 50% of 30,000,000 = 15,000,000 shares
3. Boeing: 50% of 10,000,000 = 5,000,000 shares

Determine how many creation units can be made from each stock:

1. Apple: **20,000,000 shares ÷ 100 shares (per creation unit) = 200,000 creation units**

2. Microsoft: **15,000,000 shares ÷ 150 shares = 100,000 creation units**

3. Boeing: **5,000,000 shares ÷ 50 shares = 100,000 creation units**

Since the number of creation units is determined by the stock with the fewest possible units, the limiting factor in this case is Microsoft and Boeing, which both allow for 100,000 creation units.

That is an easy calculation showing a truth that might be overlooked at first glance. Whether or not an ETF has good trading volume, the liquidity comes from the stocks behind it. This adds another layer of market depth, which can be accessed through creation or redemption. For example, a fund manager or large trader. Understanding this hidden liquidity is important for big-time buyers who want to place large orders all at once without negatively impacting the market

Chapter 3

Proprietary indices

3.1 The procedure for creating and implementing index strategies.

Proprietary indices are created by an investment bank or one of its specialized advisors, who manage the entire investment process. Unlike traditional stock market indices, they do not aim to reflect the state of a market or economy. Their role is more as financial instruments, used as underlying assets in products like futures contracts or options. They also serve as the basis for investment funds that seek to replicate their composition and strategy in order to achieve identical returns.

However, unlike standard indices that capture the performance of a specific market, proprietary indices are specially designed for investors whose expectations go beyond what traditional markets can offer.

The creation of an index follows several key steps, each playing a crucial role in ensuring that the index meets the needs of investors and provides opportunities for gains. First, the needs analysis is a critical phase. This involves understanding investor expectations and identifying market opportunities that will make the index attractive.

Next comes the selection of the index components and the determination of their respective weights, based on the defined strategy. Another essential step is drafting the "Index Rules." This document is fundamental as it records, in detail, all the rules related to the index: the calculation frequency, revision and rebalancing dates, the calculation agent, associated fees, asset class, strategy, rebalancing method, and the calculation formulas. It also includes provisions for exceptional events that could disrupt the index's calculation.

3.2 Presentation of the VolSpot Index Strategy

Our VolSpot Index leverages market volatility to generate gains. It is built on a strategy that uses swaptions as the primary product and swaps as hedging instruments. In other words, the index consists of portfolios of swaptions and swaps, with varying strikes and maturities.

In practical terms, it is made up of portfolios of straddles, meaning the purchase of both a receiver swaption and a payer swaption. Each straddle is paired with its corresponding hedging product, the swap.

3.2.1 Swap Pricing

Since the swap is the combination of the fixed leg and the floating leg, the value of the swap is equal at each instant to the difference between the values of the two legs. It then becomes essential to determine the value of each of the two legs, fixed and floating, in order to determine the Net Present Value (NPV) of the swap.

Valuation of the Fixed Leg:

The fixed leg is assimilated to a bond that pays fixed-rate coupons. Its valuation thus amounts to valuing a fixed-rate bond whose face rate is the fixed rate, and whose discount rates are retrieved from the reference yield curve corresponding to the swap's currency.

$$J_{\text{fixed}} = \textstyle\sum_{i=k}^{n} \frac{F_{fi}}{(1+ZCi)^{(Dcf_i - D_{val})}}$$

Where:

- J_{fixed} is the value of the fixed leg.
- Dcf_i is the date of the i^{th} fixed coupon.
- D_{val} is the valuation date.
- ZC_i is the reference zero-coupon rate corresponding to the period between the valuation date and the date of the i^{th} fixed coupon.
- n is the number of coupons of the fixed leg.
- k is the index of the date of the next fixed coupon after the valuation date.

Valuation of the Floating Leg:

The floating leg is assimilated to a bond that pays variable-rate coupons. This rate is indexed to the market rate corresponding to the swap's currency. We will thus value this leg in the same way as a variable-rate bond.

$$J_{\text{var}} = \textstyle\sum_{j=l}^{n'} \frac{F_{\text{var}j}}{(1+ZC_i)^{(Dcf_i - D_{val})}}$$

Where:

- J_{var} is the value of the floating leg.
- Dcf_i is the date of the j^{th} variable coupon.
- D_{val} is the valuation date.
- $F_{\text{var}j}$ is the variable flow paid on date Dcf_{var}.

NPV of the Interest Rate Swap:

For a Swap Seller, i.e., payer of the variable and receiver of the fixed:

$$\text{NPV} = J_{\text{fixed}} - J_{\text{var}}$$

Which gives:

$$\text{NPV} = \sum_{i=k}^{n} \frac{F_{fi}}{(1+ZC_i)^{(Dcf_i - D_{val})}} - \sum_{j=l}^{n'} \frac{F_{\text{var}j}}{(1+ZC_i)^{(Dcf_i - D_{val})}}$$

For a Swap Payer, i.e., payer of the fixed and receiver of the variable:

$$\text{NPV} = J_{\text{var}} - J_{\text{fixed}}$$

Which gives:

$$\text{NPV} = \sum_{j=l}^{n'} \frac{F_{\text{var}j}}{(1+ZC_i)^{(Dcf_i - D_{val})}} - \sum_{i=k}^{n} \frac{F_{fi}}{(1+ZC_i)^{(Dcf_i - D_{val})}}$$

Determination of the Swap Rate:

The valuation of an interest rate swap comes down to determining the Swap rate; this is the fixed interest rate required by the buyer of the swap and paid by its seller. This rate, involved in the calculation of the fixed flows F_{fi}, is determined by equalizing the two legs.

We have:

$$F_{fi} = N\frac{(t_i - t_{i-1})}{\text{base}}\tau_{\text{fixed}} \tag{3.1}$$

Where:

- N is the notional amount.
- t_i is the time of payment of the fixed interests.
- n is the number of payments.
- base is the number of days of the day-count basis (360, 365, or 366).

We deduce the Swap rate from the following equality:

$$J_{\text{fixed}} = J_{\text{var}} \tag{3.2}$$

Which is equivalent to:

$$\sum_{i=k}^{n} \frac{N\frac{(t_i - t_{i-1})}{\text{base}}\tau_{\text{fixed}}}{(1+ZC_i)^{(Dcf_i - D_{\text{val}})}} = \sum_{j=l}^{n'} \frac{N\frac{(t_j - t_{j-1})}{\text{base}}(\tau_{\text{fwd}j} + \text{spread})}{(1+ZC_j)^{(Dcf_j - D\text{val})}} \tag{3.3}$$

We find:

$$\tau_{\text{fixed}} = \frac{\displaystyle\sum_{j=l}^{n'} \frac{N\frac{(t_j - t_{j-1})}{\text{base}}(\tau_{\text{forward}j} + \text{spread})}{(1+ZC_j)^{(Dcf_j - D\text{val})}}}{\displaystyle\sum_{i=k}^{n} \frac{N\frac{(t_i - t_{i-1})}{\text{base}}}{(1+ZC_i)^{(Dcf_i - D_{\text{val}})}}} \tag{3.4}$$

Note: Generally, the swap rate is calculated for a zero spread.

Spread: or additive margin, it represents the amount added to the variable rate for calculating the floating leg's flows. The spread is also used in the credit risk part and reflects the degree of confidence in the counterparty to honor its commitments.

Determination of the Spread:

From equality (3.4), we obtain:

$$\text{Spread} = \frac{\sum_{i=k}^{n} \frac{N\frac{(t_i - t_{i-1})}{\text{base}}\tau_{\text{fixed}}}{(1+ZC_i)^{(Dcf_i - D_{val})}} - \sum_{j=l}^{n'} \frac{N\frac{(t_i - t_{i-1})}{\text{base}}\tau_{\text{var}}}{(1+ZC_i)^{(Dcf_i - D_{val})}}}{\sum_{i=k}^{n} \frac{N\frac{(t_i - t_{i-1})}{\text{base}}}{(1+ZC_i)^{(Dcf_i - D_{val})}}}$$

3.2.2 Martingale Representation Theorem

In probability theory, the martingale representation theorem states that a random variable measurable with respect to the filtration generated by a Brownian motion can be expressed in terms of an Itô integral with respect to this Brownian motion.

The theorem only asserts the existence of the representation and does not help to find it explicitly; in many cases, it is possible to determine the form of the representation using Malliavin calculus. Similar theorems also exist for martingales on filtrations induced by jump processes, for example, Markov chains. Following Baxter (1966), Hull (2011), and Burgess (2014), we established the martingale representation theorem, which provides us with a framework for evaluating the price of an option using the formula below, in which the price V_t at time t of such an option with a payoff X_T at time T is evaluated with respect to a tradable asset or numéraire N with the corresponding probability measure Q_N.

$$\frac{V_t}{N_t} = E^{Q_N}\left[\frac{X_T}{N_T} \mid \mathcal{F}_t\right]$$

Which is equivalent to:

$$V_t = N_t E^{Q_N}\left[\frac{X_T}{N_T} \mid \mathcal{F}_t\right]$$

Where V_t is the price of the option evaluated at time t, N_t is the numéraire evaluated at time t, E^{Q_N} is an expectation with respect to the numéraire measure Q_N, and X_T is at time T.

A European option with a payoff X_T at time T takes the following form for a European call:

$$X_T = \max\left(S_T - K, 0\right) = \left(S_T - K\right)^+$$

And similarly for a European put option:

$$X_T = \max\left(K - S_T, 0\right) = \left(K - S_T\right)^+ \tag{3.5}$$

Present Value of the Swap

An interest rate swaption is an option on an interest rate swap (IRS). To evaluate the payment of the swaption, we need to understand the IRS instrument and how to determine its price or present value.

In an interest rate swap transaction, a series of fixed cash flows is exchanged for a series of floating cash flows. A swap can be considered as an agreement to exchange a fixed-rate loan for a variable or floating-rate loan. Burgess (2017a) provides an in-depth examination of interest rate swaps, their pricing, and the risks they present.

The net present value (PV) or the price of an interest rate swap can be evaluated as follows:

$$\begin{aligned} \mathrm{PV}^{\text{Swap}} &= \varphi\left(\mathrm{PV}^{\text{Fixed Leg}} - \mathrm{PV}^{\text{Float Leg}}\right) \\ &= \varphi\left[\sum_{i=1}^{n} N r^{\text{Fixed}} \tau_i \rho(t_E, t_i) - \sum_{j=1}^{m} N(l_{j-1}+s)\tau_j \rho(t_E, t_j)\right] \end{aligned} \tag{3.6}$$

Where $\mathrm{PV}^{\text{Fixed Leg}}$ refers to the present value of the fixed-rate swap payments. Receiver swaps receive fixed coupons (and pay floating coupons), and payer swaps pay fixed coupons (and receive floating coupons). $\mathrm{PV}^{\text{Float Leg}}$ refers to the present value of the floating or variable Libor coupon swap payments. Each coupon is determined by the Libor rate at the beginning of the coupon period. When the Libor rate is known, it is said that the rate has been fixed or reset, and the corresponding coupon payment is known.

In the swap market, investors wish to conduct transactions at zero cost. On the effective date of the swap, the swap has zero value, but over time, this is no longer the case, and the swap becomes profitable or unprofitable. To this end, investors want to know what fixed rate must be used to make the fixed and floating legs equal in a swap transaction, which we call P^{Market}. This fixed rate is called the swap rate or par rate. Interest rate swaps are generally quoted and traded in financial markets as par rates, i.e., the rate that corresponds to the net present value (NPV) of the fixed leg being equal to the NPV of the floating leg. Thus, swaps executed with the fixed rate at the par rate are called at-par swaps and have a net NPV of zero.

$$\mathrm{PV}^{\text{Swap}} = \varphi\left[\sum_{i=1}^{n} N r^{\text{Fixed}} \tau_i \rho(t_E, t_i) - \sum_{j=1}^{m} N(l_{j-1}+s)\tau_j \rho(t_E, t_j)\right] \tag{3.7}$$

Since at-par swaps have a zero PV, we deduce:

$$\sum_{i=1}^{n} N r^{\text{Fixed}} \tau_i \rho(t_E, t_i) = \sum_{j=1}^{m} N(l_{j-1}+s)\tau_j \rho(t_E, t_j) \tag{3.8}$$

Furthermore, at-par swaps have a fixed rate equal to the par rate, i.e., $r^{\text{Fixed}} = p^{\text{Market}}$.

Following Burgess (2017a), we can represent the floating leg as a fixed leg traded at the market par rate p^{Market}, and thus equation (3.7) becomes:

$$
\begin{aligned}
\mathrm{PV}^{\mathrm{Swap}} &= \varphi \left[\sum_{i=1}^{n} N(r^{\mathrm{Fixed}} - p^{\mathrm{Market}})\tau_i \rho(t_E, t_i) - \sum_{j=1}^{m} N s \tau_j \rho(t_E, t_j) \right] \\
&= \varphi \left[(r^{\mathrm{Fixed}} - p^{\mathrm{Market}}) A_N^{\mathrm{Fixed}} - s A_N^{\mathrm{Float}} \right]
\end{aligned} \tag{3.9}
$$

Where:

- $A_N^{\mathrm{Fixed}} = N \sum_{i=1}^{n} \tau_i \rho(t_E, t_i)$

In the case where there are no Libor spreads on the floating leg, this simplifies to:

$$
\begin{aligned}
\mathrm{PV}^{\mathrm{Swap}} &= \varphi \left[\sum_{i=1}^{n} N r^{\mathrm{Fixed}} \tau_i \rho(t_E, t_i) - \sum_{j=1}^{m} N l_{j-1} \tau_j \rho(t_E, t_j) \right] \\
&= \varphi \left[(r^{\mathrm{Fixed}} - p^{\mathrm{Market}}) A_N^{\mathrm{Fixed}} \right]
\end{aligned} \tag{3.10}
$$

3.2.3 Pricing the Swaption

In a receiver swaption, the holder has the right to receive the fixed cash flows of the underlying swap at an exercise rate agreed upon today and to pay the floating leg flows. A rational option holder will only exercise the option if the fixed flows to be received are greater than the floating flows to be paid. The corresponding option payoff X_T can be represented as follows:

$$
\begin{aligned}
X_T &= \max \left(\sum_{i=1}^{n} N K \tau_i \rho(t_E, t_i) - \sum_{j=1}^{m} N l_{j-1} \tau_j \rho(t_E, t_j), 0 \right) \\
&= A_N^{\mathrm{Fixed}} \max \left(K - p^{\mathrm{Market}}, 0 \right) \\
&= A_N^{\mathrm{Fixed}} (K - p^{\mathrm{Market}})^{+}
\end{aligned} \tag{3.11}
$$

As can be seen by comparing equations, the payoff of a receiver swaption reproduces the payoff of a put option scaled by the annuity of the fixed leg of the swap A_N^{Fixed}.

Similarly, a payer swaption gives its holder the right to receive the floating cash flows of the underlying swap and has a payoff X_T.

It is easy to see from the swaption payoff that a payer swaption represents the payoff of a call option, and a receiver swaption represents the payoff of a put option.

Both options give the right, but not the obligation, to enter into a swap contract in the future to pay or receive fixed cash flows in exchange for floating cash flows, with the fixed rate set today at the strike rate K.

In the general case, we can represent the payoff of a swaption as follows:

$$
X_T = A_N^{\mathrm{Fixed}} \left(\varphi \left(p^{\mathrm{Market}} - K \right)^{+} \right) \tag{3.12}
$$

Where $\varphi = 1$ for a payer swaption and $\varphi = -1$ for a receiver swaption.

By applying the martingale representation theorem from the previous section, we can determine the price of the swaption using the swaption payoff equation, which gives:

$$V_t = N_t E^{Q_N}\left[\frac{X_T}{N_T} \mid \mathcal{F}_t\right] = N_t E^{Q_N}\left[\frac{A_N^{\text{Fixed}}\left(\varphi\left(p^{\text{Market}} - K\right)^+\right)}{N_T} \mid \mathcal{F}_t\right] \tag{3.13}$$

Following Burgess (2017a), we can choose a convenient numéraire to simplify the expectation term. In this case, we choose the annuity measure A_N^{Fixed} with the corresponding probability measure Q_A, which leads to:

$$V_t = A_N^{\text{Fixed}}(t) E^{Q_A}\left[\left(\varphi\left(p^{\text{Market}} - K\right)\right)^+ \mid \mathcal{F}_t\right] \tag{3.14}$$

We might observe that the expectation term can be evaluated using the generalized Black-Scholes formula. However, for completeness, we replace the annuity measure Q_A with the more familiar risk-neutral measure Q. This helps to identify the Black-Scholes expectation.

By applying the Radon-Nikodym derivative, which allows us to change the numéraire and the associated probability measure, we get:

$$\left(\frac{dQ_M}{dQ_N}\right) = \frac{\frac{M_t}{M_T}}{\frac{N_t}{N_T}} = \frac{N_T}{N_t}\frac{M_T}{M_t} \tag{3.15}$$

Using the Radon-Nikodym derivative to change from the annuity measure Q_A to the risk-neutral measure Q, we derive an expression similar to the Black-Scholes formula:

$$V_t = A_N^{\text{Fixed}}(t) E^{Q}\left[\left(\varphi\left(p^{\text{Market}} - K\right)\right)^+ \mid \mathcal{F}_t\right] \tag{3.16}$$

3.2.4 Generalized Black-Scholes and Black-76 Formulas

The generalized Black-Scholes formula for the valuation of European options (Black-Scholes, 1973) is popular among traders and practitioners due to its analytical tractability. The formula heavily relies on dynamic delta hedging (see Derman and Taleb, 2005). It values the price V_t at time t of a European option expiring at time T as:

$$V_t^{BS} = \varphi e^{-r(T-t)}\left[S_t e^{-b(T-t)} N(\varphi d_1) - K N(\varphi d_2)\right] \tag{3.17}$$

Where:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(b + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{3.18}$$

Setting the cost of carry term $b = 0$ leads to the Black-76 formula for the valuation of interest rate options (Black, 1976):

$$V_t^{BS76} = \varphi e^{-r(T-t)} \left[S_t N(\varphi d_1) - K N(\varphi d_2) \right] \tag{3.19}$$

Comparing this with our swaption pricing formula, we can see that the swaption pricing formula is the Black-76 formula scaled by the annuity factor $A_N^{\text{Fixed}}(t)$. In this case, the underlying asset is an interest rate, so we adapt the formula accordingly.

By setting the zero rate $r = 0$, we eliminate the extra discounting term, leading to:

$$V_t = A_N^{\text{Fixed}}(t) \text{Black-76}(p^{\text{Market}}, K, N(T-t), \sigma(K,t), r = 0) \tag{3.20}$$

Explicitly, we have:

$$V_t = \varphi A_N^{\text{Fixed}}(t) \left[p^{\text{Market}} N(\varphi d_1) - K N(\varphi d_2) \right] \tag{3.21}$$

With:

$$d_1 = \frac{\ln\left(\frac{p^{\text{Market}}}{K}\right) + \left(\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{3.22}$$

And $\varphi = 1$ denotes a payer swaption and $\varphi = -1$ a receiver swaption. In the case where our underlying swap has a floating Libor margin, we adjust the strike by replacing K with K':

$$K' = K - s\left(\frac{A_N^{\text{Float}}(T)}{A_N^{\text{Fixed}}(t)}\right) \tag{3.23}$$

3.2.5 Conclusion

In conclusion, we have examined the martingale representation theorem for option valuation, which allows us to price options based on a numéraire of our choice. We have also reviewed the classical results of European call and put option valuation to help identify that payer swaptions are comparable to call options and receiver swaptions are comparable to put options.

Since interest rate swaptions are options on interest rate swaps, we have also studied how to evaluate and price an interest rate swap to better understand the swaption payoff. In particular, we have highlighted that a key component of the underlying swap price is the annuity duration, which was instrumental in selecting a numéraire to evaluate the expected payoff of the swaption.

We have examined how to value interest rate swaptions using the martingale representation theorem to derive a closed-form analytical solution. We chose the annuity measure to simplify the expected payoff of the swaptions. This reduced the price calculation to an expression similar to that of Black-Scholes (1973). To make this calculation more transparent, we applied the Radon-Nikodym derivative to change the probability measure from the annuity measure to the risk-neutral measure, which is more classical and recognizable, to arrive at a swaption pricing formula expressed in terms of the Black-Scholes (1973) formula.

We have shown that the interest rate swaption pricing formula is essentially the Black-76 formula scaled by the annuity factor of the underlying swap.

3.2.6 Types of Index Returns

We can distinguish two types of returns for proprietary indices. Thus, these indices can be either Total Return or Excess Return indices.

Total Return Indices:

These indices take into account and reflect the movements of the levels of the underlying assets as well as the distributions generated by these underlying assets. The distributions generated by the underlying assets can be in the form of interest or dividends. Investors generally prefer to reinvest these cash flows into the index product or other relatively less risky products, notably treasury bonds.

Excess Return Indices:

In contrast to total return indices, these indices do not take into account cash payments related to investment interest and dividends. Therefore, these indices reflect the value of the index relative to the market.

We can distinguish these two types of returns by the following function:

$$TR(t) = ER(t) + i\frac{\mathrm{act}(t, t-1)}{360} \tag{3.24}$$

Where:

- $TR(t)$: 'Total Return' at date t.
- $ER(t)$: 'Excess Return' at date t.
- i: the interest rate (of the underlying assets).
- $\mathrm{act}(t, t-1)$: the number of days between dates t and $t-1$.

3.3 Implementation of the Swaption Roll Strategy

3.3.1 Calculation, and Expiration Dates

The first step is generating a calendar of calculation dates, composed of business days while omitting holidays, covering dates between the backtest start date and the end date. This interval represents the period during which we are interested in observing the strategy's behavior with historical data.

This calendar is based on imported data corresponding to national calendars of various countries: a European calendar for the EUR swaption roll, a U.S. calendar for the USD swaption roll, and so on.

3.3.2 Interest Rate Swaption Valuation Method

Valuing an interest rate swap is critical for all counterparties dealing with this type of financial product.

Before the financial crises, the valuation of interest rate swaps was straightforward: the bootstrapping process was used to calculate the discount factors of future cash flows from a risk-free rate curve.

3.3.3 The Single-Curve Stripping Method

The classic method used to evaluate an interest rate swap was the single-curve stripping approach.

This methodology involves selecting the most liquid vanilla interest rate instruments to construct a single rate curve, which is used both as the discount rate and as the forward rate. This was possible because, at the time, the LIBOR rate was the reference for most interest rate derivatives, considered a risk-free rate.

The single-curve approach can be summarized by the following steps:

- Select a set of interest rate derivatives quoted on the market with increasing maturities. For example, a very common procedure is to choose a combination of short-term EUR deposits, medium-term FRA/Futures on the 3M Euro LIBOR, and medium to long-term swaps on the 6M Euro LIBOR.
- Using the classic bootstrapping technique, construct the zero-coupon (ZC) rate curve from these instruments.
- Use the ZC curve to extract both the forward rates for calculating future cash flows and the discount factors.
- With the elements calculated in point 3, compute the price of the derivative product by summing all the discounted cash flows.

3.3.4 The Dual-Curve Stripping Method

The financial crises experienced by the market have shown that assumptions considered given until 2007 should be revisited to build a technical framework as coherent as possible with current market conditions and the changes occurring.

This necessary revision involves transitioning from the traditional single-curve approach to the new multiple-curve (dual-curve) approach to address inconsistencies in the frameworks used previously.

In this methodology, the discount curve has become the OIS rate curve, while the forward rate curve is calculated separately from deposit rates, FRA rates, and interest rate swap rates.

Constructing a Zero-Coupon Discount Curve from OIS Rates

1. **Short Term (maturities up to 1 year):** OIS rates can be directly used to construct the curve since OIS with maturities less than one year do not pay periodic interest. Interpolation can be used for intermediate dates.

   For example, a cash flow to be received in 6 months will be discounted using the 6-month OIS rate quoted on the market.

2. **Long Term (beyond 1 year):** We need to use OIS with maturities greater than one year that pay intermediate interest.

   In this case, a traditional bootstrapping process can be used.

Consequently, a risk-free rate curve has been constructed from OIS rates, which will be used for discounting cash flows. Using interpolation (linear or cubic) allows obtaining discount rates for intermediate dates.

Constructing Zero-Coupon Rates from Deposit Rates, FRA Rates, and Interest Rate Swaps

When valuing an interest rate swap, a forward curve is used to calculate forward rates—that is, the rates we must use as the best predictions for future cash flows generated by the swap's floating leg. In reality, unlike the fixed leg, forward rates are unknown at the beginning of the contract.

The new approach to constructing a forward curve must consider one of the main consequences triggered since mid-2007: the new and strong segmentation of the interest rate market into sub-areas corresponding to instruments characterized by different underlying maturities.

This means, for example, that the risk of a 6-month loan was higher than that of a loan renewed every 3 months, which has obvious consequences on lending costs and thus on product pricing.

3.3.5 Pricing Methodology

Interest Rates

1. Construct the rate curve for discounting cash flows using OIS-type rates.
2. Select different sets of interest rate instruments with increasing maturities for each period.
3. Generate the X-tenor interest rate corresponding to each instrument.
4. Construct a rate curve for calculating future cash flows with LIBOR-type rates.

5. Use the LIBOR rate curve to calculate the forward rate for generating the expected future cash flows.

6. Discount the cash flows using the OIS curve.

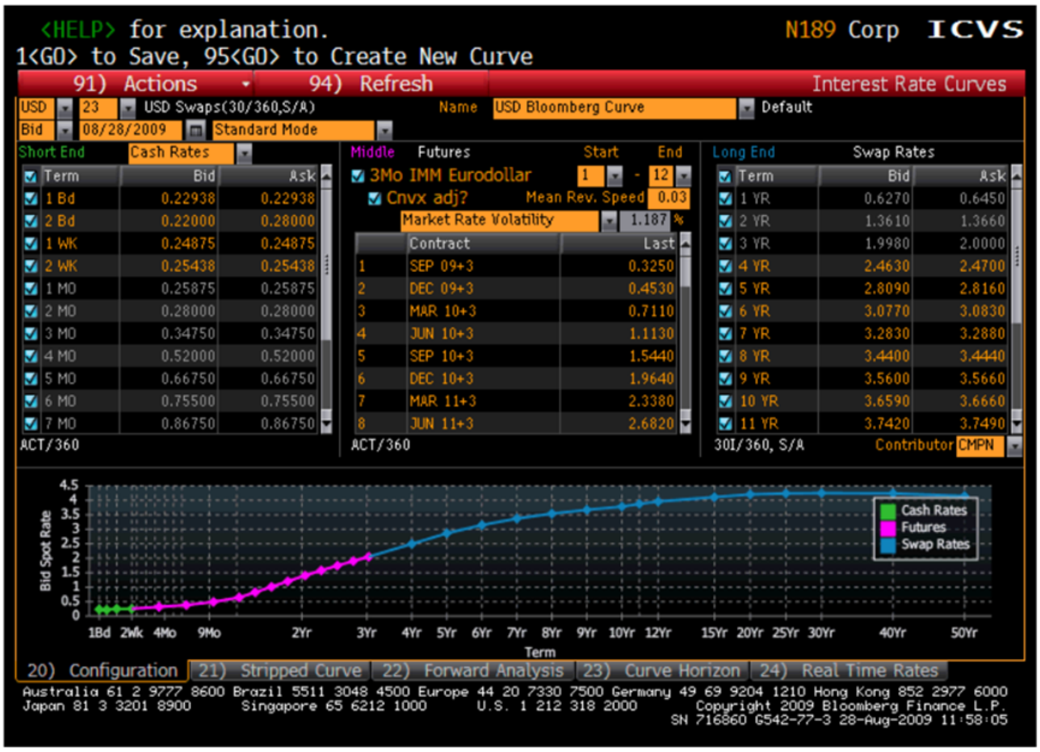

Figure 3.1: Example of a Bloomberg screen allowing the selection of a set of instruments to construct the USD 23 IR curve.

Implied Volatility

1. Estimate the implied volatilities of unlisted (OTC) swaptions using the Newton-Raphson method.

2. Interpolate the implied volatilities of swaptions.

3. Interpolate missing points.

4. Calculate the ATM volatility of the swaption.

5. Price the swaptions using the Black model.

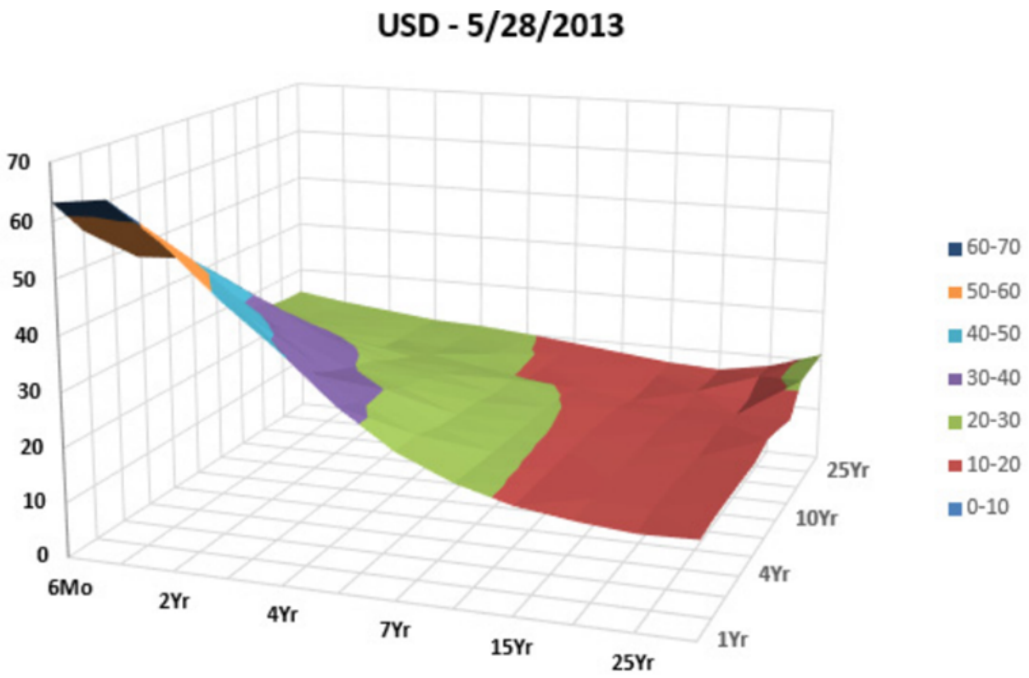

Figure 3.2: Example of a volatility surface of a swaption product.

3.3.6 Selection of Swaptions Entering the Portfolio

When selecting swaptions, the choice of strike and maturity/tenor (option and swap) is generally determined based on the investment strategy. For the strike choice, it is common to use ATM straddles with an exercise price equal to the swap rate. Regarding maturity, it depends on the nature of the strategy. For spot volatility strategies, it is common to select predetermined maturity/tenor dates, such as 12 years/30 years. Conversely, for forward volatility strategies, the logic is somewhat different because the objective is to maximize forward volatility. To achieve this, intermediate calculations are necessary, including the calculation of a metric that maximizes volatility. Based on this value, we then refer to predefined universes to calculate the exact period of each maturity/tenor. This approach allows selecting the most suitable maturity tenors for the investment strategy, depending on the desired forward volatility.

Option Pricing:

The price variations for each group of straddles is different aach group is characterized by different maturities/tenors and exercise prices, as explained in the previous selection figure 3.3. It is important to note that each group of straddles can have a different price dynamic, depending on the chosen maturity and exercise price. This analysis allows for a better understanding of the price variations of the straddles and to select the groups best suited to the investment strategy.

Calculating the Quantities of Straddles

The calculation of the quantities of the products that make up the group is generally determined by two methods. The first method, called "FIXED," involves taking the

Ticker 1 Grp_0	Ticker 1 Grp_1	Ticker 1 Grp_2	Ticker 1 Grp_3	Ticker 1 Grp_4
<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	None	None	None	None
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<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	None	None	None
<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	None	None	None
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<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	<STRADDLE 0,0299 2022-06-07 00:00:00 2042-06-09 00:00:00>	None	None
<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	<STRADDLE 0,0299 2022-06-07 00:00:00 2042-06-09 00:00:00>	None	None
<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	<STRADDLE 0,0299 2022-06-07 00:00:00 2042-06-09 00:00:00>	<STRADDLE 0,0294 2022-06-14 00:00:00 2042-06-16 00:00:00>	None
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<STRADDLE 0,0295 2022-05-24 00:00:00 2042-05-26 00:00:00>	<STRADDLE 0,0303 2022-05-31 00:00:00 2042-06-02 00:00:00>	<STRADDLE 0,0299 2022-06-07 00:00:00 2042-06-09 00:00:00>	<STRADDLE 0,0294 2022-06-14 00:00:00 2042-06-16 00:00:00>	None
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Figure 3.3: Method for entering straddle groups

initial invested amount (for example, 100 dollars) and multiplying it by a leverage effect. The result obtained will correspond exactly to the quantity of the group throughout the entire backtest period. The second approach, "IL," assumes that the index level will be reinvested at each roll date. This level is multiplied by a leverage effect, which also allows determining the exact quantity of the straddle group. These two approaches help determine the quantities of products to purchase for each group of straddles, depending on the chosen investment strategy. It is important to note that the choice of calculation method can have a significant impact on the backtest results and must therefore be made carefully.

Mark to Market Value

Marking to market (Mark to Market) is a method of measuring the value of accounts that can fluctuate over time, such as assets and liabilities.

Valuation at market price aims to provide a realistic assessment of the current financial situation of an institution or a company, based on current market conditions. Marking a swaption to market is a technique that allows it to be evaluated using current market prices. This makes it possible to determine its net value, and therefore its profit or loss for a counterparty at a given time.

Marking to market can be implemented using different methods, all leading to approximate, non-identical figures, and based on the discount curve that will be used to discount its cash flows. Swaptions are marked to market according to an agreed method on a periodic basis (daily, weekly, monthly, or whenever gains and losses exceed a specific minimum amount), using the new pricing methodology presented in the previous chapter.

Calculation of the MVHP (Market Value of Hedging Portfolios)

In general, the hedging mechanism in volspots relies on buying and selling underlying swaps at well-determined frequencies. Based on the hedging procedure explained in the index rules related to swaptions, we calculate the market value of the hedged portfolios, also called Market Value of Hedging Portfolios (MVHP).

Calculation of Cash Component Level (CCL):

The CCL is calculated as follows:

$$CCL(t) = CCL(t-1) + OPPL(t) + HPPL(t) \tag{3.25}$$

with:

- $OPPL(t)$: the Profit & Loss of the swaption portfolio at date t.
- $HPPL(t)$: the Hedging Portfolios Profit & Loss.

Option Portfolio Profit and Loss (OPPL):

OPPL is defined as follows:

$$OPPL(t) = \sum \text{quantity}_i(t) * MtM(t) \tag{3.26}$$

where:

- $\text{Quantity}_i(t)$: the quantity of a position in the global portfolio.
- MtM: the Mark-to-Market Value of the position.

Calculation of the Index Level:

After implementing numerous methods or functions that calculate the parameters entering our strategy, we reach the final step by calculating the level of the index in question.

The index level at date t is given by the following formula:

$$IL(t) = IL(t-1) + \text{Performance}(t) \tag{3.27}$$

with a base level of $\text{IL}(t_0) = 100$.

Algorithm 1: Example of an algorithm to calculate an index level

```
for all t in series_dates do
    Position ← univers(t)
    OPPL(t) ← quantite(Position) * MtM(Position)
    if t = start then
        CC(t) ← il_initial_value
    end
    else
        T ← t − 1
        CC(t) ← CC(t) * capitalisation(t)
        if t is roll_date then
            Position ← expired positions of date t
        end
        if t is the expiry date of Position then
            CC(t) ← CC(t) + quantite(Position) * MtM(Position)
        end
    end
    IL(t) ← CC(t) + OPPL(t)
end
```

3.4 Conclusion

After studying in detail the theory of swaptions and their pricing, we focused on the practical part of the strategy, which involves implementing a the swaptions using the option rolling strategy.

The option rolling strategy is applied to swaptions, which are agreements between a potential buyer and a potential seller of an asset who commit to buy (for the buyer) or sell (for the seller) a certain quantity of underlying assets at a specified price and date. Given that most swaptions have a series of monthly or quarterly expirations, it is not possible to hold a swaption that only "matures" in a few years. Therefore, the solution is to "roll" onto swaptions with successive expirations. This means that when a swaption's expiration approaches, it is sold to simultaneously acquire another swaption on the same asset but with a longer maturity. This approach allows simulating strategies over given durations.

Chapter 4

Missions and Add-On Value

4.1 Analysis of missions and results achieved

To present my missions, I have chosen to divide them into four sub-sections. Firstly, we will look at responses to requests from our internal collaborators and external databases. Next, we will take a look at our control assignments. Thirdly, we will discuss my development assignments and longer-term projects. Finally, we'll look at any difficulties encountered.

4.1.1 Internal and external requests

Let us begin with the recurring requests we receive from external sources. These daily inquiries are numerous and diverse, depending on the external databases involved. They may pertain to specific data, such as the net asset value of a fund, the value of its holdings, its performance, or even its static data. Additionally, while most documents related to Amundi's funds are available on the Amundi ETF website, some may be missing or outdated. In such cases, we receive specific document requests that we need to address.

There are also instances where inquiries focus on the confirmation of data that appears abnormal. In these situations, we must verify the information, correct any errors if necessary, or provide justification for the accuracy of the data sent. More broadly, when external partners have questions regarding our ETF range, we respond to their inquiries and process their requests accordingly.

In addition to external requests, we frequently receive internal inquiries. Data on our funds is of paramount importance to the Business Line teams with whom I have had the opportunity to collaborate. It enables these teams to make informed decisions regarding the issuance and management of ETFs by analyzing historical performance, market trends, and volatility. Through these analyses, our teams are able to optimize ETF composition, adjust strategies based on market conditions, and thus ensure competitive and high-performing financial products for investors.

Throughout my internship, I was particularly engaged with internal teams, especially those in contact with potential clients and investors, such as our Sales teams.

4.1.2 Control missions

To present the control tasks I carried out, I will first discuss the regular verifications, followed by the exceptional ones.

Let us begin with the recurring checks, notably the "Morning Check." This report, initiated every morning, is distributed to the entire Data-Digital team. Its purpose is to verify several key elements: first, ensuring that the automatic data transfer processes to the main external databases are functioning correctly; next, confirming that the most recent data on our funds is properly integrated into our database, allowing for the quick detection of any missing data; further, ensuring that the Amundi ETF website is updated with the latest data; and finally, identifying funds whose net asset value shows an abnormal variation from one day to the next. In the event of anomalies, it is necessary to identify the cause, correct the error in collaboration with the relevant teams, and update the

"Morning Check" in accordance with process changes and new verification needs. As an intern, I was responsible for regularly updating the code for this report.

Another recurring task I handled was the "Primary Daily Report," which is sent twice daily. The first version, dispatched at 6:15 p.m., is intended for the co-directors and summarizes all ETF share transactions on the primary market, highlighting the quantities issued and canceled since the beginning of the year. As an intern, my role was to ensure that all the day's transactions were accurately reflected in the report before its dispatch. To achieve this, I verified the integration of files received throughout the day by locating them in our internal system and manually correcting any errors. Then, using SQL queries, I ensured the consistency of the obtained results. The following morning, the report is sent again to the entire Business Line and the co-directors, with the expectation that the figures remain consistent with those of the previous day.

Throughout the year, Amundi's funds undergo numerous operations, such as launches, mergers, liquidations, changes in management fees, or index adjustments. While our team does not directly manage these operations, it plays a crucial role in communicating the associated data. We are responsible for updating and transmitting relevant information to market platforms, thereby ensuring transparency and control over the data. Effective organization is essential to maintain clear communication with market platforms and ensure that databases are kept up-to-date.

For instance, consider the amalgamation of a newly created Amundi fund absorbing another existing Amundi fund. In this case, the intern is responsible for several key steps: First, communication must be initiated in advance (at D-30, D-5, and D-1) with external databases. Then, all static data and characteristics of the newly created, launched, and listed fund must be transmitted. Next, the static data on the Amundi ETF website, as well as on the main market platforms, must be updated. Finally, it is crucial to verify that the new fund's daily data is correctly disseminated while ensuring the preservation and display of the historical data from the old fund, should we decide to retain the "Track Record."

Lastly, there are exceptional control tasks. To ensure the quality of disseminated data, our team performs "Quality Checks." This involves comparing the data from our database with that of an external database. In case of discrepancies, it is necessary to inform the relevant contacts to correct the error. These checks are conducted under specific circumstances or for particular reasons, such as when we need to verify the quality of external database data following a prolonged period of erroneous data transmission.

4.1.3 Projects and Achievements

In my role within the digital team of the Amundi ETF department, my team is responsible for producing various reporting tools for the capital markets and sales teams, as well as internal reports aimed at ensuring the quality of data integrated into the system overnight. Due to the diversity of these tasks, we manage several files and VBA macros scattered across different platforms, each associated with specific SQL queries for each report. This process, particularly time-consuming, complicates the rapid generation of reports.

It is within this context that my project was initiated: the development of a centralized tool in Python capable of generating all these reports, verifying the data, and distributing them to the relevant entities via email. In essence, this project entails a complete migration of VBA processes to a local web interface, reducing report generation time by 90%. In the following sections, we will discuss the main processes that have been successfully migrated to this Python-based solution.

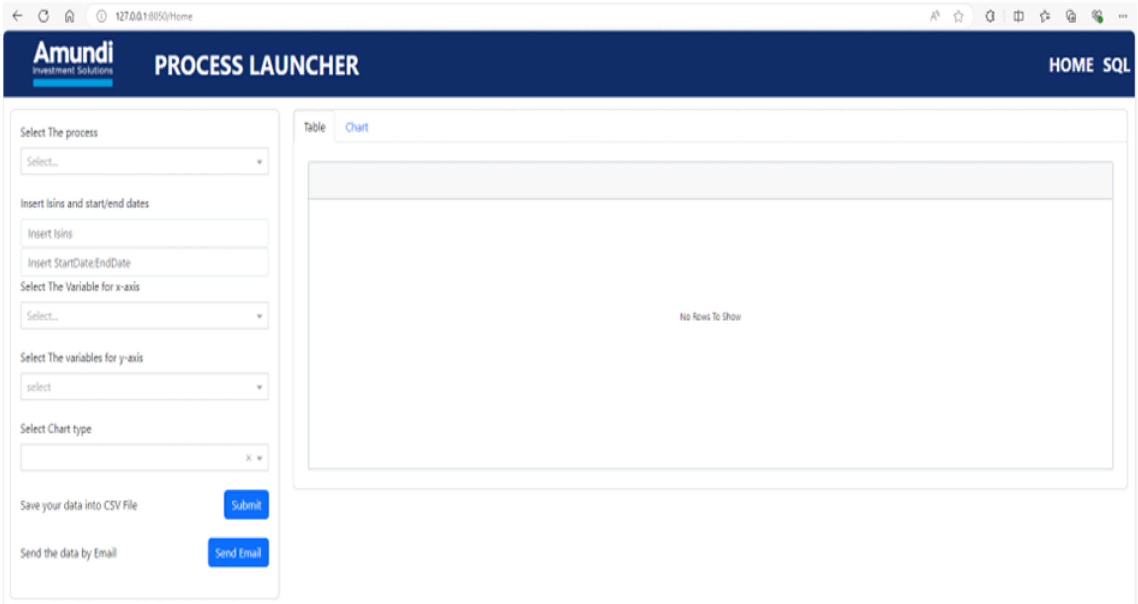

Figure 4.1: Process Launcher Tool

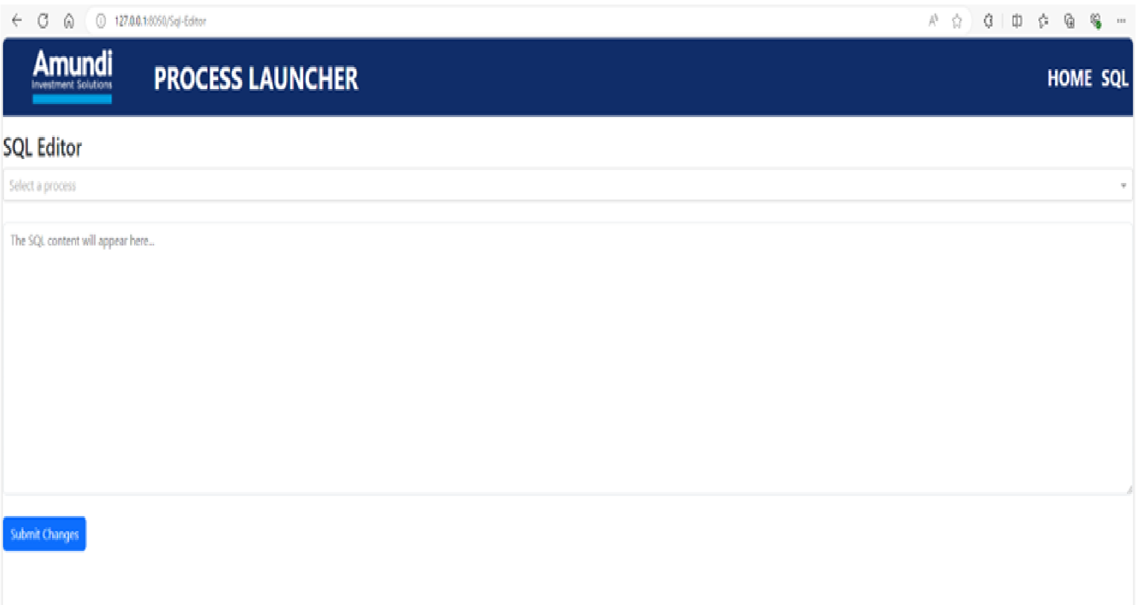

Figure 4.2: Process Launcher Tool

Morning Check

This verification involved a daily check performed each morning to ensure the smooth operation of processes related to data import (such as index compositions, fund compositions, etc.), as well as the calculation of performance metrics and other similar information processed overnight. Previously, a similar check existed but was carried out manually. This required manually executing SQL queries and verifying the results to ensure that the correct data was returned.

This method had several drawbacks: first, the time constraint. Each morning, manually executing SQL queries one by one in Microsoft SQL Server, followed by checking the results, consumed considerable time. This hindered progress on other ongoing projects. Second, there was an issue of reliability. Manual verification of query results could lead to misinterpretations by the person in charge of the check. Finally, the manual nature of the process limited the number of data points that could be verified, as the more queries there were to process, the more time was needed for verification.

This project was particularly instructive, as it required me to design a coherent and easily interpretable interface for other team members. I took the initiative to automate the execution of SQL queries by integrating them into the ”Process Launcher” tool using the corresponding templates. I also developed logic within the tool to ensure it could adapt to any template being used.

Below is a preview of the template I integrated into the process launcher, which was automatically filled out and e-mailed to the team every morning.

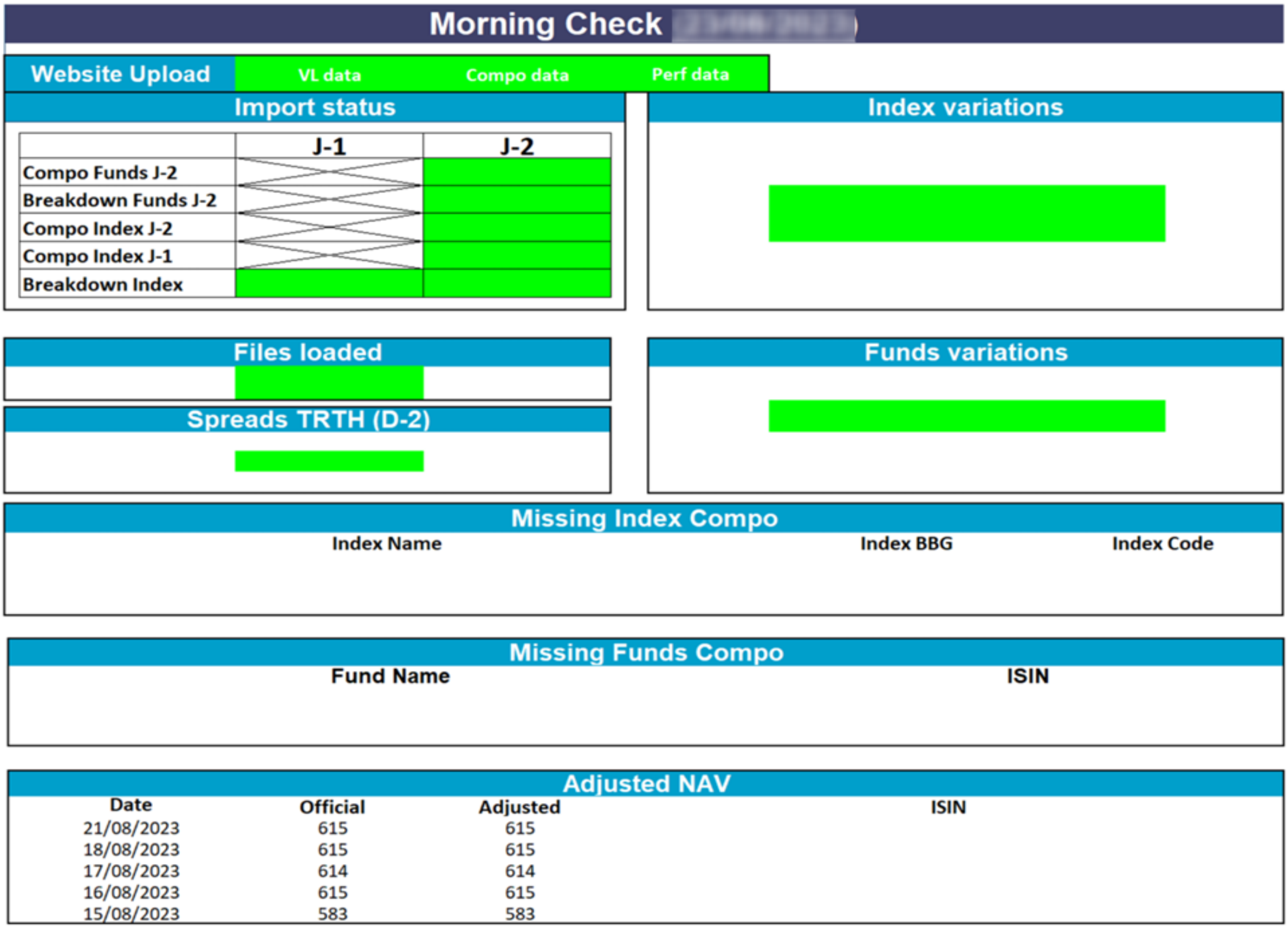

Figure 4.3: Screenshot of Morning Check template

Primary Daily Report

Another analysis that we conduct on a daily basis is the examination of trades in the primary market of Amundi ETFs.

In fact, the summary of our primary market trades exists in two formats: one is a report of net creations and redemptions since the beginning of the year, month, or week, relative to the current date. This report is sent via email every morning and evening, thanks to a VBA macro. This macro runs SQL queries in our database to retrieve all relevant figures and format them for easy and quick interpretation. The data stored in our database comes from various sources that send us daily summaries of trades.

The second format is a screen on an intranet site that also summarizes all creations and redemptions since the beginning of the year, but is sourced from a different table than the one used for the previous summary mentioned.

Below is the format of the report sent via email:

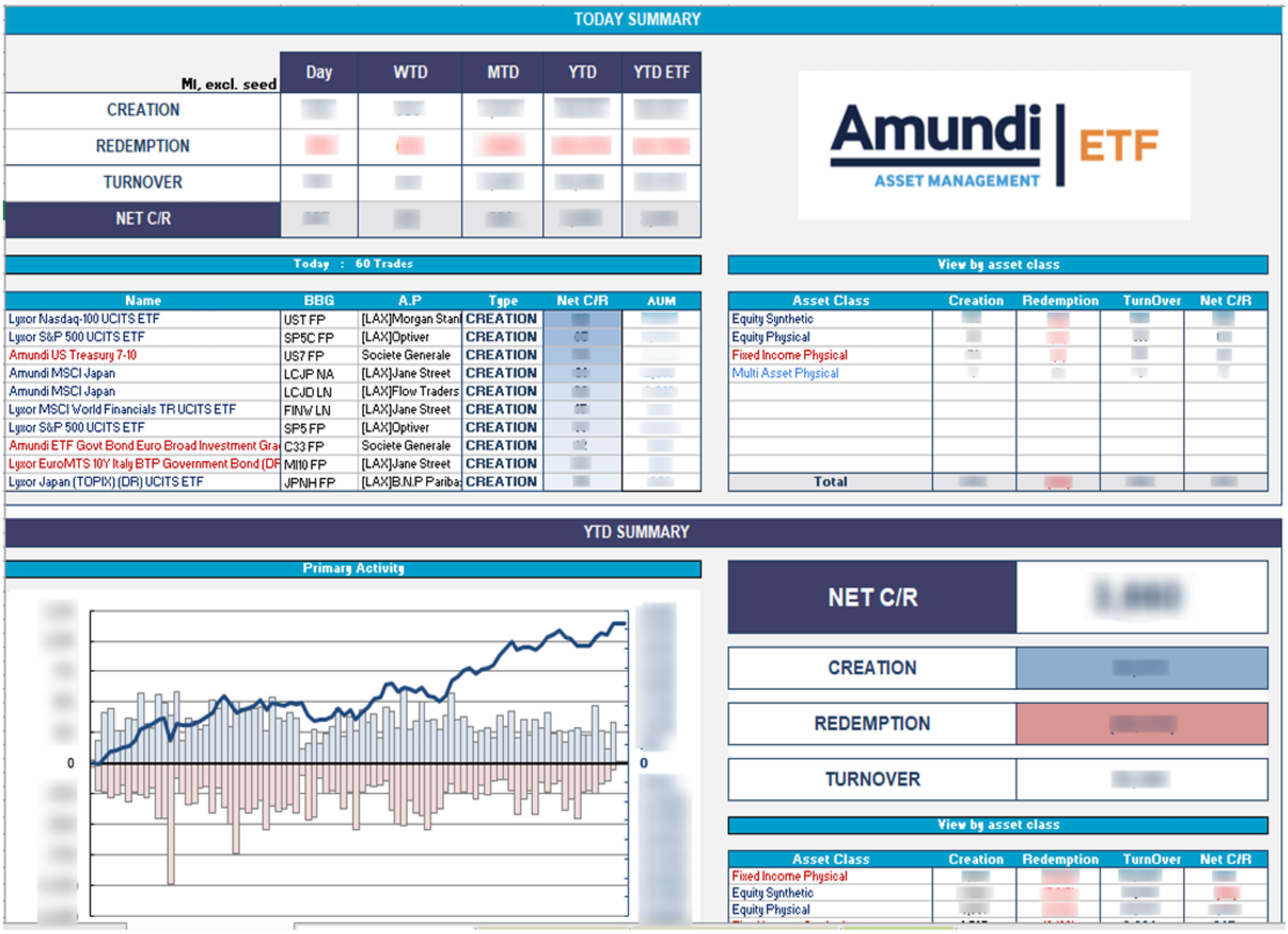

Figure 4.4: Screenshot of Primary Daily Report

The mission involved comparing two data sources: one derived from external files that aggregated all orders placed on the primary market, and the other sourced internally from within the company. The objective was to understand the origin of discrepancies observed between these two data sets.

To achieve this, it was necessary to gain a deep understanding of how operations function in the primary market and to identify the various sources involved. This task proved to be particularly time-consuming and complex, requiring the involvement of multiple teams, both internal and external to the company.

In summary, the primary market is where ETFs (Exchange Traded Funds) are created for the first time by asset management companies. In this context, new shares of ETFs are issued and offered to investors. This issuance occurs through the creation of blocks of shares, known as "creation units," which are exchanged for underlying assets such as stocks or bonds. These creation units are then traded between investors and ETF creators, contributing to maintaining the ETF's price in line with the value of its underlying assets.

In parallel, the secondary market for ETFs allows investors to buy and sell ETF shares after their initial issuance on the primary market. The secondary market operates similarly to the trading of stocks on traditional stock exchanges, with prices fluctuating based on supply and demand. It offers significant liquidity, enabling investors to carry out transactions at any time during market hours. The diagram below illustrates the differences between the primary and secondary markets.

After thoroughly understanding the mechanism of fund creation and redemption, we proceeded to integrate this process into my tool, 'Process Launcher'. This tool leverages various data sources from APIs to calculate the total amount of creations and redemptions carried out during the day in the primary market. The results are then emailed to all Business Line.

4.2 Assessment of the internship

4.2.1 Developed skills

My internship at Amundi as a Data Analyst was a valuable opportunity for developing and acquiring a wide range of professional skills essential to my future career.

On the technical side, I significantly enhanced my programming skills, particularly in VBA, Python, and SQL. SQL, which I used daily, played a key role in my responsibilities, allowing me to explore Amundi's database and extract relevant information. This language was essential for responding to external partner requests, as well as for data analysis and process automation. VBA and Python were also particularly useful for automating various tools and interpreting data. While VBA remains heavily used in Amundi's processes, I occasionally had to rely on Python to automate tasks that were too complex for VBA, providing me with an opportunity to deepen my knowledge of this language.

Moreover, my internship allowed me to strengthen my financial data analysis skills. I had the opportunity to handle complex financial data, extract meaningful insights, and provide strategic information to the team, as well as to other stakeholders involved in ETFs at Amundi. I found data analysis both challenging and enriching, requiring a thorough understanding of the subjects before even beginning data processing. This process often involves adjustments and reevaluation, a skill I had touched upon during my academic training but significantly refined during this internship.

Additionally, I learned to manage my time efficiently, prioritize tasks, and follow rigorous action plans to meet deadlines. Every project I was assigned needed to be completed within the set timelines while also accommodating additional requests from external partners. This level of organization, crucial for avoiding dispersion, mirrors the experience gained during my studies, where deadlines are strict. However, in a corporate setting, it is also necessary to account for the time required for a project's production phase, a factor over which we have limited control but must anticipate in the planning process.

Lastly, teamwork proved to be a central aspect of this experience. Over the six months, I integrated into a team of seven people where communication was critical. I sought advice, took initiative, and ensured that team cohesion was maintained. Working collaboratively with another Data Analyst on a project strengthened my team project management skills, particularly in terms of task delegation, scheduling regular progress updates, and communicating our progress to the entire team.

As for skills I did not develop during this internship, I regret not gaining a deeper mastery of Bloomberg. Receiving training on this tool during my studies would have been a significant asset for my professional development.

4.2.2 Challenges Faced and Solutions

During my internship, I encountered several challenges that allowed me to acquire new skills and reinforce those I already possessed. One of the first obstacles was gaining proficiency with ETFs, financial products, I had to quickly familiarize myself with these products to interact effectively with data vendors and the ETF department teams at Amundi. Mastery of ETFs was also essential during data extraction and validation processes, as a solid understanding of these products enabled me to interpret my analysis results accurately.

Moreover, the use of SQL presented another challenge. Despite having taken database courses, I found the language difficult to master. To overcome these difficulties, I dedicated time outside of my internship hours to reviewing SQL concepts and seeking solutions independently. This approach not only allowed me to progress quickly but also taught me the importance of self-reliance rather than consistently relying on support from team members.

Integrating into the team posed an additional challenge due to the diversity and constant evolution of processes. This dynamic environment pushed me to quickly adapt to new tools and procedures, including Tableau Desktop and the wide range of possibilities offered by SQL. However, thanks to the continuous learning mindset instilled during my academic journey, I was able to overcome this adaptation phase and contribute meaningfully to the team's projects.

Time management was another significant constraint. Projects had strict deadlines, and it was not uncommon for multiple tasks to overlap. Fortunately, my academic experience had already prepared me to handle such situations, which helped me organize my work efficiently and meet deadlines consistently.

Finally, having encountered difficulties at the start of my internship, I took the initiative to create guides to ease the integration of future interns. These guides detail processes, tools used, and the solutions I implemented to ensure a smooth transition and help new arrivals quickly find their footing. My objective was to facilitate their integration and allow my successors to focus on their responsibilities from day one.

In summary, these various challenges served as valuable learning opportunities, enhancing my ability to adapt and learn independently.

Conclusion

This internship as a data analyst in the financial sector was an incredibly enriching experience, providing a wide range of professional perspectives and genuine development opportunities. The financial industry is undergoing a profound digital transformation, and the ability to effectively leverage data has become a critical driver of competitiveness. In this context, as a data analyst, I played a key role in helping teams interpret and analyze data to make informed decisions.

Beyond technical skills, such as proficiency in data visualization tools, programming, and database management, this internship enabled me to develop a solid understanding of global financial dynamics, particularly through exposure to fascinating financial products like ETFs and indices. These insights have proven essential for my future professional growth.

I joined a dynamic and welcoming team, where I quickly gained expertise in the topics assigned to me. The trust placed in me from the early weeks allowed me to fully invest in diverse tasks, ranging from tool development to direct participation in key business processes. The tools I developed not only met the team's expectations but also contributed to improving their operational efficiency.

In retrospect, I am highly satisfied with the goals achieved and the adherence to the project timeline set at the beginning of the internship. This experience has truly allowed me to acquire essential practical and technical skills, while deepening my understanding of the financial world.

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Appendices