

ECSE 543A NUMERICAL METHODS IN ELECTRICAL ENGINEERING

Assignment 3

Set: 11-Nov-2016

Due: 05-Dec-2016

1. You are given a list of measured BH points for M19 steel (Table 1), with which to construct a continuous graph of B versus H.
 - (a) Interpolate the first 6 points using full-domain Lagrange polynomials. Is the result plausible, i.e. do you think it lies close to the true B versus H graph over this range?
 - (b) Now use the same type of interpolation for the 6 points at $B = 0, 1.3, 1.4, 1.7, 1.8, 1.9$. Is this result plausible?
 - (c) An alternative to full-domain Lagrange polynomials is to interpolate using cubic Hermite polynomials in each of the 5 subdomains between the 6 points given in (b). With this approach, there remain 6 degrees of freedom - the slopes at the 6 points. Suggest ways of fixing the 6 slopes to get a good interpolation of the points. Test your suggestion and comment on the results.
 - (d) The magnetic circuit of Figure 1 has a core made of M19 steel, with a cross-sectional area of 1 cm^2 . $L_c = 30 \text{ cm}$ and $L_a = 0.5 \text{ cm}$. The coil has $N = 800$ turns and carries a current $I = 10 \text{ A}$. Derive a (nonlinear) equation for the flux ψ in the core, of the form $f(\psi) = 0$.
 - (e) Solve the nonlinear equation using Newton-Raphson. Use a piecewise-linear interpolation of the data in Table 1. Start with zero flux and finish when $|f(\psi) / f(0)| < 10^{-6}$. Record the final flux, and the number of steps taken.
 - (f) Try solving the same problem with successive substitution. If the method does not converge, suggest and test a modification of the method that *does* converge.

B (T)	H (A/m)
0.0	0.0
0.2	14.7
0.4	36.5
0.6	71.7
0.8	121.4
1.0	197.4
1.1	256.2
1.2	348.7
1.3	540.6
1.4	1062.8
1.5	2318.0
1.6	4781.9
1.7	8687.4
1.8	13924.3
1.9	22650.2

Table 1: BH Data for M19 Steel

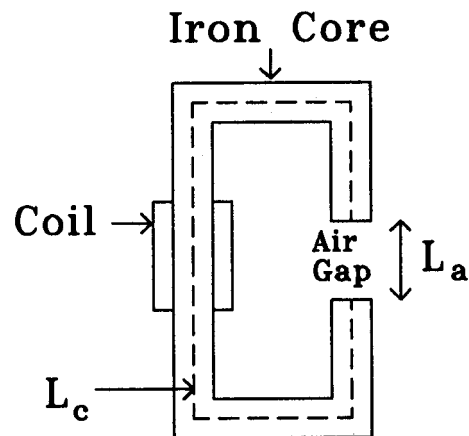


Figure 1

2. For the circuit shown in Figure 2 below, the DC voltage E is 200 mV, the resistance R is $512\ \Omega$, the reverse saturation current for diode A is $I_{sA} = 0.8\ \mu\text{A}$, the reverse saturation current for diode B is $I_{sB} = 1.1\ \mu\text{A}$, and assume $kT/q = 25\ \text{mV}$.

- Derive nonlinear equations for a vector of nodal voltages, \mathbf{v}_n , in the form $\mathbf{f}(\mathbf{v}_n) = 0$. Give \mathbf{f} explicitly in terms of the variables I_{sA} , I_{sB} , E , R and \mathbf{v}_n .
- Solve the equation $\mathbf{f} = 0$ by the Newton-Raphson method. At each step, record \mathbf{f} and the voltage across each diode. Is the convergence quadratic? [Hint: define a suitable error measure ε_k].

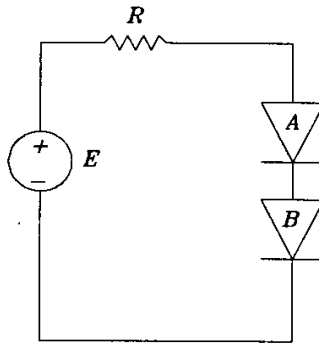


Figure 2

3. Write a program that accepts as input the values for the parameters x_0 , x_N , and N and integrates a function $f(x)$ on the interval $x = x_0$ to $x = x_N$ by dividing the interval into N equal segments and using one-point Gauss-Legendre integration for each segment.

- Use your program to integrate the function $f(x) = \sin(x)$ on the interval $x_0 = 0$ to $x_N = 1$ for $N = 1, 2, \dots, 20$. Plot $\log_{10}(E)$ versus $\log_{10}(N)$ for $N=1, 2, \dots, 20$, where E is the absolute error in the computed integral. Comment on the result.
- Repeat part (a) for the function $f(x) = \ln(x)$, only this time for $N = 10, 20, \dots, 200$. Comment on the result.
- Repeat part (b) for the function $f(x) = \ln(0.2 |\sin(x)|)$. Comment on the result.
- An alternative to dividing the interval into equal segments is to use smaller segments in more difficult parts of the interval. Experiment with a scheme of this kind, and see how accurately you can integrate $f(x)$ in part (b) and (c) using only 10 segments. Comment on the results.